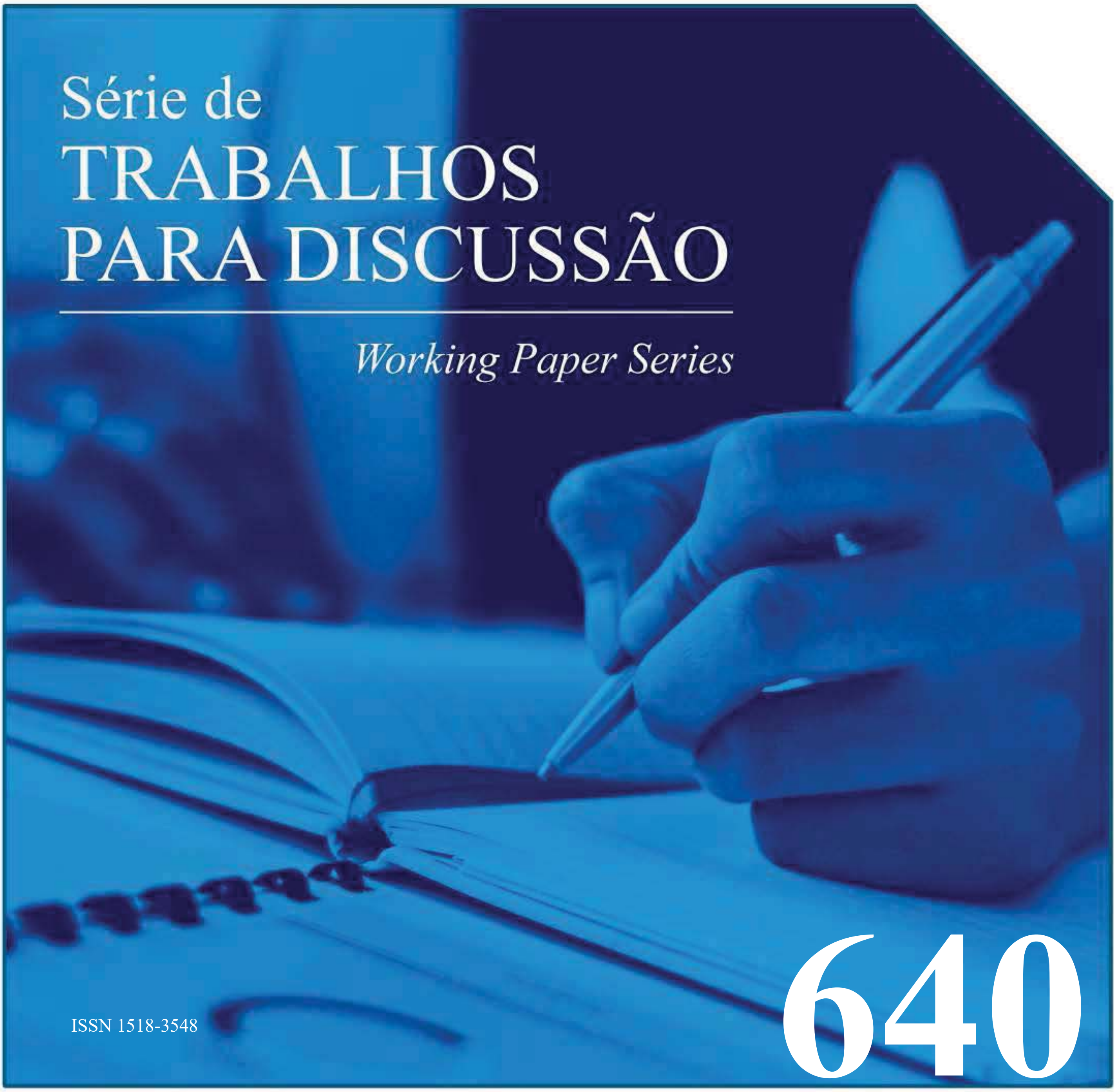

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Following the Informational Footsteps of the Supply Chain Tracks

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Non-technical Summary

This paper investigates the role of information in the formation of input-output production networks and its implications for economic misallocation. In modern economies, firms interplay through complex networks of suppliers and customers, where they typically know only their direct suppliers and lack information about firms further upstream, pattern described in the literature as small-world networks. Although this informational issue is often overlooked in the literature on macroeconomics of networks, it plays a crucial role in shaping production networks and determining shocks propagation.

To investigate the role of information in this environment, we develop a decentralized search model in which firms search for suppliers conditional on their information-level. A firm's information level is defined by how many layers of the supply chain it knows in her information set: firms with 1-level of information know only their direct suppliers, 2-level of information means know who are the suppliers of her suppliers, and so on until the full information case that corresponds to common knowledge. This informational issue in the decentralized search procedure is embedded in the minimization problem of the firms, where they first find the set of potential suppliers (those that could be chosen) and then select the effective suppliers (those that actually provide the inputs and minimize production cost). This framework creates an endogenous input-output network economy in which information plays a central role. To complement the model, we also introduce a representative consumer, under regular assumptions, who buys the final goods. To investigate the interplay between information and misallocation in this environment, we define two endogenous wedges: an information wedge, due to incomplete information, and a network wedge, due to the network structure.

On the theoretical side, we derive results that connect information, network formation and misallocation. First, the set of potential suppliers is an increasing function of the information set of the firms. Second, information impacts the moments of the network distribution, in which firms better informed transform networks on less dense but more stable ones, increasing their resilience to economic shocks and disruptions. From a policy perspective, this result sheds a light on the role of information as a tool for reducing misallocation, acting as a stabilizing force particularly in sectors where firms are poorly informed, where information-enhanced policy may be more effective than financial subsidies. Third, we show that incomplete information produces a cascade effect of productivity shocks in this environment. The spillover effects of these shocks increase with the level of information, such that better informed firms allow shocks to propagate more effectively through the network, amplifying their aggregate impact on output and consumption.

On the empirical side, we use a dataset of Brazilian firm-to-firm financial transactions to replicate the

Brazilian production network economy. From the configuration of the production network, we develop an algorithm to estimate the level of information at firm-level and test the theoretical predictions of the model. We find that information is heterogeneous across sectors and position in the network, where firms typically know only their direct suppliers, with intermediate firms possessing more information than both upstream and downstream producers.

Sumário Não-Técnico

Este artigo investiga o papel da informação tanto na formação do desenho de uma economia de rede estruturada em uma cadeia de produção quanto suas implicações para o *misallocation* na economia. Economias modernas são caracterizadas por complexas redes de ofertantes e demandantes nas quais as firmas costumeiramente têm informação sobre as empresas com as quais elas interagem enquanto desconhecem o restante do desenho da rede, assim como suas interrelações. Esse padrão é conhecido na literatura como *small-world networks*. Por mais que essa característica informacional seja usualmente negligenciada na literatura de macroeconomia de redes, a incompletude informacional exerce papel crucial na determinação da configuração das redes de produção e na propagação dos choques econômicos.

Para investigar o papel da informação neste ambiente, o artigo desenvolve um modelo de procura descentralizado no qual as firmas procuram seus ofertantes de maneira condicional ao seu conjunto informacional. O nível de informação de uma firma é definido pelo número de camadas da rede conhecidas no conjunto informacional dela. Logo, grau 1 de informação significa que a firma só conhece quem são seus ofertantes, enquanto grau 2 representa ter conhecimento sobre quem são os seus ofertanes assim como os ofertantes dos seus ofertantes, e o mesmo de forma análoga ocorre até a situação de informação completa, equivalente a ter *common knowledge*. Esta construção informacional no modelo de procura descentralizada é embutida em um problema de minimização das firmas, onde primeiro encontra-se o conjunto de ofertantes potenciais (aqueles possíveis de serem escolhidos) para então selecionar os ofertantes efetivos (aqueles efetivamente escolhidos e que minimizam o custo das firmas). Este ambiente endogeniza a construção da rede econômica, na qual a informação torna-se fundamental. Para complementar este modelo, o artigo introduz um consumidor representativo, sob hipóteses usuais, que compra os bens finais produzidos na rede. Para investigar a interação entre informação e *misallocation*, define-se dois *wedges*: *wedge* informacional, devido as firmas terem informação incompleta, *wedge* da rede, devido a estrutura da rede.

Do lado teórico, derivam-se resultados que conectam informação, construção da rede e *misallocation*. Primeiro, encontra-se que o conjunto de ofertantes potenciais é uma função crescente do nível de informação das firmas. Segundo, a informação impacta os momentos da distribuição da rede, de tal forma que firmas mais informadas transformam a rede em um ambiente menos denso, porém mais estável, aumentando a resiliência da economia a choques e disrupções. Sob uma perspectiva de política pública, esse resultado direciona os holofotes para o papel da informação como um instrumento para reduzir *misallocation*, atuando como uma força estabilizadora particularmente em uma rede na qual as firmas são pouco informadas, tal que políticas de aumento informacional podem ser mais efetivas do que subsídios financeiros. Terceiro, mostra-se que a existência de informação incompleta produz um efeito em cascata, sob as diferentes camadas da rede, quando choques de produtividade afetam a economia. Essa externalidade é proporcional ao nível de informação dado que firmas mais bem informadas possibilitam com que choques econômicos se propaguem mais efetivamente através da rede, amplificando os efeitos agregados na atividade econômica e no consumo.

Do lado empírico, o artigo usa uma base de dados brasileira, sobre transação financeira entre as firmas, para replicar a cadeia de produção da rede econômica brasileira. A partir da configuração da rede de produção, desenvolve-se um algoritmo para estimar o nível de informação para cada firma da economia brasileira e testar as predições do modelo teórico. Com isso, encontra-se que a informação é heterogênea a nível de setores e posição na rede, firmas produtoras de bens intermediários possuem mais informação do que firmas de bens finais ou de bens iniciais, e, na sua maioria, firmas conhecem tipicamente só os seus ofertantes.

Following the Informational Footsteps of the Supply Chain Tracks

Victor Monteiro*, Diogo Abry Guillen†and Thiago Christiano Silva‡§

16 de dezembro de 2025

Abstract

We develop an endogenous production network economy model coupled with incomplete information, where the degree of information at the firm-level is the engine of the network formation and distorts both producers' decision and the aggregate allocations of the economy. To tie this relationship, we consider that producers find their suppliers through a decentralized search given their level of information, in which firms are more or less informed depending on how many linkages of the production network they know. In our model, we establish the existence, uniqueness and efficiency of the network equilibrium for a given level of information, and show that the higher the level of information, (i) the more stable the network, (ii) the lower the density of the network, and (iii) the higher the spillover impact of a productivity shock on the aggregate output. We also design an optimal contract to show that the combination of information-enhancing policies and tax-subsidies is able to mimic a Walrasian full information equilibrium. Finally, we use a proprietary dataset that covers a large share of Brazilian financial transactions to investigate stylized facts about information and network formation as well as test empirically the implication of our model.

JEL Classification: C67, D83, D85, E32

Keywords: Macroeconomics of Networks, Incomplete Information, K-Level of information

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1 Introduction

The study of networks is an important topic in different veins of economic literature. From the analysis of human behavior in social environments to the production chain of input-output companies, networks are a cornerstone that helps us to understand how a group of individuals or firms interplay with each other and how it affects the whole economy, dampening or amplifying individual shocks.[1]

In this paper, we investigate the role of information by assuming that producers learn through their supply-chain interactions. By adding such incompleteness, we can discuss the network formation, firm-level allocation, macroeconomic distortion and the appropriate policy to recover a first-best allocation. In contrast to papers on macroeconomics and networks, we build an endogenous input-output production network economy under incomplete information, based on the empirical evidence of navigation models literature, as Kleinberg (2000) and Watts and Strogatz (1998), that networks are characterized by the combination between clusters and sparse connections, such that a firm can better predict the behavior of closest firms than distant ones, as explored by Lipnowski and Sadler (2019) and Breza et al. (2018).

We join two strands of the literature: macroeconomics of networks and information theory. On the macro side, we build an input-output network model, à la Oberfield (2018) and Boehm and Oberfield (2020), but, at the micro level, firms optimally find their set of suppliers according to their level of information, and then decide who are the ones that will effectively be chosen to trade. We embed this setup in a general equilibrium model, with a representative household with standard preferences, as in Acemoglu and Azar (2020), that consumes the goods bought from the industries of the network. From this setting, we illustrate information as a source of the firms' link-formation incentives and derive a closed form solution for the relationship between network formation, information and aggregate allocation.

On the information side, we built a k-level approach to represent the knowledge that producers have about the network, borrowing this notion from papers such as Farhi and Werning (2019), Crawford et al. (2013), Allen et al. (2006), and Kajii and Morris (1997). This approach allows us to endogenously tie the relationship between the linkage-formation of the production network and the information set of the producers in a way akin to the idea of an Erdos number, as in Watts and Strogatz (1998).

In our model, following empirical evidence,[2] producers only observe the nearest firms on the production network and are unaware of the existence of possible suppliers or intermediate clients further away in the

[1] This network agenda is reviewed and explained by Carvalho and Tahbaz-Salehi (2019), which covers different strands of network models, such as Boehm and Oberfield (2020), Oberfield (2018), Acemoglu and Azar (2020) and others

[2] As Bloch and Dutta (2011) and Easley and Kleinberg (2010).

network due to limited information. Thus, the k-level gives that level-1 of information is tantamount to a producer who knows only who are her suppliers, level-2 characterizes the producers who know who are the suppliers of her suppliers, and so on until round-k, which illustrates complete information.

To establish how the network is assembled, we first create a decentralized search producer model based on the degree of information and the position in the network to endogenously derive the set of potential suppliers of a firm. From this set, the producers choose who are the suppliers that effectively maximize their profits.[3]

In order to study how information and network interconnections matter in the aggregate environment, we highlight two sources of misallocation to be addressed [4].

First, there is a network reason: does a given network formation give the optimal allocation? To answer this question, we create a wedge given by the difference of aggregate output between the central planner allocation of a standard Real Business Cycle model and a decentralized network allocation under perfect information, which illustrates the misallocation provided by network formation.[5] Second, there is an informational reason: does the level of information affect the allocation? Following the same notion, we create a wedge given by the difference of aggregate output due to information incompleteness on the network setup.

Our first set of theoretical results establishes that more information: (i) improves the allocations, (ii) reduces the network wedge and the information wedge[6] and (iii) generates an amplification impact of the productivity shock on the consumption of the household,[7] such that (iv) information co-moves with the productivity shock.

Our second set of results establishes the relationship between information and the network structure, which dialogs with the literature on diffusion network models and their empirical patterns, as in Banerjee et al. (2013) and Akbarpour et al. (2020). We find that as one increases the level of information, network (i) becomes less dense, (ii) becomes more stable, and (iii) the individual centrality of the firm increases.

[3]The set of effective suppliers is a subset of the potential suppliers. The potential suppliers represent the suppliers that a firm may transact. Although it does not necessarily provide the maximal profit to who bought the inputs, we create the set of effective suppliers that provides the maximal profit for who is buying the inputs

[4]Aligned with Oberfield (2018), Boehm and Oberfield (2020), Bigio and La'o (2020), and Liu (2019), we approach that using the notion of incentive distortions in a production network in the form of wedges; however, we consider those wedges endogenously.

[5]In order to be clear about the assumptions behind the statement. To disentangle the network effect, we compare a network environment under common knowledge to a standard RBC without network. On the other hand, to disentangle the information effect, we compare the network environment with incomplete information with a network environment with common knowledge.

[6]When a producer acquires a new level of information, her output rises through two channels, changes in the set of effective suppliers and the reallocation of the inputs, which consequently increases her profit. Now, from the higher production, the difference according to the complete information case decreases as the information wedge, thus improving the welfare of the economy. For the household, from the market clearing, his consumption rises by the increase in output.

[7]Where the consumption is affected through a partial (competitive) equilibrium channel, which comes from the inputs of the firm, and a general (network) equilibrium, which comes from the propagation according to the network, such that an industry with higher levels of knowledge jolt more layers in the network, amplifying even more the impact of a given productivity shock

Therefore, instead of adding new firms to lead to more diffusion in the network we obtain higher diffusion through the provision of more information to some firms.

In terms of public policy, this theoretical setup brings a new guideline, stressing the importance of information provision, about the network relationship between firms and their position, as a source of policy in a network economy.[8] Information enhancing policies are more valuable in low information networks, but they can reach zero effect in incomplete information networks with large enough information and we could still reach Walrasian efficiency.

Furthermore, we show that the optimal design of a public policy is able to lead the equilibrium allocation to its efficient level, even under incomplete information. This happens when the combination of the provision of information with a monetary subsidy is tantamount to the deviation of the incomplete information output with respect to the central planner. In our case, optimality is based on a Ramsey approach, and it does not take into account budgetary costs of the policies to the government, such that we are looking only to conditions to attain first best allocation.

Empirically, we employ proprietary microdata from the Central Bank of Brazil, covering approximately 1.6 billion bilateral transactions among more than 9.6 million firms, classified into over 450 sectors of the Brazilian economy between 2019 and 2023. By matching all transactions, this dataset allows us to reconstruct the input–output production network of Brazil for all major sectors of the economy.

Empirically, we provide new evidence on stylized facts regarding the relationship between information and network formation and test the main implications of our theoretical framework. We find that, in general, firms are only aware of their direct suppliers, while the rest of the network remains unknown to them. However, it is not homogeneously distributed, the degree of information varies across sectors and network positions. For instance, intermediate firms typically possess more information about the overall network structure than either upstream or downstream producers.

At the firm level, we estimate the set of potential suppliers and find a positive relationship between the level of information and the number of potential suppliers, consistent with our theoretical predictions. Furthermore, we document a positive relationship between the degree of information and a firm's network centrality, providing additional support for the hypothesis that information is a key determinant in the formation and structure of production networks. This stylized fact highlights the potential role of information as a policy instrument to mitigate misallocation in networked economies.

The paper is structured as follows: Section 2 discusses the input-output network model. Section 3 provides

[8] This dialogs with Banerjee et al. (2019) on information diffusion in social network economies.

the cost minimization setting and highlights the decentralized search procedure. Section 4 illustrates the endogenous wedges as the sources of misallocation of this network economy. In Section 5, we derive the theoretical results about the existence, uniqueness and efficiency of our equilibrium. Section 6 provides some counterfactual exercises, Section 7 describes the empirical investigation and in Section 8 we conclude the paper.

2 Input-Output Network Model

In this section, we build a network setup under incomplete information to investigate how the knowledge about the linkage formation affects the allocations of the firms and the household of this economy. To do that, we create a firm-level two-sided input-output model based on the manifold cost and information for an endogenous production network, where the firms at the final nodes sell their goods to a representative household.

In the following subsections we state the structure of the network, explain the preferences of the players of this game (firms and the representative household), and characterize the information-knowledge process of the producers and its relationship with the network connection, respectively. In order to provide some intuition, before we enter deeper into this framework, we propose the following simple example.

Suppose a network economy with six nodes and three layers, as depicted by figure 1, where a household C buys a bookcase from an industry I that uses a shelf of wood and spike as inputs bought from the effective suppliers S and W, while W buys the wood from two initial producers W_1 and W_2. Hence, layer one of the network is composed of firms W_1 and W_2, whilst the second layer has producers W and S, and the third layer has only I. Figure 1 illustrates that each producer in each layer is involved in a subnetwork with their direct edges and suppliers.

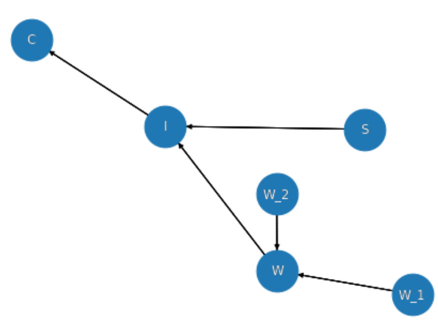

Figura 1: Complete Network

From the draw of this network, each firm has a unique level of information, which means industry I does not know the existence of firms W_1 and W_2, for instance. Therefore, the shape of the production network from the perspective of industry I, as the fixed point with a unique degree of information, is given by:

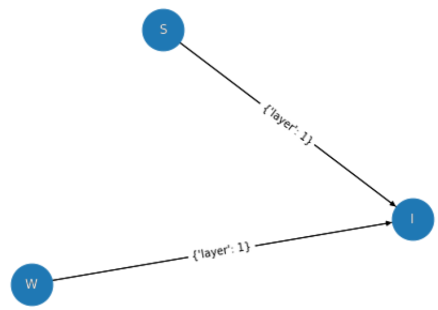

Figura 2: Production Network Under 1-Level of Information

Now, suppose industry I discovers the existence of producers W_1 and W_2 through advertising;[9] thus, with that, he mapped a new layer of the network in his information set, and the number of potential suppliers increased from two to four. After the informational increment, he has two options of shopping; buy the shelf of wood from firm W or go directly to the initial producers and purchase the wood from new potential suppliers W_1 and W_2. Industry I chooses the effective suppliers that provide the higher profit, and if it means the second option, Figure 3 illustrates the reshaping of the network as a consequence of the acquisition of more information.

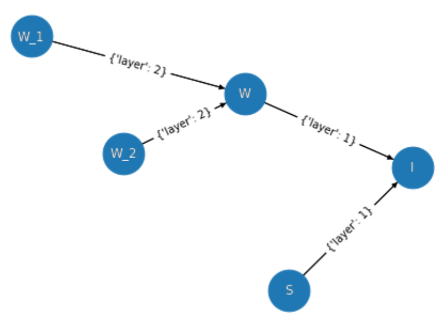

Figura 3: Production Network Under 2-Levels of Information

[9]It is important to mention that information is exogenous in our model, therefore we do not model the reason behind changes in the information sets of the firms, we only model it affects the network economy.

2.1 Network Environment

We consider an input-output environment for a network G that characterizes the strategic interactions between the producers and their suppliers, such that each producer i is encoded in a subnetwork G_i with her suppliers, where the complete network economy is formed by the union of all the subnetworks, such that $G = \bigcup_i G_i$ for $i \in \{1, 2, ..., K\}$, where G_K is the subnetwork that firm K is encoded. Thus, each producer[10] is a fixed point of a small network, such that each subnetwork is a subgraph, in which its total number is akin to the number of firms in the economy. Moreover, each subnetwork is compounded by the union of layers, $G_i = \bigcup_i L_i$, such that i gives the level of the layer that a given producer has in her information set and $L_i \in L$, where each layer is composed of the direct edges between two individual nodes. We call an individual node as g_i, such that $g_i \subset L$, where it gives the node g in which firm i belongs.

In terms of notation, we call the final layer of the network $\overline{L}$ such that $\overline{L} \in L$, while the remaining layers are L_i, such that i is akin to the number of layers that exist in the network. This notation allows us to define the position of a given layer in the network by their geodesic path, i.e., the smallest path to the final path, described by $|L_i - \overline{L}|$.

We assume that an individual node represents a given firm i, and, consequently, at each layer L_i there are S firms, with $i \in S^{L_i} \subset S^L$. In order to disentangle the final producers from the intermediates, we consider that, at the final layer $\overline{L}$, each final producer is tantamount to an individual industry w, where $w \in W$. Hence, the number of final nodes of the network shows how many industries exist in this input-output economy.

Moreover, the edges of the network have a direction that indicates who supplies from whom over the production chain. Thus, the network linkages represent the interplay of firms from different nodes and layers, given by a sequential strategic interaction, where the same firm can buy a good through an edge and in another one be a supplier that sells her good.[11] Therefore, depending on the configuration of the interactions, our model is flexible enough to match different shapes of networks such as star, line, ring, regular and other topological structures.

Following an extensive literature on networks,[12] we consider that the payoff provided by trade is the profit of the firm, $\pi_i(g_i, k_i)$, which is an indirect function of the network environment that incorporates the number of potential suppliers of the firm, its connections, its position in the network, through the node position g_i, and the degree of information k_i about the production chain.[13]

[10] Except for the initial producers that we assume are the initial nodes of a network.

[11] Thus, one example could be a firm that consumes an input from two suppliers, but sells the output to another firm.

[12] Kranton and Minehart (2001) is one example.

[13] Since the degree of information has a straightforward relationship with the nodes of a network, we divide it into two

Now, we should characterize firms' production, describing the inputs, the output and how it is encoded in the network. To address this issue, we assume that each firm produces a unique good given by the production function y_i, through the allocations of inputs x_{is}, labor h_i, productivity shock z_i, how much information k_i the firm has about the network. Furthermore, we consider that inputs x_{is} are the goods bought from the suppliers, as in Baqaee and Farhi (2020), where the subscripts i and s represent the producer and the goods, respectively.

In order to tie the transaction between a producer and her suppliers, we consider that the intermediate market is characterized by perfect competition, as in Acemoglu and Azar (2020). Then, this production function must exhibit a constant return to scale and all inputs should be complementary, as characterized by the following assumption:

Assumption 1. *In order to find the output of this network economy we consider the following conditions:*

i) The production function is nonempty.

ii) Labor is an essential factor in the production function.

iii) The set of potential suppliers of the producer is a function of the level of information k_i, and the payoff of firm $\pi_i(g_i, k_i)$.

iv) The inputs bought from each of those suppliers have an augmenting productivity shock term, transcribed by z_{is}, such that it is composed of a common component to the goods summed with an idiosyncratic component.

v) Analogously, for labor allocation, there is a labor-augmenting productivity shock z_i.

Then, the output of firm i that hires h units of labor and purchases $\{x_{i1}, ..., x_{is}\}$ units of intermediate inputs from s different suppliers, according to the k_i levels of information of the producer, is:[14]

$$y_i = f_i(z_i h_i, z_{i_1} x_{i1}, ..., z_{is} x_{is}, k_i) \tag{1}$$

To obtain tractability and guarantee an analytical solution, we follow extensive literature, such as Long Jr and Plosser (1983), Acemoglu et al. (2012), Acemoglu and Azar (2020) and others, assuming a Cobb-Douglas to represent the production function of firms according to the previous assumption.

Assumption 2. *In order to get tractability, we assume a logarithmic function form for the production*

subsections to avoid confusion in the notation.

[14]The assumption of purchase goods from from intermediate firms guarantees to us a network that describes a production chain, instead of just a circular network.

function, where all lowercase letters describe the log-variables, as follows:

$$f_i(n_i^*, z_i, h_i, x_{is}, k_i) = (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i + \sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is} \tag{2}$$

Where n_i^* represents the set of effective suppliers, z_{is} is the productivity shock of the inputs, z_i is the productivity shock of the labor, h_i is the labor force, x_{is} is the inputs, k_i is the level of information, α_i is the network elasticity, such that all lowercase letters represent the logarithm form of the variables.

2.2 Preferences of the Household

To conclude the characterization of our network environment, in this subsection, we consider that there is a representative household that interacts with the industries of the production chain and extracts utility from a weighted sum of his consumption basket, which is composed of the goods from all those w industries. Thus, assuming regular assumptions, the utility function is represented as:

Assumption 3. *The household utility function is continuous, differentiable, increasing, and strictly quasi-concave and is represented by the following:*

$$U(c_1, c_2, ..., c_w) = \sum_{i=1}^{w} \xi_i c_i \tag{3}$$

Where c_i is the consumption of good i from industries i to w, and ξ_i is the weight given to industry i in the household utility function.

Now, to state the budget constraint of this representative household, we consider that he is endowed with h units of labor, which is supplied inelastically, and receives a wage W_i. Thus, indicating p_w and c_i as the price paid by the final good and the consumption of this good, respectively, we define the budget constraint as:

$$\sum_{i=1}^{w} p_i c_i \leq W_i \tag{4}$$

In order to characterize the price that the household pays by the final goods, we consider a fixed mark-up rule, which is a natural consequence of the monopolistic competition environment of the industries, where each industry w sets the selling price of the good through the following equation:

$$p_w = (1 + \mu_w) C_i(n_i^*, p_l, p_{is}, k_i) \tag{5}$$

Where p_w is the selling price of the good produced by the industry w, μ_w is the fixed mark-up of the final firms and C_i is the production cost of the good i, which is a function of the number of effective suppliers, price of labor, price of the inputs and his information set, respectively.

Finally, after defining the network formation, the players, the payoffs, and their interplay, it is now time to characterize the transaction contract to attach the trade arrangement between two agents in the network.[15] We assume that there exists a unique bilateral contract, which is depicted by the pair $\{p_{is}, x_{is}(g_i, n_i^*, k_i)\}$, the price paid by the inputs and the quantity bought. This contract illustrates that both the linkage formation and the position in the network matter to set the contract.[16] Therefore, an optimal contract is a consequence of the optimal choices of the agents given the network and the degree of information.[17]

2.3 Information

Usually, macroeconomic networks are modeled assuming complete information, where each agent knows all the nodes of the network and the beliefs associated with all the other players, as illustrated by Carvalho and Tahbaz-Salehi (2019). In the context of input-output networks, this assumption means that a producer knows all the production chain and is able to point-out all the suppliers-customers connections in the network. However, an extensive literature, such as Watts and Strogatz (1998) and Kleinberg (2000), discusses the topological validity and empirical applicability of this assumption in real economies. Their main take-ways are that networks are outlined by high clustering and short paths, which does not condone with the assumption of complete information. Thus, incomplete information is a relevant assumption for both theory and empirical features.

Then, in order to address this empirical content, we drop the usual network assumption about complete information and build our setup under incomplete information, where a producer may not know all the layers and firms in the production chain. To do that, we borrow the notion of k-information as in Farhi and Werning (2019) to define the information set of each firm in an input-output economy. We consider that the degree of information k_i gives the number of layers that a producer knows about the network. Thus, if a producer has one layer of information, $k_i = 1$, she only knows who are her suppliers that belong to the previous layer,

[15] This transaction contract holds for trade between a producer and her supplier as the household and an industry.

[16] In the paper, we overlook the existence of different production techniques. Implicitly, we are saying that there is a unique production technique for the production of all the goods in this economy.

[17] Which can be viewed as states of the economy.

while if $k_i = 2$, she knows who are the suppliers of the suppliers and the firms in the two layers before her. Consequently, implicitly to this notion, the higher the number of layers in the information set, the greater the information about the network, such that the complete information case is tantamount to the producer knowing all nodes of the production chain.

Therefore, our framework provides a clear relationship between network formation and allocation based on two sources, cost and information, where more information implies more potential suppliers and reduces the inefficiency of the allocation. Thus, the following definition summarizes how we model the information.

Definition 1. *We assume that the level of information k_i is given by the number of layers that a firm i knows in her information set, such that $k_i \in [1, K]$. The $k_i = 1$ means the firm only knows the previous layer, i.e., its direct suppliers, while $k_i = K$ means complete information, where the firm knows all the nodes of the network as well as their interactions.*

Hence, from this construction, we also explicitly relate the concept of incomplete information with the position in the network and the set of potential suppliers through how many layers a firm knows in her information set. Thus, we define the relationship between the degree of information and the network linkages, which is a useful tool for the endogenous search and matching setup derived in Section 3, in the following way.

Definition 2. *For a given producer i, with k_i levels of information, the distance between the final layer and the level of information, $|\overline{L} - k_i|$, provides the degree of incompleteness of her information set. Therefore, the higher the value of k_i is, the greater the number of layers that a producer could search for a potential supplier.*

3 Cost Minimization Problem

In this section, we derive the cost minimization problem for a given producer, highlighting the differences between our setup and the procedures well defined in the literature, as in Boehm and Oberfield (2020), Oberfield (2018) and Acemoglu and Azar (2020), and emphasize the relationship between cost and information.

Then, in the following subsections we define the cost problem of the firm, tie the interaction of the firms modeling a decentralized search problem to endogeneize the linkage formation, find optimally the sets of potential and effective suppliers, and derive the minimal cost of a given firm according to her level of information, number of effective suppliers, and position in the network.

3.1 Cost Problem Setting

We consider that each producer minimizes a cost function for a given degree of information k_i, in which we state this problem by the following equation:

$$\min_{n_i^* \in S^k} \min_{h_i \in L, x_{is} \in X} C_i(n_i^*, p_l, p_{is}, k_i) \tag{6}$$

Where $C_i(n_i^*, p_l, p_{is}, k_i)$ is the cost of producing one unit of output for the set of effective suppliers n_i^*, such that $n_i^* \in S^k$, S^k is the set of all the suppliers for a given degree of information k, p_l is the price of labor, and p_{is} is the price that producer i pays by the input s. Then, we proceed with a two-step decision to solve the cost problem, as depicted by the following definition.[18]

Definition 3. *The cost problem of the firm is represented by a two-step procedure, where in the first stage a given producer i should backup the set of potential and effective suppliers from the number of actual ones, firm specific characteristics, network connections, and the degree of information. Then, after that, in the second stage she chooses the allocation of labor and inputs that minimizes her cost function according to the effective suppliers.*

3.2 Decentralized Search Model

In this subsection, we explore the process of endogenous matching of the firms through a decentralized search,[19] where we borrow the notion of direct search models from Wright et al. (2017),[20] to derive the set of potential and effective suppliers of a producer as a function of the network environment, illustrated by firm-level and suppliers' profit, information degree, firm-level cost, position in the network and mark-up of effective suppliers.

To do that, we build a decentralized search model for a network, written as a sequential game, where a producer i searches her potential suppliers, from as many layers as levels of information k_i there are in her information set, according to the the set of her actual suppliers[21] and their profits. Hence, this approach disentangles from which layer each supplier comes and stresses information as the engine of the formation of the network.

[18] In fact, as a consequence of the regular assumption of the cost function as convexity and continuity, if we interchange the stages of the cost problem, the allocation remains optimal, meaning that choosing first the inputs and then the effective suppliers gives the same result as that described in Definition 3.

[19] Analogous to navigation models literature as in Kleinberg (2000) and Watts and Strogatz (1998).

[20] Other insightful papers in this literature are Lagos and Wright (2005) and Corbae et al. (2003).

[21] The set of actual suppliers is viewed through a participation constraint that is always binding since it is better for the supplier to sell the good to a producer than his outside option to produce the desired good.

Thus, at this point, it is important to consider the differences between actual, effective and potential suppliers in our theoretical model. The actual suppliers represent the firms that we see in the data supplying a producer, the effective suppliers illustrate the best choice of suppliers effectively chosen,[22] and the potential suppliers represent the set of firms optimally found in the search procedure. All these measures depend on the degree of information and the position of the firm in the network.

To build the search procedure, we define N_s and N_c as the measures of suppliers and customers in this economy. From that, we consider $n_i(g_i, k_i) = \frac{n_s(g_s,k)}{n_c(g_i,k)}$, for $s \neq i$, as the ratio between suppliers and customer, and we normalize $n_c(g_i, k_i) = 1$[23] to represent $n_i(g_i, k_i)$ as the number of potential suppliers of a producer i at node g_i with k_i degrees of information. Moreover, $V_s(g_i - k_i, n_i, k_i)$ is the value function of a supplier and $V_c(g_i, n_i, k_i)$ the same for a customer, where the arguments are position in the network, number of potential suppliers and the degree of information.

We consider a probability function $\gamma_i(g_i, n_i, k_i)$ that gives the meeting probability of firm i, at position g_i, with her potential suppliers n_i, from all the k_i layers incorporated in her information set, and $\frac{\gamma_i(g_i,n_i,k_i)}{n_i(g_i,k_i)}$ the analogous meeting probability for the suppliers. In order to derive analytical solutions, we assume that the probability function is concave according to the number of potential suppliers, as seen by the following assumption.

Assumption 4. *We assume that a given producer i, at node g_i, with degree of information k_i and n_i potential suppliers, has a continuously and differentiable quadratic meeting probability function with positive constants a_j and b_j that represent the sensibilities according to the position of producer i in the network as:*

$$\gamma_i(g_i, n_i, k_i) = a_j + b_j n_i(g_i, k_i)^2$$

In order to derive the optimal number of potential suppliers, we consider a network with L layers and S^{L_i} firms at each layer, such that each one has k degrees of information, and, without loss of generality, a unique industry i at the final node g that has K layers in his information set,[24] such that $K \leq L$. Under this environment, the maximization of industry[25] is given by:

[22] Sometimes actual and effective suppliers are the same, but it is not necessary.

[23] In addition, it also translates the notion of market tightness of the network.

[24] In the appendix, we derive a closed form solution for a general probability function, and derive three cases of this decentralized search procedure, starting with the simple as possible case with two nodes, two players, a bilateral trade and evolve the generalization of the framework to obtain insights about this procedure.

[25] For the remaining derivations of the paper we use the industry as a fixed point, deriving the results for the final producer, although the derivation is analogous for an intermediate producer and if we look to the problem from the suppliers perspective, it is analogous, because we are in a symmetric setup.

$$\max_{\{n_i, p_{is}\}} \gamma(g, n_i, K)\pi_i(g, K) \tag{7}$$

Subject to the incentive constraint of being a supplier, written as a function of the actual suppliers:

$$\sum_{s=1}^{K} \frac{\gamma_i(g, n_i, k)}{n_i(g-s)} \pi_s(g-s, n_i, k) = V_s(g-1, g-2, ..., g-K, K) \tag{8}$$

Where $\gamma(g_i, n_i, K) = \sum_{s=1}^{K} \frac{\gamma_i(g_i - s, n_i, k)}{n_i(g-s)}$

Then, under this context, industry i maximizes, with respect to the number of potential suppliers and the price paid by the inputs x_{is}, her profit weighted by the meeting probability function subject to the actual suppliers' constraint. Hence, to derive the optimal number of potential suppliers, we state the following proposition:

Proposition 1. *In order to find the set of potential suppliers, industry i solves a decentralized search problem where he maximizes his expected payoff constrained to the incentive constraint, as described by equations (7) and (8). Then, the set of potential suppliers is given by:*

$$n_i(j, K) = \sum_{i}^{K} a_i + \sum_{i}^{K} b_i n_i(j-i, k)\left[\frac{1}{b_i} \frac{\mu_j \pi_i(j, K)\epsilon_i}{mc_i \pi_s(j-1, k) - \mu_{j-1}\pi_i(j, K)\epsilon_i} - a_i\right]^{\frac{1}{2}} \tag{9}$$

Where μ_i is the markup of firm i, while μ_{j-1} is the markup of the suppliers of firm i that live in the predecessor layer of the network, $n_i(j-i, k)$ is the set of potential suppliers of the suppliers of the firm i, mc_i is the marginal cost of firm i and ϵ_i is a constant term.

From the proposition above, we note that the set of potential suppliers is a function of the network environment, the payoff of the actual suppliers, which is a proxy for how attractive a supplier is, and the choice of her suppliers. Then, the number of potential suppliers is determined mainly by three factors.

First, the general equilibrium factor, illustrated by $\sum_{i}^{K} b_i n_i(j-i, k)$, which describes the fact that at some sense even though firm i knows suppliers from K layers of the network, she must compete with the remaining firms to find her suppliers. Then, the choice of the suppliers according to who will be their suppliers affects her decision. Second, the customer's net capability to purchase the inputs, through the terms $\mu_j \pi_i(j, K)$ and mc_i, where π_i, mc_i, and μ_j give the profit, marginal cost and markup of firm i, respectively. Finally, the positional factor, since the components of the probability function, such as a_i and b_i, affect the optimal decision of firm i, means that her position in the network, in addition to the knowledge about its shape, impacts the formation of the set of potential suppliers and; hence, the draw of the network.

From the potential suppliers' equation, we can establish our first result: the set of potential suppliers is monotonically increasing according to the degree of information, as illustrated in the following proposition. This is the first step to answer questions about what happens if a producer enhances her information set.

Proposition 2. *For a given network, the number of potential suppliers has a positive relationship with the degree of information.*

Now, from the derivation of the potential suppliers we calculate the set of effective suppliers n_i^*, for a given producer i such that $n_i^* \subset n_i$, in order to set who the suppliers are effectively chosen by a firm according to the formation of the network and her degree of information. Thus, to model this choice, we consider that from the set of potential ones, the producer picks the suppliers that maximize her profit,[26] as stated in the following definition.

Definition 4. *In order to choose the set of effective suppliers a given producer picks from the set of potential suppliers the ones that provide her maximal profit, $\pi^* > \pi_i$ for all possible combination. Therefore, this optimal decision can be written as the following constrained optimization.*

$$\begin{aligned} n_i^*(g,k) &\in \arg\max \pi_i(g,k) \\ s.t.\ n_i^*(g,k) &\leq n_i(g,k) \end{aligned} \tag{10}$$

Since n_i^* gives the best combination of suppliers from the set of potential suppliers, we can state a useful result about the uniqueness of the number of effective suppliers, as illustrated in the following proposition.

Proposition 3. *From the problem of the minimization cost of the firm, there exists a unique composition of effective suppliers that a firm i chooses to maximize her profit such that $n_i^* \leq n_i$ and any other combination of inputs leave to a Pareto-inferior profit equilibrium.*

3.3 Inputs Minimization Problem

Now, in this subsection, we derive the second step of our cost procedure, where the industry chooses the optimal level of labor allocation l_i^* and the optimal level of input x_{is}^* that minimizes its cost function. To do

[26] Where the marginal effect of a new supplier is increasing if $n' < n^*$, while when $n' > n^*$ we have the converse impact

that, we assume that the cost function is given by the sum of the inputs bought from the effective suppliers n_i^*, and labor allocation. We assume that industry i has k_i degrees of information, take as given the price p_{is} of the input x_{is}, and normalize the price p_l of the labor. Thus, the cost minimization problem is depicted by:[27]

$$C_i(n_i, p_l, p_{is}, k_i) = \min_{\{h_i, x_{is}\}} \{h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is}(j - k_i, n_i, k_i)\} \tag{11}$$

Subject to the production function:

$$f_i(n_i^*, z_i, h_i, x_{is}, k_i) \leq y \tag{12}$$

Then, the following proposition describes the general form for the optimal cost for a given producer.

Proposition 4. *The optimal cost for a given producer with n_i^* effective suppliers is described by the following equation:*[28]

$$C_i^* = \{l_i^* + \frac{1}{n_i^*} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{(1 - \sum_i^{n_i^*} \alpha_i) z_i} x_{is}^*\} \tag{13}$$

Therefore, the optimal cost equation reveals two important insights: the number of effective suppliers directly impacts the total cost of the producer because when a firm decides the optimal quantity of input she must bought them from the effective suppliers. Thus, it corroborates the idea that allocation depends, indirectly, on the information, since the set of effective suppliers is a function of the degree of information. Furthermore, through the elasticity term, $\frac{1 - \sum_i^{n_i^*} \alpha_i}{\sum_i^{n_i^*} \alpha_i}$, the position in the network and, consequently, the connections, affect the cost allocation too.

Finally, after the previous characterization of the environment of the network, information and firms minimization cost, we can define the equilibrium of this input-output economy, based on the productivity, unitary cost, level of information, allocation of inputs, and their interplay with the sets of potential and effective suppliers, beyond the arrangement contract.

Definition 5. *An equilibrium is a tuple $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ associated with the arrangement profile $\{p_{is}, x_{is}(g - k_i, n_i, k_i)\}$ for each intermediate producer and $\{p_w, x_{is}(g - k_i, n_i, k_i)\}$ for the industries, the producers' choice*

[27] In the appendix, we lead with a robustness analysis considering how the information setup changes if there is a fixed cost F_i to search the suppliers, translating the idea of cost to gathering the information, which is increasing with the distance between the producer and supplier nodes.

[28] In the appendix we provide the details about the derivation

about $\{x_{is}, h_i\}$, and household's choice about consumption $\{c_i\}$ such that:

(i) Each producer solves the decentralized search problem stated in equations (7) and (8) to find the set of potential suppliers n_i, and then, they decide who are the effective suppliers in a procedure as stated in definition (4).

(ii) After the decentralized search procedure, firms minimize the cost function as described in equations (11) and (12) to find the optimal allocations of x^*_{is} and l^*_i.

(iii) Each industry produces output $y_i(g,k)$ according to the production function characterized in Definition 1. Then, the total output of this network economy with k-orders of information is given by y^k_n, such that $y^k_n = \sum_{i=1}^{w} y_i(g,k)$.

(iv) Each industry sells the good to the household under a fixed markup rule such that price p_w is given by $p_w = (1+\mu_w)C_i(n_i, p_l, p_{is}, k_i)$.

(v) The representative household maximizes his utility function, given by equation (3), subject to the budget constraint in equation (4), given the price p_w and the labor;

(vi) Final good and labor market clear conditions hold such that:

$$
\begin{aligned}
\sum_{i=1}^{w} c_i = \sum_{i=1}^{w} y_i(g,k) = y^k_n \\
\sum_{i=1}^{n^*} h_i = W_i
\end{aligned}
\tag{14}
$$

(vii) For intermediate goods, the market clearing condition holds such that:

$$
\sum_{i=1}^{S^G} \sum_{s=1}^{n^*_i} x_{is} = \sum_{i=1}^{S^{G \setminus J}} y_i \tag{15}
$$

4 Misallocation Sources

In order to investigate the distortions generated by network formation and incomplete information, we consider two sources of wedges. However, in contrast to papers such as Oberfield (2018), and Boehm and Oberfield (2020), we work with endogenous wedges. Thus, the network wedge, w_n, and the informational wedge, w_k are given respectively by:

$$w_n = y_p - y_n \tag{16}$$

Where: y_p is the central planner output for a standard representative agent economy (without network) and y_n is the network output, under complete information, for a given network formation.

The network wedge captures the difference in terms of output given by the existence of a network with a given linkage formation when compared with a standard central planner solution for a simple two agent bilateral trade economy. Therefore, the idea behind this wedge is to translate the fact that the formation of the network may distort the economy because sometimes network frictions[29] do not allow an agent to directly choose the optimal suppliers to buy the inputs and produce a good as in a central planner decision.

$$w_k = y_n - y_n^k \tag{17}$$

Where: y_n^k is the output of a network given a k-level of information.

Now, the second source of misallocation illustrates that the linkage formation of the network may not be optimal when we allow for incomplete information. Thus, the target of this wedge is to endogeneize how information about the network matters in allocative terms, showing that knowing distant firms in the network might provide better connections and consequently better allocations.

This construction allows us to disentangle the effect of each order of information on welfare and answer questions such as what happens if the producer has one degree of information and increases her information set in one order, which sheds light on the importance of the provision of information as an instrument of public policy to reduce misallocation.

To obtain a closed-form solution to the wedges, we calculate the output value under the three different contexts: the central planner, the network under complete information and the network under incomplete information. Then, the network wedge can be rewritten as the aggregation of how the individual allocations differ from the central planner case by the network distortions:[30]

$$w_n = \sum_{i \in G} [(\overline{h} - \hat{h}_i) + (\overline{x} - n_i^* \hat{x}_{is})] \tag{18}$$

Where subscript i represents the network producers for all the producers represented by the G nodes in

[29] This may happen due to contractual causes, nature-drawn matching of the firms, and several other reasons that do not depend on the information and profit of the firm

[30] The derivation of all the wedges is in the appendix.

the network, while those with overline represent the central planner.

The network wedge is affected by five components, labor allocation, h_i, elasticity of the network, α_i, inputs x_{is} allocation, and productivity z_i. Now, the informational wedge can be rewritten as:[31]

$$w_k = \sum_{i \in G} \{\Delta \hat{h}_i^k + \Delta(n_i^{*k} \hat{x}_{is}^k)\} \tag{19}$$

Where the subscript i represents the producers of the network, for all the producers represented by the G nodes in the network, the subscript s represents the goods used as input by the producers and the superscript k represents the degree of information for the degree of information.

Then, the informational wedge is a function of the set of effective suppliers, productivity and elasticity according to the network. From the previous equation, we note that what governs how this wedge evolves is the degree of information, where the higher the information is, the greater the number of effective suppliers and consequently the lower the difference between complete and incomplete information equilibria allocations of the inputs.

5 Equilibrium Characterization

In this section, we derive results about the existence, uniqueness and efficiency of our equilibrium definition. In order to do that, we illustrate the assumptions previously defined that permeate those results and highlight the relevance of incomplete information and network formation.

Then, starting with the conditions to have the existence of an equilibrium $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ in our production network under incomplete information. Under assumptions such as continuity and quasi-concavity about the production function, the preferences of the household, the decentralized search procedure, and the cost function, there exists an equilibrium in this network economy. Furthermore, as a natural consequence of the uniqueness of the cost minimization problem, the equilibrium $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ is uniquely determined as illustrated by the following theorem.

Theorem 1. *Suppose Assumptions* 1, 2, 3, *and* 4 *hold. Then, there exists a unique equilibrium* $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ *for a given level of information and a network formation.*

Moreover, we also study the conditions under which the equilibrium attains efficiency. In order to do that, in the following result, we highlight the importance of information and linkage formation as sources of

[31] Details about this derivation can be found in the appendix.

misallocation, as described in the previous section, through endogenous wedges.

Theorem 2. *Suppose Assumptions* 1*,* 2*,* 3*, and* 4 *hold and there exists an informational wedge* w_k *and a network wedge* w_n*. Then, an equilibrium* $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ *is efficient if, and only if, information and network wedges are equal to zero.*

Theorem 2 states that the equilibrium is inefficient if either the linkages of the production chain provide a different allocation than the central planner solution or whether incomplete information provides a different allocation than the complete information case or even whether both cases occur simultaneously. Moreover, the reverse also holds: when information and network sources do not generate misallocation the equilibrium is efficient.

Therefore, Theorem 2 allows an equilibrium under incomplete information for a given linkage formation to be efficient. Intuitively, this happens whenever the configuration of the network is optimal, coinciding with a central planner case, and the marginal value of the information is null; thus, rising the information set does not improve the allocations. Even though increments in the information set improve the set of potential suppliers, the set of effective suppliers does not necessarily change; hence, allocations cannot change after a growth in the information.

6 Network Counterfactual Exercises

In this section, we proceed with some theoretical exercises to stress all the key issues of the proposed incomplete information network mechanism and its implications. We follow three venues to explore the theoretical input-output network analysis.

First, we establish the relationship between information and network formation through the analysis of how information impacts density, the degree of centrality and betweenness centrality in the network economy. Thus, this exercise shows to us that information-enhancing policies reduce the density of a network and increase the stability of a network, while focal policies increase individual centrality.

Second, we derive the amplification effect of a productivity shock on the consumption of the household and point-out its comovement with the level of information. This exercise illustrates the relationship between information, network and economic business cycle, where information-enhancing policy amplifies even more the effect of a productivity shock.

Third, we layout under what conditions an information-enhancing policy is more valuable than a subsidy as a source of public policy and derive the optimal design of a policy based on the combination of both instruments. We consider that a subsidy raises the profit of a given firm, while more information improves her decentralized search, making better matches, and consequently increasing her profit.

6.1 Information and Network Formation

We start this subsection by deriving the impact of the degree of information on the shape of a network using three metrics: (i) the density, (ii) the degree of centrality and (iii) the stability.[32] Our goal is to understand how the distribution of information and, consequently, the application of information-enhancing policies could change the configuration of a network.

Density - Following Jackson (2010), we use the concept of density of a network to illustrate how far the empirical network is in comparison with its optimal draw, thus giving a flavor about the relationship between information, network and misallocation. This is a consequence of the fact that density is defined[33] as the ratio between the number of actual edges and the number of potential edges. Therefore, we can translate this general definition to our input-output environment, writing it in terms of the sets of potential and effective suppliers.

Definition 6. *The density of a network G is given by:*

$$D(G) = \frac{\#Actual\ Edges}{\#Potential\ Edges} = \sum_{i \in G} \frac{n_i^*}{n_i} \tag{20}$$

As the concept of density is tantamount to the ratio of suppliers, we have that information directly affects the level of the density of our network economy. Therefore, in the following proposition, we illustrate that information-enhancing policies reduce the degree of density through changes in the set of suppliers.

Proposition 5. *The provision of information-enhancing policies to the firms of a network economy G reduces the density of the network such that:*

$$\frac{D(G)}{\frac{\partial D(G)}{\partial k_i}} \leq D(G) \tag{21}$$

Therefore, economies with higher levels of information should have a small proportion of effective suppliers

[32]Details can be found in the appendix.

[33]The main reference in this graph theory is Jackson (2010), although Bloch et al. (2016) also reviewed those network metrics.

compared to potential ones, because those producers have more information to choose better who their suppliers are, thus filtering their choices, as in Oberfield (2018), and reducing misallocation.

Centrality - In order to measure the centrality of a given producer i, we follow Jackson (2010) and consider that it is equivalent to the ratio of the number of edges that involve firm i and the total edges of the network G. Again, we can adapt this graph-analysis to our input-output environment, where the level of centrality of a producer is given by the proportion of her potential suppliers with respect to the total, as depicted in the following definition.

Definition 7. *The degree of centrality of a given producer i in a network G is given by his weight on the set of suppliers of the network:*

$$d_i(G) = \frac{n_i}{\sum_{i \in G} n_i} \tag{22}$$

Since we already documented that information-enhancing policies increase the set of potential suppliers, this result is applied here for the centrality of the producers. Therefore, policies focused on the provision of information improve the degree of centrality of the producer.

Proposition 6. *Focal information-enhancing policies improve the centrality of a given producer i, as illustrated by:*

$$\frac{\partial d_i(G)}{\partial k_i} = \frac{\partial n_i}{\partial k_i} \geq 0 \tag{23}$$

Even though information-enhancing policies could be locally applied to improve the centrality of a given producer and reduce the distortions of the network, centrality could also be understood as a collective concept, where some firms are more central than others. Then, the use of a second centrality measure allows us to establish the relationship between the information, the stability and the draw of the network. This is the goal of our last exercise in this subsection.

Stability - Now, we consider the concept of betweenness centrality from Freeman (1978), which is given by the ratio of the number of edges that link producers j and k through firm i in comparison with all the edges that link producers j and k. This notion establishes the dependence of the network on producer i. Hence, we treat this concept as the inverse of the concept of stability of the network for the remainder of the paper.

Definition 8. *The degree of stability of network G with respect to a given producer i is given by:*

$$d_i^e(G) = \sum_{\{j,k\}\neq i} [\frac{2}{(N-1)(N-2)} \sum_{(j,k)} \frac{v_G(i:j,k)}{v_G(j,k)}]^{-1} = \sum_{\{j,k\}\neq i} [\frac{2}{(N-1)(N-2)} \frac{1}{\{n_j | k \in n_a \ \ \forall a \in n_j\}}]^{-1} \quad (24)$$

Where N is the total number of nodes (firms) of network G, $v_G(i : j,k)$ is the number of edges (suppliers) between producers j and k that pass by producer i, $v_G(j,k)$ is the total number of edges (suppliers) between producers j and k such that the distance between producers j and k is $|j-k| = 2$, and $\{n_j | k \in n_a \ \ \forall a \in n_j\}$ represents the set of all the suppliers of producer j that has firm k as a supplier.

Then, this definition illustrates that the higher the centrality of producer i is: (i) the higher the dependence of the network on her, and consequently (ii) the lower the stability of the network economy. In the following proposition, we show that an information-enhancing policy improves the stability of the network because this kind of policy reduces the betweenness centrality such that economies with high information have less dependence on a few firms.

Proposition 7. *As the provision of information reduces the betweenness centrality of each producer in a network G, for all $k < k'$ levels of information:*

$$\{n_j | k \in n_a \ \ \forall a \in n_j\} \subset \{n_j | k' \in n_a \ \ \forall a \in n_j\} \quad (25)$$

Then, information-enhancing policies improve the stability of a network economy G, where:

$$\frac{\partial d_i^e(G)}{\partial k_i} \geq 0 \quad (26)$$

This result shows that economies with higher information have lower dependence on more connected producers; thus, for them, the social value of a connection is less valuable than in economies with low information. Therefore, in addition to the impact of information-enhancing policies in the configuration of the network, it also acts as insurance, preserving the production chain with respect to occasional shocks on the economy and smoothing its effect across the production chain.

6.2 Counterfactual Exercise

Now, we derive two results about the relationship between information, allocation and network. First, we derive the amplification effect of the productivity shock on the consumption of the household, and second, we establish how its general equilibrium effect comoves with the degree of information.

Proposition 8. *The elasticity of the household's consumption with respect to the productivity shock of a given producer i is given by:*

$$\frac{\partial c_i}{\partial z_i} = (1 - \sum_{i}^{n_i^*} \alpha_i)h_i + \sum_{i}^{n_i^*} \alpha_i x_{is} + \sum_{j \neq i}^{w} (\frac{\partial z_j}{\partial z_i}(1 - \sum_{j}^{n_i^*} \alpha_j)h_j + \frac{\partial z_{js}}{\partial z_i} \sum_{j}^{n_j^*} \alpha_j x_{js} \geq 0 \tag{27}$$

Where $\frac{\partial z_j}{\partial z_i} > 0$ *for all* i *and* j.

Now, the effect of information on this elasticity is given by:

$$\frac{\partial^2 c_i}{\partial z_i \partial k_i} = \frac{\partial h_i}{\partial k_i} \frac{1}{1 - \sum_i \alpha_i} + \frac{\partial x_{is}}{\partial k_i} \frac{1}{\sum_i \alpha_i} \geq 0 \tag{28}$$

Where $\frac{\partial z_{js}}{\partial z_i} > 0$.

The first equation of this proposition disentangles the impact of the productivity shock on the consumption of the household into two components: (i) the partial (competitive) equilibrium effect and (ii) the general (network) equilibrium effect. The indirect (network equilibrium) effect of this shock depicts its propagation according to the other industries of this network economy. Since productivity has a common component, when industry i is hit by a shock it propagates amplifying the effect through the production chain, enhancing the productivity of the other industries and increasing the impact on consumption. Now, from the second equation of Proposition 8, we learn about the (i) comovement between the impact of productivity and information and that (ii) information-enhancing policy amplifies the effect of the productivity shock.

6.3 Information x Subsidy

In order to investigate a public policy implication of our incomplete information network setup, we establish the following proposition based on two sources of analysis. First, we analyze under what conditions providing information about the network to an industry is more efficient than a monetary subsidy in terms of individual profit and aggregate welfare. Second, exploring the complementarity between information and monetary subsidies, we study the optimal design of a public policy under both instruments.

Proposition 9. *For a given network economy, information-enhancing policies are more effective than a subsidy to raise profit if the following inequality holds:*

$$|\tau_i^y| < |\frac{\Delta(p_w y_i)}{p_w}| \tag{29}$$

Now, in terms of welfare, providing information is better than an income subsidy if the following inequality holds:

$$|\tau_i^y| < |\Delta y_i| \tag{30}$$

Where all Δ terms give the difference under two different arbitrary levels of information, for revenue, $p_w y_i$, and output y_i.

Therefore, the combination of the subsidy $|\tau_i^y|$ with the provision of information $|\tau_i^I|$ guarantees optimality if, and only if,

$$|\tau_i^y| + |\tau_i^I| = y_p - y_n^k \tag{31}$$

Therefore, Proposition 9 elucidates that information could be a powerful tool of public policy for network economies. It provides the conditions where the tax-subsidy policy is less efficient than the information-enhancing policies for profit and welfare. These conditions set that monetary transfer is a suboptimal policy whenever the size of the subsidy is lower than (i) the variation of the real revenue of the firm, and (ii) the increment of the production function generated by the information policy, respectively.

Proposition 9 also depicts the optimal design of a public policy that is given by the sum of a monetary transfer with an information-enhancing policy, where this policy has a net effect that is tantamount to the size of the sum of network and information wedges. Therefore, it sheds light that the provision of information is more valuable in low information networks, while the subsidy acts as a complementary instrument to provide an exogenous upward drift in the production of distorted environments.

Finally, in the proposition below, we study the impact of an enhancing-information policy on the aggregate output of this network economy, such that:

Proposition 10. *The impact of an enhancing-information policy for a given sector, with a k-level of information, in the network economy is tantamount to the increase in the Domar-weights (λ) provided by the increment in the information set of the producers as illustrated by:*[34]

$$\frac{\Delta Y_i}{Y_i} = \sum_{i \in I} [\lambda_{i,k+1} - \lambda_{i,k}] \tag{32}$$

Proposition 10 states that the higher the informational distortion of a given sector (or producer) is, the better the capability of a policy based on the provision of information to raise the GDP of the network economy. Therefore, we can evaluate the impact of the information as a tool of public policy to correct distortions through the measure of Domar-weights.

7 Empirical Analysis

In this section, we take our model to the data to investigate the stylized facts regarding the relationship among information, network structure, and misallocation using a firm-level Brazilian financial transaction dataset. The subsequent subsections first describe the dataset, then explain the empirical algorithm used to estimate the firms' level of information, and finally present the empirical results.

7.1 Dataset

We construct the Brazilian production network using proprietary data on firm payments and transfers from the Central Bank of Brazil (BCB).[35] Table 1 provides descriptive statistics characterizing the granularity of this dataset, which spans the period 2019 to 2023. It comprises over 9.6 million firms across more than 450 sectors and records approximately 1.5 billion bilateral transactions. Specifically, we observe about 124 million bilateral payments between non-financial firms[36] using financial instruments such as invoices and wire transfers. These instruments are extracted from the BCB's payment system databases: *Sistema de Transferências de Reservas*, *CIP-Sitraf*, and *Siloc*.

[34] The result is analogous for a given producer instead of a given sector

[35] Firms can utilize various instruments for inter-firm payments. We construct the quarterly aggregate of all firm-to-firm transfers, without distinguishing the specific transfer instrument.

[36] We exclude the financial sector and focus solely on traditional business sectors.

Tabela 1: Descriptive Statistics

Statistic	**N**	**Mean**	**Standard Deviation**	**Median**
Firm Age (years)	9,605,131	13.311	9.957	10
Payment (BRL)	124,397,900	50,849.54	10,375,589.00	2,478.13
Number of Payments	124,397,900	12.025	86.126	4

7.2 Algorithm Estimation

Following our theoretical mechanism, we estimate the level of information of firms within the Brazilian production network. To accomplish this, we propose a two-stage estimation algorithm, where we first define the position of the firms in the network to then estimate their level of information.

The first stage defines the network layers based on the empirical network structure found from the bilateral transactions in our dataset, which allows us to identify each firm's position. We begin by setting the most downstream vertices of the production chain (final producers) as Layer 1. We then recursively map the previous neighbors (the firms that directly interact with firms in the preceding layer) under the assumption that they represent the effective suppliers. These suppliers are assigned to the next subsequent layer (Layer 2). This approach continues recursively until we reach the most upstream producers (initial suppliers) of the supply chain.

The second stage of our algorithm determines the firms' information level. We define a given firm's information level as being proportional to the norm (distance) between its assigned layer and the layer of its furthest supplier. Hence, a firm that belongs to Layer 1 and has suppliers from Layers 2 and Layer 3 has two degrees of information.

7.3 Empirical Results

What is the level of information of the firms? Our first empirical result documents the firms' level of information and its evolution between 2019 and 2023. Figure 4 depicts the distribution of the information level for the top 20 sectors of the Brazilian economy, across the 30 largest supply chains within each sector.

Two key findings emerge from this preliminary analysis. First, sectors generally exhibit a low level of information regarding their supply chain, with the vast majority characterized by an information depth of less than two levels. This suggests that firms typically only know their immediate suppliers, corroborating our theoretical setup in which firms lack common knowledge of the network structure. Second, the temporal variation in the information set is highly heterogeneous across sectors. This heterogeneity supports the hypothesis that both idiosyncratic (sector-specific) and aggregate variables influence a firm's decision process for identifying suppliers within the production network.

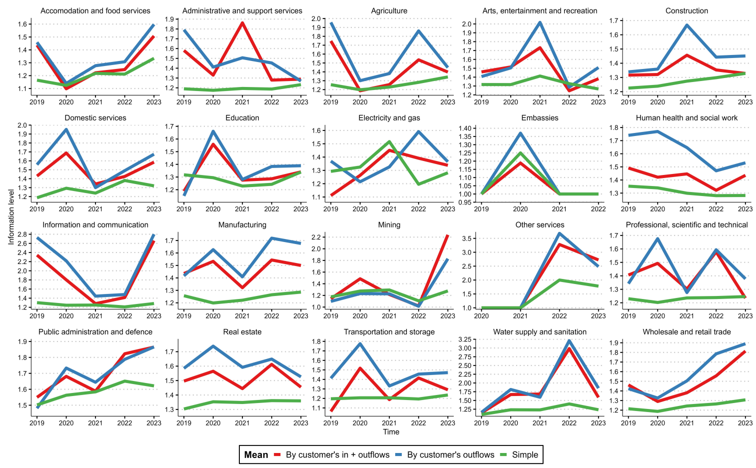

Figura 4: Information Level by NAICS

Another source of heterogeneity in the distribution of information among firms in a network economy may stem from the fact that the information set changes according to the position in the production network. This can also explain the stylized fact that networks are characterized as small path networks, highlighted by Watts and Strogatz (1998), given the existence of clusters where some firms interact with many others, while others maintain only a few connections.

Does Position in the Network Change Firm Information Level? Figure 5 links the relationship between the information level and network position. Our estimation reveals a pattern where firms in the core of the network exhibit a higher level of information compared to both the most upstream firms of the production economy and the most downstream firms. One conjecture for this finding is related to competition: in order to survive as a successful producer of intermediate goods, a firm must maintain more connections, thus creating an incentive for firms to actively increase their information. Another possibility relates to geographical transaction costs. Various explanations could emerge from this feature, as explored by Jackson and Rogers (2005).

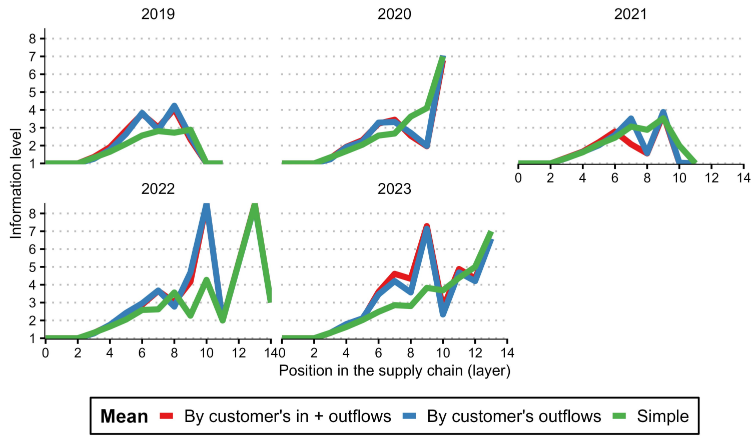

Figura 5: Information Level and Position in the Network

Does more information really mean more potential suppliers? To empirically test an implication of our theoretical mechanism regarding the relationship among network position, information, and network formation, we estimate the level of firms' potential suppliers. Theoretically, we derived that a higher degree of information corresponds to a greater number of potential suppliers. Figure 6 presents our estimates for the sets of potential and actual suppliers under two specifications:[37] a simple average within the supply chain, and a weighted average based on the size of the financial transaction between firms. This figure also highlights how these sets change according to the firm's position in the network.

Figure 6 clearly shows that firms in the core of the network, which possess more information, are the same firms that have a larger pool of potential suppliers. Empirically, this result strongly corroborates our theoretical mechanism, allowing us to argue empirically too that a higher information set for a given firm leads to a larger set of potential suppliers. This pattern suggests that information could be used as a policy instrument to improve matching efficiency within the network and, consequently, reduce economic misallocation.

[37] It is important to clarify the empirical definitions of potential and actual suppliers to maintain coherence with the theoretical framework. We define a firm's potential suppliers as all active suppliers that have interacted with the firm during the entire sample period. The actual suppliers are those that interact with the firm in the given year of analysis. For example, when estimating the sets for firm i in 2020, all firms that interacted with i between 2019 and 2020 are considered potential suppliers, while the set of actual suppliers is restricted to firms that interacted with i only in 2020.

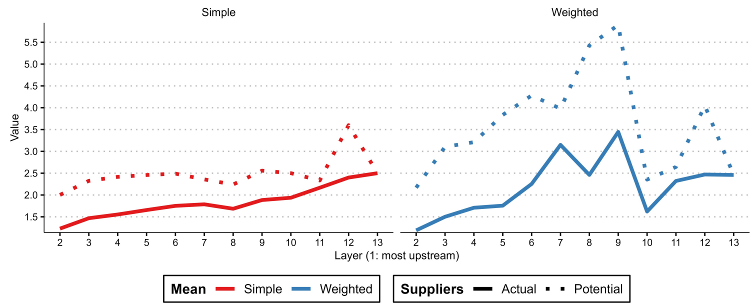

Figura 6: Distribution of Actual and Potential Suppliers

In our final result, we empirically test our theoretical proposition on the relationship between a firm's information level and its network centrality. Theoretically, we posited that a higher level of information correlates with greater firm centrality within the economy. This is a direct consequence of the firm's larger set of potential suppliers, which facilitates a higher number of connection and, therefore, a more central position in the network, reducing its vulnerability to idiosyncratic supplier shocks.

Figure 7 illustrates that centrality exhibits a non-linear behavior across network layers, one notable aspect warrants mention: within the core of the network, where the degree of information is the highest, the level of firm centrality increases, which corroborates our theoretical intuition. This suggests that information is an important factor for enhancing a firm's connections with the remaining participants of the production economy. Consequently, managing or manipulating the availability of information within specific sectors could serve as a useful policy lever for improving network stability and mitigate the risk of negative cascade effects.

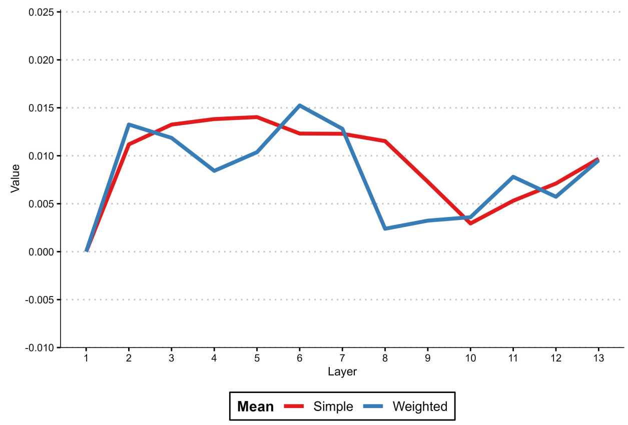

Figura 7: Centrality vs Position in the Network

8 Conclusion

In this paper, we have provided a new general equilibrium input-output network model, under incomplete information, that engages the literature on the macroeconomics of networks, such as Acemoglu and Azar (2020), with k-level information, such as Crawford et al. (2013), to explain the relationship between information, firm dynamics and misallocation in a network economy in terms of three strands of this setup: (i) network formation, (ii) network allocation, and (iii) public policy. Then, at the network level, our first contribution is embedding this setup into a decentralized search procedure, where each firm chooses, endogenously, her sets of potential and effective suppliers from the empirical draw of the network and the degree of information. From that, we build an endogenous production network, where its linkage-formation depends directly on how much information each producer has about the production chain.

On the theoretical side, in terms of network configuration, we find that network economies with higher information have less density and more stability. In terms of network allocation, we find that network economies with higher information have a higher spillover effect of the productivity shock on household consumption. In terms of public policy, we find that information-enhancing policies are more valuable for economies with low information than a tax-subsidy policy, while the converse is true for economies with high information. Furthermore, we design an optimal contract based on both instruments that provides efficient allocations even under incomplete information.

On the empirical side, we use proprietary microdata from the Central Bank of Brazil to investigate new stylized facts in the literature of macroeconomics of networks and test the predictions of our model. First, we draw the production chain of the Brazilian economy and estimate the level of information for each firm, classifying their position in the network. Second, we find that information is distributed heterogeneously according to the major sectors and the position of the firms in the network. Furthermore, we find the existence of a positive relationship between information and centrality, where, consistent with our setup, the higher the level of information of the firm the larger is her set of suppliers.

Finally, we leave as future research a deeper analysis of the dynamic interaction within firms in the network environment, where would be interesting to investigate the relationship between the network formation and the dynamic update of firms' information set.

Appendix

8.1 Solving the Decentralized Search Procedure

In this subsection, we solve the decentralized search procedure, deriving the set of potential suppliers and effective suppliers for a given firm. To do that, we divide the analysis into two cases, where we start by the simplest network environment and then we increase the complexity of the network.

Case 1: We start assuming a network marked by two layers, two nodes, one link and two firms. There is a unique producer and one actual supplier, symbolizing a unique transaction between the firms, as illustrated in Figure 8.

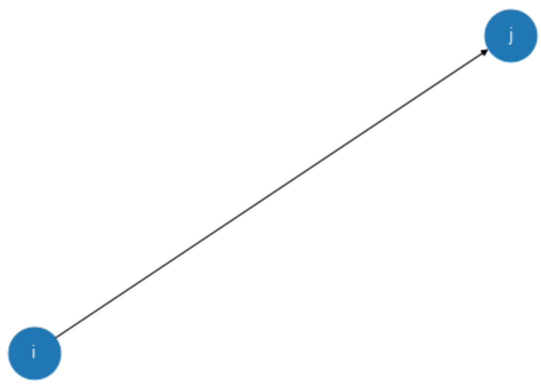

Figura 8: Simple Network

In this context, the decentralized search procedure is characterized as a maximization problem, where producer i maximizes his profit function times the matching probability function subject to the incentive constraint. It is important to highlight that since there is a unique transaction in this economy and two players, the degree of information is one, naturally.

$$\max_{\{n_i, p_{is}\}} \gamma_i(j, n_i)\pi_i(j, 1) \tag{33}$$

Subject to the incentive constraint given as a function of the suppliers:

$$\frac{\gamma_i(j, n_i)}{n_i(j-1)}\pi_s(j-1, 1) = V_s(j-1) \tag{34}$$

To solve this maximization problem, we write the Lagrangian as follows:

$$L = \gamma_i(j,n_i)\pi_i(j,1) - \lambda[\frac{\gamma_i(j,n_i)}{n_i(j-1)}\pi_s(j-1,1) - V_s(j-1)] \tag{35}$$

Calculating the first-order conditions with respect to the price paid by the input x_{is}, represented by p_{is}, and the number of potential suppliers n_i, we obtain:[38]

$$\begin{aligned} \frac{\partial L}{\partial p_{is}} = 0 \implies \gamma_i(j,n_i)\frac{\partial \pi_i(j,1)}{\partial p_{is}} = \lambda \frac{\gamma_i(j,n_i)}{n_i}\frac{\partial \pi_s(j-1,1)}{\partial p_{is}} \\ \frac{\partial L}{\partial n_i} = 0 \implies \frac{\partial \gamma_i(j,n_i)}{\partial n_i}\pi_i(j,1) = \lambda \frac{\partial(\gamma_i(j,n_i)/n_i)}{\partial n_i}\pi_s(j-1,1) \end{aligned} \tag{36}$$

Thus, combining the first-order conditions to eliminate the multiplier we obtain the following:

$$\begin{aligned} \frac{\gamma_i(j,n_i)}{\frac{\partial \gamma_i(j,n_i)}{\partial n_i}} \frac{\frac{\partial \pi_i(j,1)}{\partial p_{is}}}{\pi_i(j,1)} = \frac{\gamma_i(j,n_i)}{n_i} \frac{\frac{\partial \pi_s(j-1,1)}{\partial p_{is}}}{\pi_s(j-1,1)}[\frac{\partial(\gamma_i(j,n_i)/n_i)}{\partial n_i}]^{-1} \implies \\ \frac{n_i}{\frac{\partial \gamma_i(j,n_i)}{\partial n_i}} = \frac{\frac{\partial \pi_s(j-1,1)}{\partial p_{is}}}{\pi_s(j-1,1)} \frac{\pi_i(j-1,1)}{\frac{\partial \pi_i(j-1,1)}{\partial p_{is}}}\pi_i(j-1,1)\frac{1}{\frac{\partial(\gamma_i(j,n_i)/n_i)}{\partial n_i}} \end{aligned} \tag{37}$$

Then, using the fact that the functional form of the probability is given by : $\gamma_i = a_i + b_i n_i^2$,[39] we can rewrite equation 37, such that:

$$\begin{aligned} \frac{n_i}{2bn_i} = \frac{\mu_s}{\pi_s}\frac{\pi_i}{\mu_i}\frac{1}{\frac{a_i}{n_i^2} + b_i} \implies \\ \frac{a_i}{n_i^2} = \frac{\pi_i}{\pi_s}\frac{\mu_s}{\mu_i}2b_i - b_i \implies \\ \frac{1}{n_i^2} = \frac{b_i}{a_i}(2\frac{\pi_i}{\pi_s}\frac{\mu_s}{\mu_i} - 1) \end{aligned} \tag{38}$$

Where $\mu_s = \frac{\partial \pi_s(j-1,1)}{\partial p_{is}}$ and $\mu_j = \frac{\partial \pi_i(j-1,1)}{\partial p_{is}}$.

Then, the set of potential suppliers is given as follows:

$$n_i = (2\frac{b_i}{a_i}\frac{\pi_i}{\pi_s}\frac{\mu_s}{\mu_i} - 1)^{-\frac{1}{2}} \tag{39}$$

[38]Since the profit function only considers the set of effective suppliers instead of the set of potential suppliers, we consider that the derivative of the profit function with respect to n_i is equal to zero.

[39]Here, we drop the arguments of the probability function as well as the set of potential suppliers to avoid excessive notation in the derivation of the solution.

In this simple environment, the degree of information does not matter, because the network only has two nodes. To improve this, we now illustrate a second example below.

Case 2: Suppose there is a network g, with three nodes, i.e., three firms and two layers, where the producer at the final node has complete information, while the other two companies are her actual suppliers. Figure 9 below illustrates this new configuration.

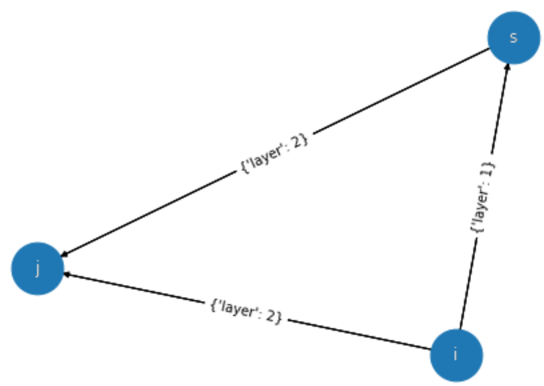

Figura 9: Network Economy

Naturally, the maximization problem of producer i, in the last node, changes, as seen below.

$$\begin{aligned} &\max_{\{n_i, p_{is}\}} \gamma(j, n_i)\pi_i(j, 2) \\ &\text{st: } \frac{\gamma_i(j, n_i)}{n_i(j-1)}\pi_s(j-1, 1) + \frac{\gamma_i(j-1, n_i)}{n_i(j-2)}\pi_s(j-2, 1) = V_s(j-1, s-2) \end{aligned} \tag{40}$$

Where $\gamma(j, n_i) = \gamma_i(j, n_i) + \gamma_i(j-1, n_i)$

For this version of a network, the effects of the presence of information become clear by the incentive constraint of the actual suppliers and the producer's profit function, where; now, unlike the previous case, producer i considers potential suppliers firms from both layers of the network since he has information about them. Hence, the participation constraint incorporates the profit and meeting probability of firm s in node $j-2$ beyond the payoff of the supplier at node $j-1$. Then, the Lagrangian is given by:

$$L = \gamma(j, n_i)\pi_i(j, 2) - \lambda_j[\frac{\gamma_i(j, n_i)}{n_i(j-1)}\pi_s(j-1, 1) + \frac{\gamma_i(j-1, n_i)}{n_i(j-2)}\pi_s(j-2, 1) - V_s(j-1, j-2)] \tag{41}$$

Calculating the first-order conditions, we obtain:

$$\begin{aligned} \frac{\partial L}{\partial p_{is}} = 0 &\implies \gamma(j, n_i)\frac{\partial \pi_i(j,1)}{\partial p_{is}} = \lambda[\frac{\gamma_i(j,n_i)}{n_i(j-1)}\frac{\partial \pi_s(j-1,1)}{\partial p_{is}} + \frac{\gamma_i(j-1,n_i)}{n_i(j-2)}\frac{\partial \pi_s(j-2,1)}{\partial p_{is}}] \\ \frac{\partial L}{\partial n_i} = 0 &\implies \frac{\partial \gamma(j,n_i)}{\partial n_i}\pi_i(j,1) = \lambda\frac{\partial(\gamma_i(j,n_i)/n_i)}{\partial n_i}\pi_s(j-1,1) \end{aligned} \tag{42}$$

It is important to highlight that we consider $\frac{\partial(\gamma_i(j-1,n_i)/n_i)}{\partial n_i} = 0$, since the set of potential suppliers of a given firm does not affect the matching probability function of the supplier of the supplier to find a new match and in order to simplify the solution, we also assume that $\frac{\partial \pi_s(j-2,1)}{\partial p_{is}} = 0$. Then, combining the first-order conditions to drop the Lagrange multiplier, we obtain:

$$\frac{\gamma(j,n_i)\frac{\partial \pi_i(j,1)}{\partial p_{is}}}{\frac{\partial \gamma(j,n_i)}{\partial n_i}\pi_i(j,1)} = [\frac{\gamma_i(j,n_i)}{n_i(j-1)}\frac{\partial \pi_s(j-1,1)}{\partial p_{is}} + \frac{\gamma_i(j-1,n_i)}{n_i(j-2)}\frac{\partial \pi_s(j-2,1)}{\partial p_{is}}][\frac{\partial(\gamma_i(j,n_i)/n_i)}{\partial n_i(j-1)}\pi_s(j-1,1)]^{-1} \tag{43}$$

If we apply the same derivation as in the first case, considering the functional form of the probability, we obtain the following:

$$\begin{aligned} &\gamma(j,n_i) = \frac{\pi_i(j,2)}{c_i\pi_i(j-1,1)}\frac{\frac{\partial \gamma(j,n_i)}{\partial n_i}}{\frac{\partial(\gamma_i(j,n_i)/n_i)}{\partial n_i(j)}}\frac{\gamma_i(j,n_i)}{n_i}\mu_j \implies \\ &\frac{\gamma_i(j,n_i)+\gamma_i(j-1,n_i)}{\gamma_i(j,n_i)} = \frac{\pi_i(j,2)}{c_i\pi_i(j-1,1)}\frac{2b_i}{-\frac{a_i}{n_i^2}+b_i}\mu_j \implies \\ &\frac{\gamma_i(j-1,n_i)}{\gamma_i(j,n_i)} = \frac{\pi_i(j,2)}{c_i\pi_i(j-1,1)}\frac{2b_in_i}{b_in_i^2-a_i} - 1 \implies \\ &a_i + b_in_i^2 = \frac{c_i\pi_i(j-1,1)\gamma_i(j-1,n_i)}{\mu_j\pi_i(j,2)}\frac{b_in_i^2-a_i}{2b_in_i} - \gamma_i(j-1,n_i) \end{aligned} \tag{44}$$

Therefore, the set of potential suppliers can be written as follows:

$$n_i = \gamma_i(j-1,n_i)(\frac{c_i\pi_i(j-1,1)}{\mu_j\pi_i(j,2)}\frac{1}{\epsilon_i} - 1) \tag{45}$$

Where $\epsilon_i = \frac{a_i+b_in_i^2}{bn_i^2-a_i}$ is a constant that belongs to the interval (0,1).

However, this equation is a function of the matching probability function $\gamma_i(j-1,n_i)$ of the suppliers that are in other layers of the network, which depicts the major difference in comparison with equation (39), i.e., the fact that, with two layers of information, the position of the actual suppliers in the network and the connections become crucial in the matching process. Thus, we need to solve recursively this problem to write the set of potential suppliers for industry i in terms of the primitives of the environment of this network; hence, we solve the problem of the suppliers in the predecessor layer, which is analogous to Case 1, as follows.

$$\max_{\{n_i, p_{is}\}} \gamma(j-1, n_i)\pi_i(j-1, 1) \tag{46}$$

Subject to

$$\frac{\gamma_i(j-2, n)}{n_i(j-2)}\pi_s(j-2, 1) = V_s(j-2) \tag{47}$$

The Lagrangian for this maximization problem is written as:

$$L = \gamma(j-1, n_i)\pi_i(j-1, 1) - \lambda_{j-1}[\frac{\gamma(j-2, n)}{n_i(j-2)}\pi_s(j-2, 1) - V_s(j-2)]$$

Then, the optimal number of potential suppliers for the producer at node $j-1$ is depicted as:

$$n_i(j-2, 1) = (2\frac{b_i}{a_i}\frac{\pi_i(j-1, 1)}{\pi_s(j-2, 1)}\frac{\mu_{j-2}}{\mu_{j-1}} - 1)^{-\frac{1}{2}} \tag{48}$$

Then, substituting equation 48 into equation 45, we obtain a closed form solution to the set of potential supplier for firm i in the final node of network g with 2 layers in her information set.

$$\begin{aligned} n_i(j-1, 1) &= [a_i + b_i n_i(j-2, 1)](\frac{c_i \pi_j(j-1, 1)}{\mu_j \pi_i(j, 2)}\frac{1}{\epsilon_i} - 1) \implies \\ n_i(j-1, 2) &= [a_i + b_i(2\frac{b_i}{a_i}\frac{\pi_i(j-1, 1)}{\pi_s(j-2, 1)}\frac{\mu_{j-2}}{\mu_{j-1}} - 1)^{-\frac{1}{2}}](\frac{c_i \pi_j(j-1, 1)}{\mu_j \pi_i(j, 2)}\frac{1}{\epsilon_i} - 1) \end{aligned} \tag{49}$$

Therefore, this equation describes that depending on the degree of information about the network, a given firm considers both her incentives, the incentives of her suppliers as well as the incentives of the suppliers of the suppliers in order to find her set of potential suppliers.

8.2 Derivation of The Minimization Cost Problem

Now, to conclude the derivation of the optimal level of unitary cost, we derive the second stage, where each producer i chooses the quantity to be bought of the inputs x_{is} and allocation of labor h_i from the effective suppliers n_i^*. We already work with the log-linearized form of this problem and normalize the labor price, where:

$$\begin{aligned} \min_{\{h_i, x_{is}\}} c(n_i^*, p_l, p_{is}, k_i) &= h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is}(j-k, n_i, k_i) \\ \text{s.t. } f_i(n_i^*, z_i, h_i, x_{is}, k_i) &= (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i + \sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is} \end{aligned} \tag{50}$$

Then, the Lagrangian is given by:

$$L = h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is} - \lambda [f_i(n_i^*, z_i, h_i, x_{is}, k_i) = (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i + \sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is}] \tag{51}$$

Calculating first-order conditions, we obtain:

$$\begin{aligned} \frac{\partial L}{\partial h_t} = 0 &\implies 1 - \lambda(1 - \sum_{i}^{n_i^*} \alpha_i) z_i = 0 \\ \frac{\partial L}{\partial x_t} = 0 &\implies \sum_{s=1}^{n^*} p_{is} - \lambda \sum_{i}^{n_i^*} \alpha_i \sum_{i}^{n_i^*} z_{is} = 0 \end{aligned} \tag{52}$$

Hence, assuming that $\sum_{s=1}^{n^*} p_{is} = p_{is} n_i^*$, as a consequence of perfect competition, we find that the price of a given input is proportional to the ratio of productivity as well as the elasticity across the network:

$$p_{is} = \frac{1}{n_i^*} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{(1 - \sum_i^{n_i^*} \alpha_i) z_i} \tag{53}$$

Therefore, the optimal cost is given by $C_i = \{h_i^* + \frac{1}{n_i^*} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{(1 - \sum_i^{n_i^*} \alpha_i) z_i} x_{is}^*\}$, where the quantity used as input and labor are optimal. Hence, implicitly to this equilibrium, any combination different of the effective suppliers must provide a higher cost, thus we have for all i that:

$$\frac{\partial C_i}{\partial n_i^*} < \frac{\partial C_i}{\partial n_i} \tag{54}$$

8.3 Deriving the Wedges

To derive the wedges, we start recovering the production function:

$$y_i = f_i(n_i^*, z_i, h_i, x_{is}, k_i) = (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i + \sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is} \quad (55)$$

Where $x_{is}(j, n_i^*, k_i)$ is the input bought by producer i from n_i^* effective suppliers from different layers, between j and $j - k_i$, and under k_i degrees of information about the network formation, $\sum_{i=1}^{n_i^*} \alpha_i$ is the network elasticity, h_i is the labor allocation, z_i is the labor-augmenting productivity shock term, and z_{is} is the input-augmenting productivity shock term for each input.

To derive the network and informational wedges, we need to provide the definitions of output under three different contexts: (i) central planner solution y_p, (ii) complete information network solution y_n, and (iii) incomplete information network solution y_n^k. Now, to simplify the calculation, we assume the following condition:

$$\sum_{s=1}^{n_i^*} x_{is}(j, n_i, k_i) = n_i^* x_{is}(j, k_i) \quad (56)$$

Hence, the output of producer i, under incomplete information, where $k_i = 1$, can be rewritten as:

$$y_n^k = (1 - \sum_{i=1}^{n_i^*} \alpha_i)(z_i h_i) + n_i^*(j, 1) \sum_{i=1}^{n_i^*} \alpha_i [z_{is} x_{is}(j, n_i, k_i)] \quad (57)$$

Now, under the same context, let us calculate the output for the case of complete information for a given network formation. Thus, consider a network G with g layers, a unique producer i, at final layer j, with $k_i = g$ degrees of information. Then, the complete information output is given by:

$$y_n = (1 - \sum_{i=1}^{n_i^*} \alpha_i)(z_i h_i) + n_i^*(j, k) \sum_{i=1}^{n_i^*} \alpha_i [z_{is} x_{is}(j, n_i, k)] \quad (58)$$

Now, recovering the fact that informational wedge is given as follows:

$$w_k = y_n - y_n^k \quad (59)$$

Then, substituting the formulations of the output in each wedge equation, we obtain that informational wedge is given as follows:

$$w_k = 1 - \sum_{i=1}^{n_i^*} \alpha_i)(z_i h_i) + n_i^* \sum_{i=1}^{n_i^*} \alpha_i (j, k_i)[z_{is} x_{is}(j, n_i, k_i)] - ((1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i) + n_i^*(j, 1) \sum_{i=1}^{n_i^*} \alpha_i [z_{is} x_{is}(j, n_i, 1)] \quad (60)$$

Therefore, the aggregate informational wedge is given by:

$$w_k = \sum_{i \in G} \{1 - \sum_{i}^{n_i^*} \alpha_i z_i [h_i(j, n_i, k_i) - h_i(j, n_i, 1)] + \sum_{i=1}^{n_i^*} \alpha_i z_{is} [n_i^*(j, k_i) x_{is}(j, n_i, k_i) - n_i^*(j, 1) x_{is}(j, n_i, 1)]\} \quad (61)$$

Calling the following terms as:

- $(1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i(j, n_i, k_i) = \hat{h}_i(j, n_i, k_i)$
- $(1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i(j, n_i, 1) = \hat{h}_i(j, n_i, 1)$
- $\Delta \hat{h}_i^k = \hat{h}_i(j, n_i, k_i) - \hat{h}_i(j, n_i, 1)$
- $\sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is}(j, n_i, k_i) = \hat{x}_{is}(j, n_i, k_i)$
- $\sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is}(j, n_i, 1) = \hat{x}_{is}(j, n_i, 1)$
- $\Delta \hat{x}_{is}^k = \hat{x}_{is}(j, n_i, k_i) - \hat{x}_{is}(j, n_i, 1)$
- $\Delta {n_i^*}^k = n_i^*(j, k_i) - n_i^*(j, 1)$

Then, the informational wedge can be rewritten as:

$$w_k = \sum_{i \in G} [\Delta \hat{h}_i^k + \Delta({n_i^*}^k \hat{x}_{is}^k)] \quad (62)$$

Now, to derive the network wedge, we set a standard production function for the central planner case, as depicted in its log-linearized form as follows:

$$y_p = \alpha_p(z_p + h_p) + (1 - \alpha_p)(z_p + x_p) \quad (63)$$

Where lowercase letters represent the log-deviation variables.

Analogously, the output for this case is a function of the productivity shock term z_p, input x_p and elasticity α. Since this equation translates the idea of bilateral trade between two firms, we use the subscript p to illustrate the planner case. Then, recovering the fact that the network wedge is given by:

$$w_n = y_p - y_n \quad (64)$$

Applying the output for complete information and the central planner in the network wedge equation implies that it can be rewritten as:

$$w_n = \alpha_p(z_p h_p) + (1-\alpha_p)(z_p x_p) - (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i) + n_i^*(j, k_i) \sum_{i=1}^{n_i^*} \alpha_i [z_{is} x_{is}(j, n_i, k_i)] \tag{65}$$

Therefore, the aggregate network wedge can be written as:

$$w_n = \sum_{i \in G} [z_p \alpha_p h_p - z_i 1 - \sum_{i=1}^{n^*} \alpha_i h_i(j, n_i, k_i) + (1-\alpha) z_p x_p - n_i^*(j, k_i) \sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is}(j, n_i, k_i)] \tag{66}$$

Considering that:

- $z_p \alpha_p h_p = \overline{h}$
- $z_i(1 - \sum_{i=1}^{n^*} \alpha_i) h_i(j, n_i, k_i) = \hat{h}_i$
- $(1-\alpha) z_p x_p = \overline{x}$
- $\sum_{i=1}^{n_i^*} \alpha_i z_{is} x_{is}(j, n_i, k_i) = \hat{x}_{is}$

We can rewrite the network wedge as follows:

$$w_n = \sum_{i \in G} [(\overline{h} - \hat{h}_i) + (\overline{x} - n_i^* \hat{x}_{is})] \tag{67}$$

8.4 Omitted Proofs From the Text

8.4.1 Decentralized Search Approach

Proposition 1:

Demonstração. We consider a network g with g_i nodes, S firms at each node, and a unique producer i with K degrees of information at layer j, while the other firms, from other layers, have k levels of information.

To obtain the closed form in a general manner, we first derive the analytical solution for a general matching probability function and then apply our assumption about concavity of this function and find the closed form equation.

$$\max_{\{n_i, p_{is}\}} \gamma(j, n_i) \pi_i(j, K) \tag{68}$$

Subject to

$$\frac{\gamma_i(j,n_i)}{n_i(j-1)}\pi_s(j-1,k)+\frac{\gamma_i(j-1,n_i)}{n_i(j-2)}\pi_s(j-2,k)+...+\frac{\gamma_i(j-K,n_i)}{n_i(j-K)}\pi_s(j-K,k) = V_s(j-1,j-2,...,j-K) \quad (69)$$

Where $\gamma(j,n_i) = \sum_{s=1}^{K}\gamma_i(j-s,n_i)$.

The Lagrangian is given by:

$$L = \gamma(j,n_i)\pi_i(j,K)-\lambda_1[\frac{\gamma_i(j,n_i)}{n_i(j-1)}\pi_s(j-1,k)+\frac{\gamma_i(j-1,n_i)}{n_i(j-2)}\pi_s(j-2,k)+...+\frac{\gamma_i(j-K,n_i)}{n_i(j-K)}\pi_s(j-K,k)-V_s(j-1,j-2,...,j-K)] \quad (70)$$

Deriving the first-order conditions with respect to the price paid by the input, p_{is}, and the number of suppliers, n_i we obtain:

$$\begin{aligned}\frac{\partial L}{\partial p_{is}} &= \gamma_i(j,n_i)\frac{\partial \pi_i(j,K)}{\partial p_{is}} - \lambda_1[\frac{\gamma_i(j,n_i)}{n_i(j-1)}\frac{\partial \pi_s(j-1,k)}{\partial p_{is}} + ... + \frac{\gamma_i(j-K,n_i)}{n_i(j-K-1)}\frac{\partial \pi_s(j-K,k)}{\partial p_{is}}] = 0 \\ \frac{\partial L}{\partial n_i} &= \frac{\partial \gamma_i(j,n_i)}{\partial n_i(j-1)}\pi_i(j,K) - \lambda_1[\frac{\partial(\frac{\gamma_i(j,n_i)}{n_i(j-1))})}{\partial n_i(j-1)}\pi_s(j-1,K)] = 0\end{aligned} \quad (71)$$

Rearranging the term of both equations to isolate the Lagrange multiplier, we have the following conditions:

$$\begin{aligned}\lambda_1 &= (\frac{\partial \gamma_i(j,n_i)}{\partial n_i(j-1)}\pi_i(j,K))[\frac{\partial(\frac{\gamma_i(j,n_i}{n_i(j-1)})}{\partial n_i(j-1)}\pi_s(j-1,k)]^{-1} \\ \lambda_1 &= \gamma_i(j,K)\frac{\partial \pi_i(j,K)}{\partial p_{is}}[\frac{\gamma_i(j,n_i)}{n_i(j-1)}\frac{\partial \pi_s(j-1,k)}{\partial p_{is}} + ... + \frac{\gamma_i(j-K,n_i)}{n_i(j-K-1)}\frac{\partial \pi_s(j-K,k)}{\partial p_{is}}]^{-1}\end{aligned} \quad (72)$$

Now, rearranging the terms we obtain an equation to represent the number of potential suppliers implicitly defined according to the matching probability function and characteristics of the firms.

$$(\frac{\partial \gamma_i(j,K)}{\partial n_i(j-1)}[\frac{\partial(\frac{\gamma_i(j,n_i)}{n_i(j-1)})}{\partial n_i(j-1)}]^{-1}\frac{1}{\gamma_i(j,K)})[\frac{\gamma_i(j,n_i)}{n_i(j-1)}\frac{\partial \pi_s(j-1,k)}{\partial p_{is}}+...+\frac{\partial \gamma_i(j-K,n_i)}{n_i(j-K-1)}\frac{\partial \pi_s(j-K,k)}{\partial p_{is}}] = \frac{\partial \pi_i(j,K)}{\partial p_{is}}\frac{\pi_s(j-1,k)}{\pi_i(j,K)} \quad (73)$$

Finally, applying our assumption about the functional form to the matching probability and the derivative of the profit function, we follow the same steps as cases 1 and 2 to derive the optimal number of potential suppliers as follows:

$$
\begin{aligned}
&\frac{\gamma_i(j,K)+\gamma_i(j-1,k)+...+\gamma_i(j-K,k)}{\gamma_i(j,K)} = \left(\frac{c_i\pi_j(j-1,1)}{\mu_j\pi_i(j,2)}\frac{1}{\epsilon_i}-1\right) \implies \\
&\gamma_i(j,K) = [\gamma_i(j,K)+\gamma_i(j-1,k)+...+\gamma_i(j-K,k)]\left(\frac{c_i\pi_j(j-1,1)}{\mu_j\pi_i(j,2)}\frac{1}{\epsilon_i}-1\right)^{-1} \implies \\
&a_i+b_in_i(j,K)^2 = [\gamma_i(j,K)+\gamma_i(j-1,k)+...+\gamma_i(j-K,k)]\left(\frac{c_i\pi_j(j-1,1)}{\mu_j\pi_i(j,2)}\frac{1}{\epsilon_i}-1\right)^{-1} \implies \\
&n_i(j,K)^2 = \frac{1}{b_i}\{[\gamma_i(j,K)+\gamma_i(j-1,k)+...+\gamma_i(j-K,k)]\left(\frac{c_i\pi_j(j-1,1)}{\mu_j\pi_i(j,2)}\frac{1}{\epsilon_i}-1\right)^{-1}-a_i\} \implies \\
&n_i(j,K) = \left(\frac{1}{b_i}\right)^{\frac{1}{2}}\{[\gamma_i(j,K)+\gamma_i(j-1,k)+...+\gamma_i(j-K,k)]\left(\frac{c_i\pi_j(j-1,1)}{\mu_j\pi_i(j,2)}\frac{1}{\epsilon_i}-1\right)^{-1}-a_i\}^{\frac{1}{2}}
\end{aligned}
\tag{74}
$$

Therefore, finding the set of potential suppliers for industry i in the final node of a network with K degrees of information is a function of the primitives of the economic environment and the set of potential suppliers of her suppliers.

$$
n_i(j,K) = \sum_i^K a_i + \sum_i^K b_in_i(j-i,k)\left[\frac{1}{b_i}\frac{\mu_j\pi_i(j,K)\epsilon_i}{mc_i\pi_s(j-1,k)-\mu_{j-1}\pi_i(j,K)\epsilon_i}-a_i\right] \tag{75}
$$

Where we consider the marginal cost $mc_i = c_i$ only to make the notation clear. □

Proposition 2:

Demonstração. In order to prove the positive relationship between the set of potential suppliers and the degree of information we proceed with an almost static comparative argument, comparing what happens with the set of potential suppliers when the information set of the producer increases. Thus, the derivative of the set of potential suppliers with respect to the degree of information is strictly positive. Then, let's calculate this derivative using the closed-form solution of the set of potential suppliers for a general network economy g where firm i has K levels of information, as described in the previous proposition.

$$
\frac{\partial n_i(j,K)}{\partial k_i} > 0 \iff \sum_i^K b_in_i(j-i,k)\frac{\partial\left[\frac{1}{b_i}\frac{\mu_j\pi_i(j,K)\epsilon_i}{mc_i\pi_s(j-1,k)-\mu_{j-1}\pi_i(j,K)\epsilon_i}-a_i\right]}{\partial k_i} > 0 \tag{76}
$$

Since the set of potential suppliers of the suppliers does not vary due to variations in the information set of firm i, as well as the profit function of the suppliers, the marginal cost of firm i and the markups, then we need to apply a quotient rule to find the derivative. In order to avoid excessive notation, when we calculate the derivative we drop the arguments of the functions, such that the condition can be written as follows:

$$\frac{\partial n_i(j,K)}{\partial k_i} > 0 \iff \sum_i^K b_i n_i(j-i,k) \frac{\mu_j \epsilon_i \frac{\partial \pi_i}{\partial k_i}(mc_i\pi_s - \mu_i\pi_i\epsilon_i) + \mu_j\epsilon_i\pi_i\mu_j\frac{\partial \pi_i}{\partial k_i}}{(mc_i\pi_s - \mu_j\pi_i)^2} > 0 \tag{77}$$

Since the denominator, the set of potential suppliers of the suppliers and the elasticity b_i are positive terms, this condition can be rewritten as follows:

$$\begin{aligned} &\frac{\partial n_i(j,K)}{\partial k_i} > 0 \iff \mu_j\epsilon_i\frac{\partial \pi_i}{\partial k_i}(mc_i\pi_s - \mu_i\pi_i\epsilon_i) + \mu_j\epsilon_i\pi_i\mu_j\frac{\partial \pi_i}{\partial k_i} > 0 \iff \\ &\mu_j\epsilon_i\frac{\partial \pi_i}{\partial k_i}mc_t\pi_s + \mu_j^2\epsilon_i\pi_i[\frac{\partial \pi_i}{\partial k_i} - \epsilon_i\frac{\partial \pi_i}{\partial k_i}] > 0 \end{aligned} \tag{78}$$

Therefore, since the coefficient $\epsilon_i \in (0,1)$ and by assumption the derivative of the profit function with respect to the degree of information is positive, we have that the set of potential suppliers is a monotonic increasing in the degree of information. □

Proposition 3:

Demonstração. From the statement, the set of effective suppliers provides the highest profit, such that there is a unique $\pi^* > \pi_i \ \forall i$. The first step to prove this proposition is to recover the profit function, where:

$$\pi_i = p_i y_i - c_i$$

Now, we recover the cost problem of the firm, where:

$$C_i = \min_{\{h_i, x_{is}\}}\{h_i + \sum_{s=1}^{n_i^*} p_s x_{is}\} \tag{79}$$

Where n_i^* is the set of effective suppliers.

Then, the optimal profit function can be written as a function of the minimization cost problem as follows:

$$\pi^* = p_i y_i - \min_{\{h_i, x_{is}\}}\{h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is}\} \tag{80}$$

Now, from this equation, we can show that $\pi^* > \pi_i \ \forall i$, thus:

$$\begin{aligned}
&\pi^* > \pi_i \iff \\
&p_i y_i - \min\{h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is} + \sum_{i=1}^{n_i^*} F_i\} > p_i y_i - \min\{h_i + \sum_{s=1}^{n_i} p_{is} x_{is}\} \iff \\
&\min\{h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is}\} < \min\{h_i + \sum_{s=1}^{n_i} p_s x_s\}
\end{aligned} \tag{81}$$

Since by the minimization cost problem we find a unique solution that minimizes the cost, under mild conditions about convexity of the cost function, there is a unique combination of effective suppliers, allocation of inputs and labor that guarantees its inequality to hold.

Moreover, if we generalize our setup, adding a fixed cost F_i to capture the effective suppliers, which could translate the idea that some firms are distant from potential suppliers in the formation of the network due to information constraint, geographic factors or any other reason, our result still holds. In this case, our cost function becomes:

$$C_i = \min_{\{h_i, x_{is}\}} \{h_i + \sum_{s=1}^{n_i^*} p_s x_{is} + \sum_{i=1}^{n_i^*} F_i\} \tag{82}$$

Then, the profit function under the set of effective suppliers is the highest profit if, and only if:

$$\pi^* > \pi_i \iff \min\{h_i + \sum_{s=1}^{n_i^*} p_{is} x_{is} + \sum_{i=1}^{n_i^*} F_i\} < \min\{h_i + \sum_{s=1}^{n_i} p_s x_s + \sum_{i=1}^{n_i} F_i\} \tag{83}$$

This also holds since the fixed cost is at least the same for both sides of the inequality.

□

8.4.2 Equilibrium Properties

Theorem 1:

Demonstração. In order to prove Theorem 1, we proceed with two main blocks: first, we show the existence of the equilibrium, and then, we show the uniqueness.

(i) Existence - To prove existence, we show there is an equilibrium given by $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$, such that each allocation of this equilibrium is a fixed point.

Suppose there is an input x_{is}, such that $x_{is} \in X$, and there exists a function m such that $m : x_{ij} \to x_{is}$, for an arbitrary j and s. This function characterizes the input x_{is} used by supplier s_i to produce a good that is sold to producer i and used as an input x_{is} to produce a different good. Then, if we show that input x_{is} belongs to the matching function m, i.e., $x_{is} \in m$, we can apply Kakutani's fixed point theorem and conclude that m has a fixed point.

Proceeding in this way, the matching function, from the decentralized search problem of firm i that has a unique level of information in network g, is given by:

$$\max_{\{n_i, p_s\}} \gamma(j-1, n_i)\pi_i(j-1, 1) \tag{84}$$

Subject to

$$\frac{\gamma(j-1, n)}{n_i(j-2)}\pi_s(j-2, 1) = V_s(j-2) \tag{85}$$

By the duality between the cost function $C_i(n_i^*, p_l, p_{is}, k_i)$ and the profit function $\pi_i(s, k)$, we note that input $x_{is} \in C_i(n_i^*, p_l, p_{is}, k_i)$, and consequently it also belongs to the profit function, $x_{is} \in \pi_i(s, k)$. Hence, from this previous programming, we have $x_i \in m$; therefore, by Kakutani's fixed point theorem, this condition implies that there exists at least one fixed point in the matching function, which guarantees that, at optimal, x^* is a fixed point.

Now, as we have already proven for the input and the matching function, the idea for the output at the individual level is analogous, except that now we also use the continuity of the production function in order to aggregate the output y_n^k. Moreover, as labor is essential and by the fixed mark-up assumption, we have that independent of the level of x^* the selling price p_w is positive, which concludes that there exists at least one fixed point in our equilibrium definition.

Here, we assumed that firm i has a unique level of information and a unique supplier to simplify the proof, although all the results remain the same for the general case since the basis of the demonstration comes from the primitives of the economic environment.

(ii) Uniqueness - To prove uniqueness, we assume, by contradiction, that there exist two different equilibria for the same degree of information and network formation given by $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ and $(x_{ij}, n_j, n_i^*, p_{ij}, y_j)$, such that $i \neq j$. Thus, there are two fixed points to be represented by the selling price of each industry for each equilibrium, where we assume without loss of generality that $p_w < p_j$. Since the

cost function $C_i(n_i^*, p_l, p_{is}, k_i)$ is concave on the price of the input. Analogous to Acemoglu and Azar (2020), we take the minimum of the collection of cost functions in this network economy, which is also a concave function given by:

$$C_{Min}(n_i^*, p_l, p_{is}, k_i) = \min_{n_i} C_i(n_i^*, p_l, p_{is}, k_i) \tag{86}$$

Then, recovering our definition of the fixed markup, it is easy to note that the minimal price, provided by the minimal cost, is also a concave function because the markup is a constant term. Hence, taking the minimization operator on the price p_{is}, for $i = \{w, j\}$, we obtain:

$$\min p_i = (1 + \mu_i) \min_{n_i} C_i(n_i, p_l, p_{is}, k_i) \tag{87}$$

Now, let $v \in (0, 1)$, such that $vp_j < p_w$ and $vp_j = p_w$ for some w and j, by nondecreasing cost function we have that:

$$\begin{aligned} & p_w \geq vp_j \\ & \implies \min p_w \geq \min vp_j \\ & \implies \min p_w \geq \min vp_j + p_w - vp_j \\ & \implies \min p_w - p_w \geq \min vp_j - vp_j \end{aligned} \tag{88}$$

Now, by concavity, we have the following inequality:

$$\min vp_j - vp_j \geq (1 - v)C(0) + v[\min vp_j - p_j] \tag{89}$$

Since $(x_{ij}, n_j, n_i^*, p_{ij}, y_j)$ and naturally p_j is a fixed a point, we obtain that:

$$\min vp_j - vp_j \geq (1 - v) \min C(0) + v[\min vp_j - p_j] \implies \min vp_j - vp_j \geq (1 - v) \min C(0) \tag{90}$$

Finally, as there is a cost even for zero production, we verify the following:

$$\min vp_j - vp_j > 0 \tag{91}$$

Which is a contradiction since, by construction, $\min vp_j - vp_j < 0$. Therefore, for a given network formation and degree of information the allocations provided in equilibrium are unique. □

Theorem 2:

Demonstração. To prove Theorem 2, we divide our proof into two blocks, first the if part and then the only if.

If part - Assume the equilibrium $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ is efficient, which implies there is no misallocation in this economy. Hence, the network formation does not negatively affect the allocations compared to a central planner solution; thus, the following identity holds:

$$y_n = y_p \tag{92}$$

Therefore, applying it to the formula of the network wedge, we note that this source of misallocation is null when the equilibrium is efficient:

$$w_n = y_p - y_n = 0 \tag{93}$$

Now, the fact that the equilibrium $(x_{is}, n_i, n_i^*, p_w, p_{is}, y_i)$ is efficient also means that an increase in the information level cannot improve the allocations, independent of the degree of information that a firm has about the network. Therefore, the output provided under incomplete information coincides with the output provided under complete information, where:

$$y_n^k = y_n \tag{94}$$

Hence, applying this equation in the information wedge construction we note that it is null, which concludes this part of the proof.

$$w_k = y_n - y_n^k = 0 \tag{95}$$

Only if part - Assume the information and network wedges are null, thus $w_k = 0$ and $w_n = 0$, and using the fact that w_k and w_n are given, respectively by:

$$\begin{aligned} w_k &= y_n - y_n^k \\ w_n &= y_p - y_n \end{aligned} \tag{96}$$

These equations can be rewritten as follows:

$$\begin{aligned} y_n &= y_n^k \\ y_p &= y_n \end{aligned} \tag{97}$$

Therefore, combining them, we obtain $y_n^k = y_p$, i.e., network formation under incomplete information provides an output allocation equal to the central planner solution, which means that output is efficient. Now, in order to prove that other allocations are also efficient, we start recovering the production function:

$$y_p = y_n^k = (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i h_i + n_i^* \sum_{i=1}^{n_i^*} \alpha_i [z_{is} x_{is}(j, n_i, k)] \tag{98}$$

Therefore, since the output y_n^k is efficient and from the previous equation combined with the fact that the equilibrium is uniquely determined, then h_i, n_i^* and x_{is} must also be efficient, which is a natural consequence that the price paid by the input x_{is} must also be at an optimal value, implying that p_{is} is at an efficient level.

Finally, given that equilibrium is uniquely determined as shown in the previous theorem, there is a unique efficient n_i^*, and consequently, n_i must also be at an efficient level. □

8.4.3 Counterfactual Exercises:

In order to prove the proposition about the impact of a productivity shock and its relationship with the degree of information, we first state the following proposition about the marginal effects of each variable with respect to the information.

Proposition 11. *The marginal effects of the information, when all the producers alter their level of information, in the main variables of our network environment are given by:*

i) Production cost:

$$\frac{\partial C_i}{\partial k_i} = \frac{\partial h_i}{\partial k_i} - \frac{\frac{\partial n_i^*}{\partial k_i}}{n_i^{*2}} \frac{\sum_i^{n_i^*} z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} x_{is} + \frac{1}{n_i^*} \frac{\partial x_{is}}{\partial k_i} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} \tag{99}$$

ii) Selling price:

$$\frac{\partial p_w}{\partial k_i} = (1 - \mu_w)[\frac{\partial h_i}{\partial k_i} - \frac{\frac{\partial n_i^*}{\partial k_i}}{n_i^{*2}} \frac{\sum_i^{n_i^*} z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} x_{is} + \frac{1}{n_i^*} \frac{\partial x_{is}}{\partial k_i} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i}] \tag{100}$$

iii) Production Function:

$$\frac{\partial y_i}{\partial k_i} = (1 - \sum_{i=1}^{n_i^*} \alpha_i) z_i \frac{\partial h_i}{\partial k_i} + \sum_{i=1}^{n_i^*} \alpha_i z_i \frac{\partial x_{is}}{\partial k_i} \geq 0 \tag{101}$$

iv) Profit function:

$$\frac{\partial \pi_i}{\partial k_i} = [(1 - \mu_w) y_i - 1] \frac{\partial C_i}{\partial k_i} + p_w \frac{\partial y_i}{\partial k_i} \geq 0 \tag{102}$$

v) Household Consumption:

$$\frac{\partial U(c_i)}{\partial k_i} = \sum_{i=1}^{w} \xi_i \frac{\partial y_i}{\partial k_i} \geq 0 \tag{103}$$

Proposition 11

Demonstração. In order to illustrate the proof of the marginal effects, we start by reminding us that the cost function, production function and household utility function are continuous and differentiable, which allows us to focus directly on deriving the five marginal effects of information on the production cost, selling price, production function, profit function and household consumption, respectively.

i) Recovering the equation of the production cost:

$$C_i = h_i + \frac{1 - \sum_{i=1}^{n_i^*} \alpha_i z_{is}}{\sum_{i=1}^{n_i^*} \alpha_i z_i} \frac{1}{n_i^*} x_{is} \tag{104}$$

Then, the marginal effect is calculated as:

$$\begin{aligned} \frac{\partial C_i}{\partial k_i} &= \frac{\partial h_i}{\partial k_i} + \frac{\partial \frac{1}{n_i^*}}{\partial k_i} \frac{\sum_i^{n_i^*} z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} x_{is} + \frac{1}{n_i^*} \frac{\partial x_{is}}{\partial k_i} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} \implies \\ \frac{\partial C_i}{\partial k_i} &= \frac{\partial h_i}{\partial k_i} - \frac{\frac{\partial n_i^*}{\partial k_i}}{n_i^{*2}} \frac{\sum_i^{n_i^*} z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} x_{is} + \frac{1}{n_i^*} \frac{\partial x_{is}}{\partial k_i} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{1 - \sum_{i=1}^{n_i^*} \alpha_i z_i} \end{aligned} \tag{105}$$

Since $\frac{\partial n_i^*}{\partial k_i} \geq 0$ and $\frac{\partial x_{is}}{\partial k_i} \geq 0$ we have that $\frac{\partial C_i}{\partial k_i}$ has an ambiguous sign. Depending on the magnitude of the derivative terms.

ii) Recovering the equation of the selling price equation:

$$p_w = (1 - \mu_w) C_i \tag{106}$$

Then, the marginal effect is calculated as follows:

$$\frac{\partial p_w}{\partial k_i} = (1-\mu_w)\frac{\partial C_i}{\partial k_i} = (1-\mu_w)[\frac{\partial C_i}{\partial k_i} = \frac{\partial h_i}{\partial k_i} - \frac{\frac{\partial n_i^*}{\partial k_i}}{n_i^{*2}} \frac{\sum_i^{n_i^*} z_{is}}{1-\sum_{i=1}^{n_i^*} \alpha_i z_i} x_{is} + \frac{1}{n_i^*}\frac{\partial x_{is}}{\partial k_i} \frac{\sum_i^{n_i^*} \alpha_i z_{is}}{1-\sum_{i=1}^{n_i^*} \alpha_i z_i}] \tag{107}$$

Since the derivative of the price of the industries is a function of the effect on the cost function, the sign of the former is ambiguous here too.

iii) Recovering the equation of the production function:

$$y_i = \frac{z_i h_i}{1-\sum_{i=1}^{n_i^*} \alpha_i} + \frac{\sum_{i=1}^{n_i^*} z_{is} x_{is}}{\sum_{i=1}^{n_i^*} \alpha_i} \tag{108}$$

Then, the marginal effect is calculated as follows:

$$\frac{\partial y_i}{\partial k_i} = \frac{\partial z_i}{\partial k_i}\frac{h_i}{1-\sum_{i=1}^{n_i^*}} + \frac{z_i}{1-\sum_{i=1}^{n_i^*} \alpha_i}\frac{\partial h_i}{\partial k_i} + \frac{\sum_{i=1}^{*} \partial z_{is}}{\partial k_i}\frac{x_{is}}{\sum_{i=1}^{n_i^*}} + \frac{\sum_{i=1}^{n_i^*} z_{is}}{\sum_{i=1}^{n_i^*} \alpha_i}\frac{\partial x_{is}}{\partial k_i} \geq 0 \tag{109}$$

Since $\frac{\partial z_i}{\partial k_i} \geq 0$, $\frac{\partial h_i}{\partial k_i} \geq 0$ and $\frac{\partial x_{is}}{\partial k_i} \geq 0$, then $\frac{\partial y_i}{\partial k_i} \geq 0$.

iv) Recovering the equation of the profit function:

$$\pi_i = p_w y_i - C_i \tag{110}$$

Then, the marginal effect is calculated as follows:

$$\begin{aligned}
&\frac{\partial \pi_i}{\partial k_i} = \frac{\partial p_w}{\partial k_i} y_i + p_w \frac{\partial y_i}{\partial k_i} - \frac{\partial C_i}{\partial k_i} \implies \\
&\frac{\partial \pi_i}{\partial k_i} = [(1-\mu_w) y_i - 1]\frac{\partial C_i}{\partial k_i} + p_w \frac{\partial y_i}{\partial k_i} \implies \\
&\frac{\partial \pi_i}{\partial k_i} = [(1-\mu_w) y_i - 1]\frac{\partial h_i}{\partial k_i} + \frac{1-\sum_{i=1}^{n^*} \alpha_i}{\sum_{i=1}^{n^*} \alpha_i}\frac{\partial n_i^*}{\partial k_i} x_{is} + \frac{1-\sum_{i=1}^{n^*} \alpha_i}{\sum_{i=1}^{n^*} \alpha_i}\frac{\partial x_{is}}{\partial k_i} n_i^* + \\
&p_w \frac{\partial z_i}{\partial k_i}\frac{h_i}{1-\sum_{i=1}^{n_i^*}} + \frac{z_i}{1-\sum_{i=1}^{n_i^*} \alpha_i}\frac{\partial h_i}{\partial k_i} + \frac{\sum_{i=1}^{*} \partial z_{is}}{\partial k_i}\frac{x_{is}}{\sum_{i=1}^{n_i^*}} + \frac{\sum_{i=1}^{n_i^*} z_{is}}{\sum_{i=1}^{n_i^*} \alpha_i}\frac{\partial x_{is}}{\partial k_i}
\end{aligned} \tag{111}$$

Since $\frac{\partial C_i}{\partial k_i} \geq 0$, $\frac{\partial y_i}{\partial k_i} \geq 0$ and for regular conditions for the markup μ_w, then $\frac{\partial \pi_i}{\partial k_i} \geq 0$.

v) Recovering the equation of household utility:

$$U(c_i) = \sum_{i=1}^{w} \xi_i c_i \tag{112}$$

Then, from the market clearing condition, we have that $c_t = y_t$, and then the marginal effect is calculated as:

$$\frac{\partial U(c_i)}{\partial k_i} = \sum_{i=1}^{w} \xi_i \frac{\partial y_i}{\partial k_i} \geq 0 \tag{113}$$

Since $\frac{\partial y_i}{\partial k_i} \geq 0$, then $\frac{\partial U(c_i)}{\partial k_i} \geq 0$. □

Now, let us prove the counterfactual exercises about the relationship between information and network formation:

Proposition 5

Demonstração. Recovering the formula of the density of a network economy G, we have the following:

$$D_i = \frac{n_i^*}{n_i} \tag{114}$$

Suppose that all producers of this input-output network economy enhance their information level in a unique layer; thus, both the set of potential suppliers and the set of effective suppliers tend to increase, as their derivatives indicate as follows:

$$\begin{aligned} &\frac{\partial n_i^*}{\partial k_i} > 0 \\ &\frac{\partial n_i}{\partial k_i} > 0 \\ &\frac{\partial n_i^*}{\partial k_i} \leq \frac{\partial n_i}{\partial k_i} \end{aligned} \tag{115}$$

Since the set of effective suppliers is a subset of the set of potential suppliers, we have that the derivative of the former should be lower than or equal to the derivative of the latter. Then, taking the derivative of the definition of density with respect to the degree of information, we obtain:

$$\frac{\partial D_i(G)}{\partial k_i} = \frac{\frac{\partial n_i^*}{\partial k_i}}{\frac{\partial n_i}{\partial k_i}} \leq 1 \rightarrow \frac{D(G)}{\frac{\partial D(G)}{\partial k_i}} \leq D(G) \tag{116}$$

□

Proposition 6 -

Demonstração. Recovering the definition of the individual centrality measure, we have:

$$d_i(G) = \frac{n_i}{\sum_{i \in G} n_i} \tag{117}$$

Since the term $\sum_{i \in G} n_i$ gives the total number of suppliers that there exist in this network economy, the derivation of the centrality measure must be akin to the derivation of the set of potential suppliers of the firm i. This happens because the set of total suppliers already encloses all the possible suppliers, independent of the degree of information of a given firm. Then, using the relationship between the set of potential suppliers and information level as previously derived in this appendix, we obtain:

$$\frac{\partial d_i(G)}{\partial k_i} = \frac{\partial n_i}{\partial k_i} > 0 \tag{118}$$

□

Proposition 7

Demonstração. Recovering the definition of the stability of a network G, we have:

$$d_i^{\text{bet}}(G) = \sum_{\{j,k\} \neq i} \{\frac{2}{(N-1)(N-2)} \frac{1}{\{n_j | k \in n_a \ \ \forall a \in n_j\}}\}^{-1} \tag{119}$$

As a consequence of the previous proof, we have that whether the level of information k increases to k', such that $k < k'$, the set of potential suppliers of producer j should increase; thus, the set of potential suppliers under degree k is a subset of the set of potential suppliers under degree k', thus:

$$\{n_j | k \in n_a \ \ \forall a \in n_j\} \subset \{n_j | k' \in n_a \ \ \forall a \in n_j\} \tag{120}$$

Therefore, it implies that the definition of stability is increasing with respect to the degree of information, thus:

$$\frac{\partial d_i^{bet}(G)}{\partial k_i} \geq 0 \tag{121}$$

□

Proposition 9:

Demonstração. In this proposition, we prove that information-enhancing policies could be more efficient than a monetary transfer in terms of profit and welfare. In order to show that, we start by the profit function of a given firm i for both cases, where π_i^τ is the profit of a given firm that received a subsidy of size τ, while π_i is the provide in the case without subsidy. We assume that subsidy only increases the output of the firm, then $y_t^\tau = y_t + \tau$. Thus, we obtain equality for both types of profit functions in the case of an industry i if, and only if:[40]

$$\begin{aligned}
&\pi_i^\tau = \pi_i \iff \\
&p_w y_i^\tau - C_i^\tau = p_w y_i - C_i^{k+1} \iff \\
&(1+\mu_w) C_i^\tau y_i^\tau - C_i^\tau = (1+\mu_w) C_i^{k+1} y_{i+1} - C_i^{k+1}
\end{aligned} \tag{122}$$

Where C_i^τ is the cost of producer i when he receives subsidy τ, and C_i^{k+1} is the cost of producer i when he increases his information set from k to $k+1$ degrees of information. Since the subsidy does not distort the price and cost of the firm, hence $C_i^\tau = C_i^k$ and it only affects the output $y_i^\tau = y_i + \tau_i$, we obtain:

$$\begin{aligned}
&(1+\mu_w) C_i^k (y_i + \tau_i) = (1+\mu_w) C_i^{k+1} y_{i+1} \iff \\
&C_i^k (y_i + \tau_i) = C_i^{k+1} y_{i+1} \iff \\
&C_i^k y_i + C_i^k \tau_i = C_i^{k+1} y_i^{k+1} \iff \\
&C_i^k \tau_i = C_i^{k+1} y_i^{k+1} - C_i^k y_i \iff \\
&\tau_i = \frac{C_i^{k+1} y_i^{k+1} - C_i^k y_i}{C_i^k}
\end{aligned} \tag{123}$$

When we call $\Delta(C_i y_i) = C_i^{k+1} y_i^{k+1} - C_i^k y_i$ such as the deviation with respect to the increment of information and $C_i = C_i^k$ to avoid excessive notation, we obtain:

$$\pi_i^\tau = \pi_i \iff \tau_i = \frac{\Delta(C_i y_i)}{C_i} \tag{124}$$

Since $p_w = (1+\mu_w) C_i$, we can rewrite the subsidy equation as follows:

[40] Here, we are working with a final producer, although it could be a firm in any position of the network that the same result holds.

$$\pi_i^\tau = \pi_i \iff \tau_i = \frac{\Delta(p_w y_i)}{p_i} \tag{125}$$

Now, when we look to the statement about welfare, where $\tau < \Delta y_i$, we need to work with the concept of the informational wedge. Then, recovering this definition, we have:

$$w_k = y_n - y_k \tag{126}$$

Now, using the same construction for the proof of the previous inequality (the effect on the profit of the firms), we have the following two cases:

$$\begin{aligned} w_k &= y_n - y_n^{k+1} \\ w_k^\tau &= y_n - y_n^\tau \end{aligned} \tag{127}$$

Therefore, the informational wedge reduces more for an enhancing-policy in comparison with a monetary transfer if, and only if:

$$w_k - w_k^\tau < 0 \iff y_n - y_n^{k+1} < y_n - y_n^\tau y_n^{k+1} > y_n^\tau \iff y_n^{k+1} > y_n^k + \tau \iff \tau < y_n^{k+1} - y_n^k \iff \tau < \Delta y_i \tag{128}$$

Now, the last result of this proposition argues about the efficiency of allocations, which conceptually happens when public policies are able to quench wedges, which is why we have monetary transfer combined with the provision of information leading the difference between central planner output and network output under incomplete information to zero, such that $\tau_i^y + \tau_i^I = y_p - y_n^k$.

□

Proposition 10:

Demonstração. From the definition of the individual production function, we can aggregate and find the GDP of the network economy with k-levels of information as: $Y_i = \sum_{i \in G} y_i^k$. Then, suppose that all firms have 1-level of information in this economy, such that: $Y_i = \sum_{i \in G} y_i^1$. Now, suppose that through enhancing-information policy a given producer j receives a new level of information, while the remaining producers still have 1-level of information only; thus, $Y_i = \sum_{i \neq j \in G} y_i^1 + y_j^2$. Then, the effect of increasing information in

GDP is equivalent to:

$$\frac{\Delta Y_i}{Y_i} = \frac{\sum_{i\neq j\in G} y_i^1 + y_j^2 - \sum_{i\in G} y_i^1}{\sum_{i\in G} y_i^1} = \frac{\sum_{i\neq j\in G} y_i^1}{\sum_{i\in G} y_i^1} + \frac{y_j^2}{\sum_{i\in G} y_i^1} - 1 \tag{129}$$

By definition, the Domar-weight represents the sales share as a fraction of GDP for a given producer, and we have that Domar-weight is given by:

$$\lambda_{i,2} = \frac{y_j^2}{\sum_{i\in G} y_i^1} \tag{130}$$

Then, equation 129 can be rewritten as:

$$\frac{\Delta Y_i}{Y_i} = \frac{\sum_{i\neq j\in G} y_i^1}{\sum_{i\in G} y_i^1} + \lambda_{i,2} - 1 = \lambda_{i,2} - (1 - \frac{\sum_{i\neq j\in G} y_i^1}{\sum_{i\in G} y_i^1}) \tag{131}$$

Since $\lambda_{i,2} = \frac{y_j^2}{\sum_{i\in G} y_i^1} \rightarrow 1 - \lambda_{i,1} = \frac{\sum_{i\neq j\in G} y_i^1}{\sum_{i\in G} y_i^1}$. Thus, we have:

$$\frac{\Delta Y_i}{Y_i} = \lambda_{i,2} - (1 - (1 - \lambda_{i,1})) = \lambda_{i,2} - \lambda_{i,1} \tag{132}$$

Then, when we consider a sector I with different producers, we only need to aggregate for all the producers, such that:

$$\frac{\Delta Y_i}{Y_i} = \sum_{i\in I} \lambda_{i,2} - \lambda_{i,1} \tag{133}$$

Then, generalizing for a k-level of information as stated in the proposition is straightforward. □

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