Tail risk in government bond markets and ECB unconventional policies*

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*Not necessarily the views of ECB or Riksbank.

Contributions

We develop a novel observation-driven model to estimate the time-varying tail shape of time series data from an arbitrary fat-tailed distribution.

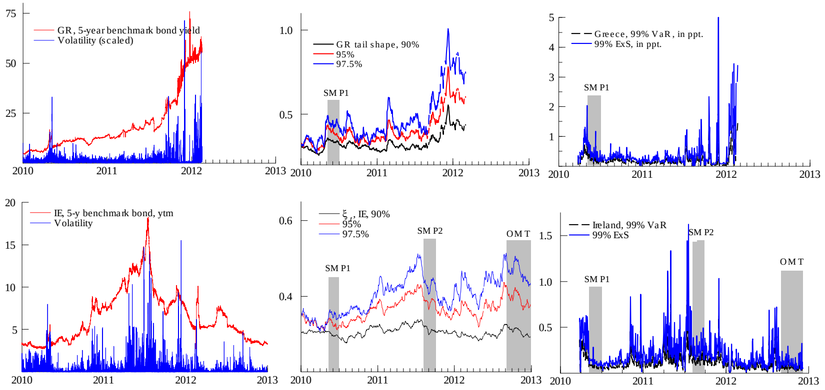
Tail shape dynamics are driven by the score of the predictive log-likelihood. Estimation is straightforward.

The model works reliably in a variety of simulation settings, and for different approaches to the modeling of non-tail observations.

ECB unconventional monetary policies, specifically the Securities Markets Programme (SMP), lowered the inventory risk of dealers making sovereign bond markets during the euro area debt crisis between 2010-2012.

Example: Tail shape and market risk estimates

For Greek & Irish 5-year benchmark bonds, 15 min frequency...



Modeling time-varying tail shape

We introduce time-variation into the tail shape parameter $\xi_t > 0$ of the Generalized Pareto Distribution (GPD).

The pdf of a GPD random variable $x_t = y_t - \tau > 0$ is

$$p(x_t; \delta, \xi_t) = \frac{1}{\delta}\left(1 + \xi_t \frac{x_t}{\delta}\right)^{-\frac{1}{\xi_t}-1},$$

where $t = 1, ..., T$, and x_t is the so-called peak-over-threshold (POT) of y_t over a predetermined value τ.

If $y_t - \tau \leq 0$, we consider x_t as missing, or censored (to come).

Setting/assumptions

1. We assume the pdf of y_t, $g(y_t)$, is fat-tailed with time-varying tail-index $\alpha_t > 0$. For example, a Student's t distribution with $\nu_t = \alpha_t = 1/\xi_t$.

Then the CDF $G(y_t)$ can be expressed approximately as

$$G(y_t) = G(\tau) + (1 - G(\tau))\, P(x_t)$$

for high values of τ. Tail behavior is captured by the Generalized Pareto distribution $P(x_t; \delta, \xi_t)$.

2. Assume pre-filtered data; fixes $\delta = 1$. Alternative one can treat a bivariate case of t.v. volatility and tail index, for instance Massacci (2015).

Score-driven tail shape

Following Creal, Koopman, and Lucas (2013), Harvey (2013), we endow ξ_t with so-called score-driven dynamics.

To ensure $\xi_t > 0$, we model $f_t = \ln(\xi_t)$. The transition dynamics for f_t are

$$
\begin{aligned}
f_{t+1} &= \omega + \sum_{i=0}^{p-1} a_i s_{t-i} + \sum_{j=0}^{q-1} b_j f_{t-j}, \\
\text{where } s_t &= S_t \nabla_t \text{ is the scaled score} \\
\nabla_t &= \partial \ln p(x_t; \delta, \xi_t) / \partial f_t \\
S_t &= E_{t-1}[\nabla_t^2 | f_t, x_{t-1}, x_{t-2}, ...]^{-1} \text{ scaling function.}
\end{aligned}
$$

Optimality: The score-driven updates always reduce the local Kullback-Leibler divergence between the true conditional density and the model implied conditional density, irrespective of the severity of model misspecification. See Blasques, Koopman, and Lucas (2015).

Treatment of non-tail observations

We consider three approaches to handling observations for which $y_t - \tau \leq 0$.

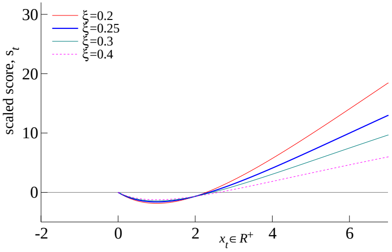
(1) Deletion of missing entries in x_t, with $t = 1, ..., T$.

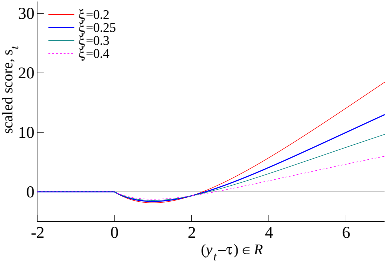
(2) Model as missing without information about the tail.

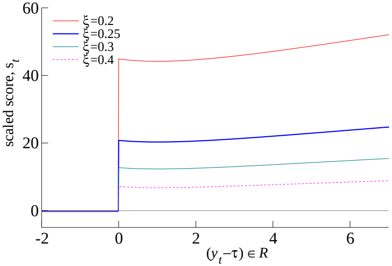
(3) Our favorite approach: model as a draw from a mixture distribution of GPD and a point mass at zero.

Each approach yields different expressions for the score ∇_t and the scaling function S_t.

News impact curves







Equations

Simulation experiments: Main lessons

- Overall, our score-driven GPD model reliably captures tail shape variation in a variety of simulation settings.
- The simple deletion of missings is appropriate if a complete time series of tail shape estimates is not required, and the tail is sufficiently fat.
- Modeling non-tail observations as missing works well iff there is strong mean reversion in tail shape.
- Modeling via the mixture distribution is most efficient. It works well if the tail is less fat, and also if mean reversion is less strong.
- (Slightly) mis-specified models still work well.

How do asset purchases impact yields?

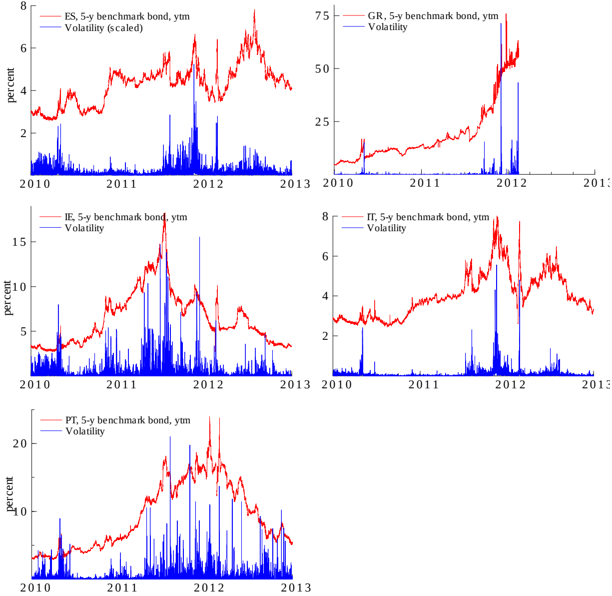
To which extent did ECB non-standard policies impact the tail shape and market risk of euro area sovereign bonds?

Elevated tail risks alone can force institutional investors and market makers to retreat; see Vayanos and Vila (2009), Adrian and Shin (2010).

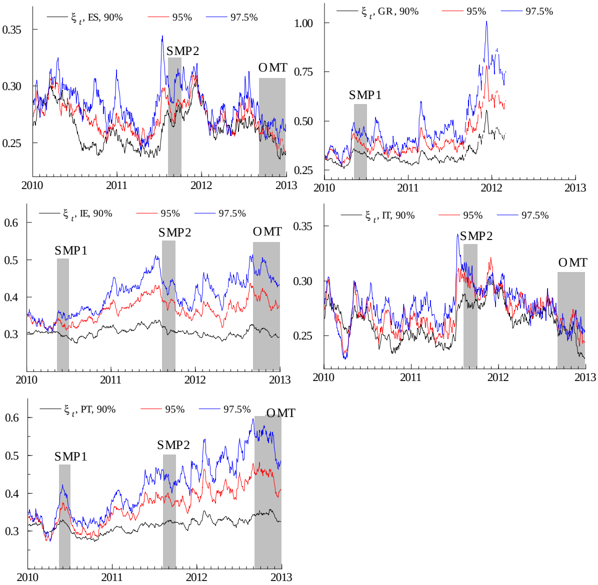
Pelizzon, Subrahmanyam, Tomio, and Uno (2013) provide anecdotal evidence that market makers withdrew from trading Italian debt securities in 2011.

Data: High-frequency bond yields between 2010-2012 for ES, GR, IE, ES, PT. Five-year benchmark bonds from Thomson/Reuters.
Sampled at 15 minute frequency as in Ghysels, Idier, Manganelli, and Vergote (2016).
SMP purchase data as in Ghysels et al. (2016).

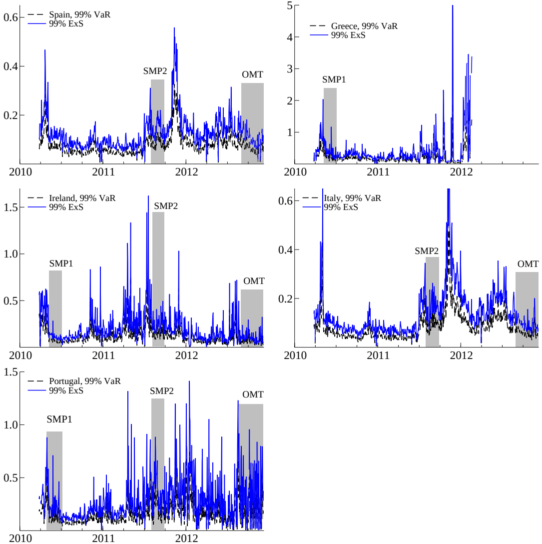
Benchmark bond yields & volatility...



… with corresponding tail shape estimates …



… and ExS, VaR as market risk measures



Impact identification

The score-driven dynamics for the tail shape parameter can be extended to include contemporaneous or lagged economic variables as additional conditioning variables.

$$f_{t+1} = \omega + a \cdot S_t \nabla_t + b \cdot f_t + c' \mathrm{X}_t,$$

where c is a vector of coefficients, and additional variables are in X_t.

We study SMP interventions in government debt securities markets in five euro area countries. Approximately €214 billion (bn) of bonds were acquired between 2010 and early 2012. The SMP announcement dates were 10 May 2010 and 8 August 2011.

Tail shape impact

	Tail(0.975) model with GAS-t filtering						
	ω	a	b	SMP$_t$	D$_{1,t}$	D$_{2,t}$	LogLik
ES	-0.001	0.002	0.999	0.013	-0.006	-0.006	-11672.1
	(0.000)	(0.000)	(0.000)	(0.006)	(0.003)	(0.003)	
	[0.000]	[0.000]	[0.000]	[0.022]	[0.036]	[0.035]	
GR	-0.002	0.006	0.999	-0.023	0.006	0.000	-8361.9
	(0.001)	(0.001)	(0.001)	(0.013)	(0.005)	(0.004)	
	[0.034]	[0.000]	[0.000]	[0.068]	[0.259]	[0.954]	
IE	-0.001	0.004	1.000	-0.031	-0.005	0.000	-11428.1
	(0.000)	(0.001)	(0.000)	(0.035)	(0.005)	(0.003)	
	[0.098]	[0.000]	[0.000]	[0.380]	[0.312]	[0.962]	
IT	-0.001	0.002	1.000	0.004	-0.009	-0.009	-11815.2
	(0.001)	(0.001)	(0.001)	(0.003)	(0.003)	(0.004)	
	[0.573]	[0.024]	[0.000]	[0.227]	[0.002]	[0.036]	
PT	-0.002	0.003	0.999	-0.006	-0.005	-0.001	-12124.2
	(0.001)	(0.001)	(0.001)	(0.025)	(0.005)	(0.003)	
	[0.148]	[0.001]	[0.000]	[0.818]	[0.339]	[0.855]	

Note: Parameter estimates for the dynamic tail model under the mixture distribution approach. ES, GR, IE, IT, and PT refer to Spanish, Greek, Irish, Italian, and Portuguese five-year bond yields (in 15mins). D$_{1,t}$ and D$_{2,t}$ are announcement dummy variables on 10/05/2010 and 08/08/2011. Standard errors in round brackets and P-values in square brackets.

Conclusion

We developed an easy-to-use statistical model to track the time-varying tail shape and market risk associated with fat-tailed time series data.

ECB unconventional monetary policies lowered the tail shape and market risk of euro area sovereign bonds during the euro area debt crisis between 2010-2012, thereby contributing towards restoring "depth and liquidity" in impaired markets.

Thank you.

Three treatments of non-tail observations.

If $y_t - \tau > 0$ denoted as $\mathrm{I}^{(+)}$ doesn't hold, we consider different treatments of x_t:

1. Deletion of missing entries,
2. Missing without information on the tail,
3. A mixture distribution of GPD and a point mass at zero, the density as mentioned before.

Then we can derive the corresponding score-driven dynamics for each treatment.

	Tail index dynamics $\xi_t = \exp(f_t)$	Score and scaling function
1.	$f_{t+1} = \omega + a \cdot S_t \nabla_t + b \cdot f_t$	$\nabla_t = \frac{1}{\xi_t} \ln\left(1 + \xi_t \frac{x_t}{\delta}\right) - (\xi_t + 1)\frac{x_t}{\delta + \xi_t x_t}$ $S_t = (1+\xi_t)(1+2\xi_t)/2\xi_t^2$
2.	$f_{t+1} = \omega + a \cdot \mathrm{I}^{(+)} S_t \nabla_t + b \cdot f_t$	$\nabla_t = \frac{1}{\xi_t} \ln\left(1 + \xi_t \frac{x_t}{\delta}\right) - (\xi_t + 1)\frac{x_t}{\delta + \xi_t x_t}$ $S_t = (1+\xi_t)(1+2\xi_t)/2\xi_t^2$
3.	$f_{t+1} = \omega + a \cdot S_t^{\phi} \nabla_t^{\phi} + b \cdot f_t$	$\nabla_t^{\phi} = \mathrm{I}^{(+)}\left[\frac{\ln(1+\tau)}{\xi_t} + \nabla_t\right] - \mathrm{I}^{(0)} \frac{(1+\tau)^{-\frac{1}{\xi_t}}}{1-(1+\tau)^{-\frac{1}{\xi_t}}} \frac{\ln(1+\tau)}{\xi_t}$ $S_t^{\phi} = [\frac{2\xi_t^2(1+\tau)^{-\frac{1}{\xi_t}}}{(1+\xi_t)(1+2\xi_t)} + \frac{(1+\tau)^{-\frac{1}{\xi_t}}}{1-(1+\tau)^{-1/\xi_t}} \frac{\ln(1+\tau)^2}{\xi_t^2}]^{-1}$

jump back