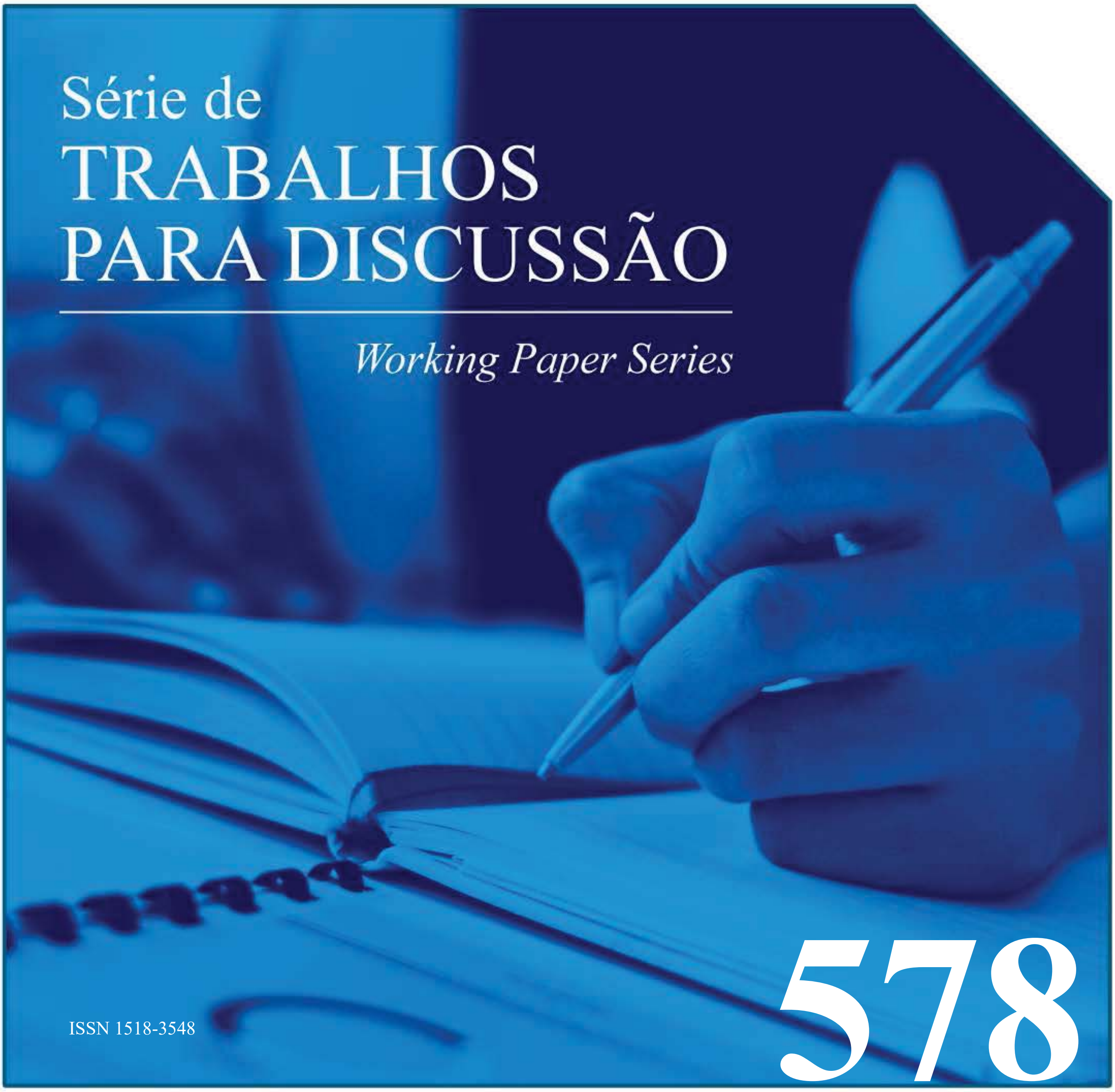

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Brazilian Macroeconomic Dynamics Redux: Shocks, Frictions, and Unemployment in SAMBA Model

Angelo Marsiglia Fasolo, Eurilton Araújo, Marcos Valli Jorge, Alexandre Kornelius, Leonardo Sousa Gomes Marinho

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Citizen Service Division

Banco Central do Brasil

Deati/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Toll Free: 0800 9792345

Fax: +55 (61) 3414-2553

Internet: http//www.bcb.gov.br/?CONTACTUS

Non-technical Summary

This paper describes extensions to the structure of the SAMBA model[1]. These additional developments include a reformulation of the labor-market block that allows for involuntary unemployment, the direct introduction of imported goods in the final consumption bundle, and a new specification for the rest-of-the-world block based on vector autoregressions (VAR). In this foreign block specification, shocks are identified by sign-restrictions and have a precise economic interpretation. These changes demand the inclusion of new observable variables and the use of a new method for estimation, which can cope with the increased number of parameters and equations. In this context, we estimate the model by Sequential Monte Carlo (SMC) Methods, leading to posterior distributions that are more stable to small perturbations of the specification of prior distributions.

The paper focuses on the dynamic properties of the model and evaluates its ability to generate moments that are close to the ones coming from the data. In addition, the paper documents the responses of important macroeconomic variables to a monetary policy shock, to a shock to the uncovered interest parity equation, and, finally, to the identified foreign shocks in the VAR specification. We compare the macro-variable responses with the ones reported in similar papers in the literature, providing economic intuition for our results. The final exercise is a historical decomposition of economic time series and concerns how the model interprets the evolution of macroeconomic variables as a combination of its structural shocks.

Summing up, the dynamic properties of the model seem reasonable and consistent economic narratives support the behavior of the impulse response functions and the results from the historical decompositions. Therefore, the model displays the essential preconditions to be employed in policy analysis and forecasting exercises, with the additional benefit of having structural features designed to analyze unemployment fluctuations in small-open economies.

[1]This model belongs to the class of Dynamic Stochastic General Equilibrium (DSGE) models and was developed by staff members at the Research Department of the Banco Central do Brasil. Indeed, SAMBA is an acronym for "Stochastic Analytical Model with a Bayesian Approach" and refers to the famous Brazilian music genre.

Sumário Não Técnico

O artigo descreve extensões feitas à estrutura do modelo SAMBA[2]. Essas mudanças incluem a reformulação do bloco que descreve o mercado de trabalho, permitindo o surgimento de desemprego involuntário, a introdução de bens importados na cesta de consumo final como também na função de produção como insumo intermediário, além de uma nova especificação para o bloco do setor externo, baseado em vetores autorregressivos (VAR). Nessa nova especificação, os choques são identificados a partir de restrições de sinais para as respostas ao impulso. Essa metodologia permite atribuir a esses choques uma interpretação econômica precisa. Por conta dessas mudanças, é preciso considerar novas variáveis observadas e um novo algoritmo no procedimento de estimação. Nesse contexto, empregamos o método de Monte Carlo Sequencial, pois este algoritmo consegue ter bons resultados para um número crescente de parâmetros e equações no novo modelo. Com efeito, as distribuições a posteriori são mais estáveis a qualquer perturbação inicial que altere as distribuições a priori.

O foco do artigo é o estudo das propriedades dinâmicas do modelo, avaliando a sua capacidade em reproduzir os momentos diretamente computados a partir dos dados. Adicionalmente, documenta-se o comportamento das funções de resposta ao impulso associadas às variáveis macroeconômicas relativamente aos seguintes choques: um choque de política monetéria, um choque à equação que descreve a paridade de juros e, por fim, aos choques externos identificados na estimação do VAR que caracteriza o setor externo. Um último exercício apresenta a decomposição histórica das variáveis macroeconômicas, ou seja, descreve como o modelo interpreta a trajetória dessas variáveis como combinação de seus choques estruturais.

Em resumo, os resultados sugerem que as propriedades dinâmicas do modelo são razoáveis. O comportamento das respostas ao impulso e o exercício de decomposição histórica podem ser racionalizados por narrativas econômicas coerentes. Desse modo, o modelo apresenta os requisitos básicos que o habilitam a ser empregado para responder questões de política econômica e em exercícios de previsão de variáveis macroeconômicas, com o benefício adicional de possuir novas estruturas que permitem uma análise mais fundamentada das flutuações no desemprego em pequenas economias abertas.

[2]O modelo pertence à família dos modelos dinâmicos e estocásticos de equilíbrio geral e foi desenvolvido por pesquisadores do Departamento de Estudos e Pesquisas do Banco Central do Brasil. O acrônimo SAMBA se refere, em inglês, ao termo *"Stochastic Analytical Model with a Bayesian Approach"*, além de ser uma alusão a um importante gênero musical brasileiro.

Brazilian Macroeconomic Dynamics Redux: Shocks, Frictions, and Unemployment in SAMBA Model

Angelo M Fasolo, Eurilton Araujo, Marcos Valli Jorge
Alexandre Kornelius, Leonardo Sousa Gomes Marinho[‡]

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Abstract

This paper documents the recent changes in the structure and estimation procedures of the SAMBA model, providing a complete description of the decision problems that each economic agent faces, the first order conditions that solve those problems, and the new techniques employed to estimate the model. This updated version of the model incorporates new features, such as involuntary unemployment, imported goods in the consumption bundle and a new identified vector auto-regressive process for the rest of the world. Reflecting these changes, the set of observables was expanded to include, for instance, participation rates in the labor market and an exogenous measure of output gap. In face of increased complexity and the large number of observables, the model was estimated using Sequential Monte Carlo (SMC) methods, allowing for a smaller sensitivity to the choice of priors.

JEL Codes: E24, E32, E52

Keywords: DSGE models; Small Open Economies; Monetary Policy; Unemployment

[‡]The authors would like to thank Carla Tito Fernandes for her support and suggestions on early versions of this paper. Corresponding Address: Banco Central do Brasil, Research Department. Setor Bancário Sul (SBS), Quadra 3, Bloco B, Edifício-Sede, Brasília-DF 70074-900, Brazil. Email: angelo.fasolo@bcb.gov.br. Dedicated to the memory of our dear colleague Leonardo Sousa Gomes Marinho.

1 Introduction

In recent years, many central banks have put continuing effort on developing and improving applied dynamic stochastic general equilibrium (DSGE) models as a tool for policy analysis and medium-term forecasting. For instance, in the European Central Bank, as in Coenen et al. (2018)[20], and in the Central Bank of Canada, as in Corrigan et al. (2021)[22], updated versions of their main DSGE models aiming at expanding the range of the economic issues analyzed and improving the models' ability to explain macroeconomic data. Banco Central do Brasil (BCB) has developed a DSGE model for the Brazilian economy: the SAMBA model[1]. This model, first documented in Castro et al. (2011, 2015)[17][18], is an open-economy DSGE model with a large set of nominal and real rigidities, such as wage and price stickiness, habit persistence in consumption, rule-of-thumb households and capital adjustment costs. The estimation used Bayesian techniques based on the Metropolis-Hastings (MH) algorithm, and the sample covered the period in which inflation targeting was adopted.

This paper outlines the description and main results of the SAMBA model, extended to address three notable features. First, to highlight the dynamics of exogenous shocks on labor market, the new specification incorporates unemployment following Galí, Smets and Wouters (2011)[28]. Their model reinterprets the staggered wage setting formulation in Erceg, Henderson and Levin (2000)[27], allowing for the emergence of involuntary unemployment while addressing the absence of economic microfoundations for unemployment in models with a conventional household specification. Second, in contrast to the original version in which only firms use imports as intermediate goods in production, households can also directly buy imported goods to compose their aggregate consumption bundle. The new structure allows for an effect of real exchange rate not only on the supply, but also on the demand of goods. Finally, the model replaces the exogenous univariate autoregressive processes characterizing the rest of the world with a structural vector autoregressive (VAR) model with sign-restricted identified shocks. The structural VAR allows for an amplification of the effects of foreign shocks in the domestic economy, alleviating the problems of small-open economy models documented in Justiniano and Preston (2010)[31].

The extended version of the model structure also affected the selection of observed variables and the estimation of the model. With respect to observed variables, the new description of the labor market allows for the inclusion of information about labor supply, measured by participation rates, while the structural VAR for the rest of the world allows for a stylized role for commodity prices to affect the domestic economy. The unbalanced growth observed in the Brazilian economy after 2014 also suggested the use of an exogenous measure of the output gap as an additional observed variable. This paper provides details on how the output gap is estimated, combining a structure like the traditional HP filter with the exogenous process of the DSGE model describing the evolution of non-stationary productivity.

[1]SAMBA is an acronym for "Stochastic Analytical Model with a Bayesian Approach" and refers to the famous Brazilian music genre.

With respect to estimation, the increased complexity of the model and the number of parameters to estimate, combined with the inclusion of more observed variables, required a new strategy to explore the posterior distribution of parameters. The conventional MH algorithm proved very sensitive to the choice of priors, while, at the same time, not properly handling discontinuities and multiple modes found in the posterior distribution. The paper will provide an example based on the distribution of parameters characterizing the cost of investment, and how it affects the inference about the effects of monetary policy shocks on investment. The model is estimated using Sequential Monte Carlo (SMC) methods, described in details in Cai et al. (2021)[14]. The use of bridge distributions, starting from a set of independent prior distributions for structural parameters, but now augmented with the so-called "system priors", to make inference about the posterior distribution brought stability across small changes in the prior distribution and allowed for a more complete assessment of the uncertainty with respect to important moments generated by the model.

The paper is organized as follows: the next section reviews the main structure of the model, with the primitive problems solved by households and firms, the characterization of monetary and fiscal policy, and the evolution of macroeconomic variables for the rest of the world; section 3 presents the changes in the estimation procedure, highlighting the main properties of the SMC methodology; and section 4 discusses some properties of the estimated model. Appendixes at the end summarize equations and present the computation of the steady state, data treatment, additional results and computational details.

2 The model

This section outlines the structure of the model that introduces unemployment and imported consumption goods into a standard small-open economy medium-scale DSGE framework for emerging economies. Because this extension of the original version of SAMBA retains most of its basic equations, the building blocks already reported in Castro et al. (2011, 2015)[17][18] are less discussed. Specifically, we focus on the mechanism that gives rise to unemployment and the intra-temporal choice between domestic and imported consumption, as well as on describing the new structural VAR process for the rest of the world.

The model comprises the following blocks: households, firms, monetary and fiscal policies, and a parsimonious description of the rest of the world. As in the original SAMBA, each component of aggregate demand corresponds to a specific sector in the economy. Monetary and fiscal policies are described as linear rules connecting a given policy instrument to the variables it responds to. Finally, the rest of the world corresponds to an identified structural VAR model, combining important foreign variables.

2.1 Households

There are two types of households in the economy: optimizing households (indexed by O) and "rule-of-thumb" households (indexed by RT, corresponding to ϖ_{RT} percentage of the population). A representative household of type O is a forward-looking agent that chooses optimal paths for consumption, savings, and investment. To smooth consumption over time, this agent uses three different assets as savings instruments: physical capital, non-contingent one-period government bonds and non-contingent one-period international bonds. By contrast, a representative household of type RT cannot access credit, capital and asset markets and does not earn dividends. This agent only spends the total amount of labor income on consumption goods.

Both types of households supply differentiated labor services. For optimizing households, monopoly unions set wages optimally according to a standard Calvo contract, while we assume RT households accept to receive the average wage of the type O households.

2.1.1 Optimizing Households

A representative optimizing household chooses consumption, physical capital, and financial assets to maximize the expected discounted flow of utility. The decision problem of a type O household is given by:

$$\max_{\{C_t, B_{t+1}, B^*_{t+1}, K_{t+1}, I_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^O, N_t^O(i))$$

The optimization problem is subject to the following flow budget constraint:

$$P_t^C C_t^O + P_t^I I_t + \frac{B_{t+1}}{S_t^B R_t} + \frac{S_t B^*_{t+1}}{R^*_t S_t^{B^*}} \leq$$
$$(1-\tau_t^N) W_t^{O,n}(i) N_t^O(i) + R_t^{K,n} K_t + B_t + S_t B_t^* + D_t^n - T_t^{Lump,n} - T_t^{D,n} + \Xi_t^n$$

where E_0 is the expectation operator as of time zero, $u(.)$ is the instantaneous utility function, $\beta \in (0,1)$ is the time discount factor, C_t^O is the consumption level, P_t^C is the price of consumption goods (used as a *numéraire* in the model), I_t is investment, P_t^I is the price of investment goods, B_t denotes government bonds, R_t is the domestic gross interest rate, S_t is the exchange rate (defined as units of domestic currency per units of foreign currency), B_t^* represents foreign-currency bonds issued abroad, R_t^* is the foreign interest rate, $W_t^{O,n}(i)$ is the household-specific nominal wage rate for the type of labor i, $N_t^O(i)$ is differentiated labor, τ_t^N is the tax rate on labor income, $R_t^{K,n}$ is the gross nominal rental rate of capital, K_t is physical capital, D_t^n denotes nominal dividends received from the firms, $T_t^{Lump,n}$ is lump-sum nominal net taxes, $T_t^{D,n}$ is a lump-sum transfer to firms and Ξ_t^n are nominal state-contingent securities. As a timing convention, B_t and B_t^* represent bonds issued in $t-1$ and maturing in t, and K_t are capital holdings from $t-1$. Thus, variables B_{t+1}, B^*_{t+1} and K_{t+1} are decided in t.

The model contains two different measures of risk premium, associated, respectively, with domestic and foreign issued bonds: S_t^B and $S_t^{B^*}$. While S_t^B is an exogenous AR(1) process, $S_t^{B^*}$ depends on aggregate variables describing both domestic and external fundamentals:

$$S_t^{B^*} = S^{B^*}\left[\exp\left(-\varphi_B^*\left(B_{t+1}^{*^x} - B^{*^x}\right) + \varphi_V^*\left(V_t^* - V\right) + Z_t^{B^*}\right)\right] \tag{1}$$

where S^{B^*} is the country risk premium in steady state, φ_B^* and φ_V^* are positive parameters, $B_{t+1}^{*^x}$ is net foreign assets-to-exports ratio[2], V_t^* represents foreign investor's attitude towards risk, and $Z_t^{B^*}$ is a shock associated with movements in the country risk related to exogenous factors. The variables B^{*^x} and V correspond to steady state values for $B_{t+1}^{*^x}$ and V_t^*. $B_{t+1}^{*^x}$ is the ratio between the nominal foreign debt in local currency and the nominal exports[3]. The net foreign assets as a proportion to exports, a new feature introduced in this updated version of the SAMBA model, is not common in DSGE models, but it is frequently considered in studies of debt sustainability. See, for instance, Reinhart et al. (2003)[37] and Kraay and Nehru (2006)[32]. In the context of the model, the country risk premium based on the net foreign assets as a proportion to exports, combined with the structural VAR characterizing the rest of the world, resulted in impulse response functions more consistent with the theory.

From Galí, Smets and Wouters (2011)[28], the optimizing representative household has a continuum of members indexed by $(i,j) \in [0,1]\times[0,1]$. Index i represents the type of labor service in which a given member of the household specializes, while the second index, j, characterizes the disutility from work for a particular member. Parameter η controls the intensity of this disutility. The time discount factor, the coefficient of relative risk aversion and the external habit persistence parameter are β, σ and κ, respectively. Also, there is full risk-sharing among members of the representative optimizing household, implying that $C_t^O(i,j) = C_t^O$. Define the expected discounted flow of utility for the optimizing household:

$$E_0\sum_{t=0}^{\infty}\beta^t Z_t^C\left[\frac{(C_t^O(i,j) - \kappa\overline{C}_{t-1}^O)^{1-\sigma}}{1-\sigma} - Z_t^{1-\sigma}1_t(i,j)Z_t^L\varphi_t^S\psi j^\eta\right] \tag{2}$$

In equation (2), the indicator function $1_t(i,j)$ equals one if the individual (i,j) is employed at time t, zero otherwise. There is a labor supply shock Z_t^L, a general preference shock Z_t^C and it is assumed that preferences over leisure fluctuate with the permanent technology shock Z_t, in order to keep the model consistent with balanced growth path.

Combined with the scale factor ψ, φ_t^S is an endogenous preference shifter, depending on aggregate consumption $\overline{C}_t^O$ for optimizing households and C_t^S, which one can interpret as a smooth trend for $\overline{C}_t^O$. This trend is designed to limit the size of the short-run wealth effect on labor supply according to the size of parameter v. Weakening the wealth effect is important to

[2]The presence of $B_{t+1}^{*^x}$ characterizes the "debt-elastic interest rate" described in Schmitt-Grohé and Uribe (2003)[39]. This specification is necessary to induce stationarity in small-open economy models.

[3]Defining $B_{t+1}^{*^y} = \left(S_t B_{t+1}^*\right)/\left(P_t^Y Y_t\right)$ as the net foreign assets-to-GDP ratio, in mathematical notation $B_{t+1}^{*^x} = \left(B_{t+1}^{*^y} Q_t^Y Y_t\right)/\left(Q_t Q_t^X X_t\right)$, where Q_t is the real exchange rate, Q_t^Y and Q_t^X are the relative prices of GDP deflator and exports with respect to the CPI, and Y_t and X_t are real GDP and exports.

ensure that employment and the labor force fluctuate procyclically as shocks disturb the economy. The limit on the size of the wealth effect on labor supply approximates the behavior of the utility function to a model with GHH preferences[4], with the advantage of allowing for a balanced growth path due to the presence of separable utility in consumption and labor. Variables φ_t^S and C_t^S evolve according to equations:

$$\varphi_t^S = \frac{C_t^S}{(\overline{C}_t^O - \kappa \overline{C}_{t-1}^O)^\sigma} \tag{3}$$

$$C_t^S = \left(C_{t-1}^S\right)^{1-v} \left[(\overline{C}_t^O - \kappa \overline{C}_{t-1}^O)^\sigma\right]^v$$

The mechanics of labor supply for the representative optimizing household goes as follows. Monopoly unions of each type of labor service i determine $W_t^{O,n}(i)$ to satisfy labor demand $N_t^O(i)$. There are members of the household specialized on type i labor, but they are heterogeneous in terms of disutilities, j, associated with providing that type of labor. The household follows a protocol that lines up workers of type i, in increasing order according to their disutilities j, to supply the employment level $N_t^O(i)$. The resulting employment level associated with workers of type i can be interpreted as labor at the extensive margin. According to this narrative, the aggregated version of equation (2), associated to a representative household, can be represented by:

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_t^C \left[\frac{(C_t^O - \kappa \overline{C}_{t-1}^O)^{1-\sigma}}{1-\sigma} - Z_t^{1-\sigma} Z_t^L \varphi_t^S \psi \int_0^1 \frac{N_t^O(i)^{1+\eta}}{1+\eta} di \right]$$

The last expression inside brackets is a consequence of the aggregation of disutility related to workers selected to satisfy employment level for type i service, $N_t^O(i)$, $\int_0^{N_t^O(i)} j^\eta dj = \frac{N_t^O(i)^{1+\eta}}{1+\eta}$. Then, the disutility is aggregated for a household across every type i service $\int_0^1 \frac{N_t^O(i)^{1+\eta}}{1+\eta} di = \int_0^1 \int_0^{N_t^O(i)} j^\eta dj di$.

A member of the optimizing household specialized on type i has an incentive to provide labor if the benefit of receiving the real wage for that work, which is $U_c(C_t^O)(1-\tau_t^N)W_t^{O,n}(i)/P_t^C$, where $W_t^{O,n}(i)/P_t^C$ is the real wage for work of type i and $U_c(C_t^O)$ is the marginal utility of consumption, compensates her disutility from work. Therefore, the following condition holds for a member with disutility index j willing to provide labor:

$$U_c(C_t^O)(1-\tau_t^N)\frac{W_t^{O,n}(i)}{P_t^C} \geq Z_t^C Z_t^{1-\sigma} Z_t^L \varphi_t^S \psi j^\eta$$

For the marginal supplier of type i labor, denoted by $L_t^O(i)$, the participation condition is:

[4]See Greenwood et al. (1988)[29]. GHH preferences are often used in models characterizing small open economies, like Schmitt-Grohé and Uribe (2003)[39], as they stress the interaction between capital accumulation and foreign bonds in the household's savings decision.

$$U_c(C_t^O)(1-\tau_t^N)\frac{W_t^{O,n}(i)}{P_t^C} = Z_t^C Z_t^{1-\sigma} Z_t^L \varphi_t^S \psi L_t^O(i)^\eta$$

Use equation (3) and the equilibrium condition $C_t^O = \overline{C}_t^O$ to obtain:

$$(1-\tau_t^N)\frac{W_t^{O,n}(i)}{P_t^C} = Z_t^{1-\sigma} Z_t^L \psi C_t^S L_t^O(i)^\eta$$

After transforming variables to render the model stationary and aggregating across labor type i, the final expression becomes:

$$(1-\tau_t^N)\tilde{W}_t^{O,n} = Z_t^L \psi \widetilde{C}_t^S (L_t^O)^\eta \tag{4}$$

2.1.2 Rule-of-Thumb Households and Labor Supply Aggregation

The RT representative household has the same preferences as a representative optimizing household and also has a continuum of members indexed by a pair $(i,j) \in [0,1] \times [0,1]$, representing the type of labor and the disutility from work for a particular member, respectively. Again, there is full risk-sharing among members of the representative rule-of-thumb household, thus $C^{RT}(i,j) = C_t^{RT}$.

This paper follows the original structure of the model in Castro et al. (2011, 2015)[17][18] and Medina and Soto (2007)[34], assuming that RT households follow the average wage set for optimizing households. Hence, for each labor type i in the representative RT household, we have $W_t^{RT,n}(i) = W_t^{O,n}$. Moreover, we postulate that the endogenous preference shifter follows $\varphi_t^{RS} = \frac{C_t^S}{(\overline{C}_t^{RT} - \kappa \overline{C}_{t-1}^{RT})^\sigma}$. By assumption, C_t^S depends only on aggregate consumption for optimizing households. Since the marginal supplier of type i compares the utility gains associated with the average real wage of the representative optimizing household with the disutility from work, in equilibrium, all labor types have the marginal supplier with the same index $j = L_t^{RT}$. These considerations lead to the equation below, after render the model stationary:

$$(1-\tau_t^N)\tilde{W}_t^{O,n} = Z_t^L \psi \widetilde{C}_t^S (L_t^{RT})^\eta \tag{5}$$

Comparing equations (4) and (5), we have $L_t^O = L_t^{RT}$. Thus, one of the consequences of these simplifying assumptions under the context of a Galí, Smets and Wouters (2011)[28] labor market is that RT households are strict followers of optimizer households, including labor supply decisions. On the other hand, in labor market negotiations, optimizer households do take into account the changes in labor supply originated from the marginal RT household. As a result, the expression for the consumption of a representative RT household is given by:

$$C_t^{RT} = (1-\tau_t^N)\frac{W_t^{O,n}}{P_t^C} N_t^{RT}$$

For each type of household, in equilibrium, we have the following aggregation:

$$\overline{C}_t^O = \int_{\varpi_{RT}}^1 \frac{1}{1-\varpi_{RT}} C_t^O ds = C_t^O$$

$$\overline{C}_t^{RT} = \int_0^{\varpi_{RT}} \frac{1}{\varpi_{RT}} C_t^{RT} ds = C_t^{RT}$$

Moreover, aggregate consumption C_t follows the equation:

$$C_t = (1-\varpi_{RT})C_t^O + \varpi_{RT} C_t^{RT} \tag{6}$$

Concerning aggregate labor supply, a household-specific employment agency uses a Dixit-Stiglitz function to aggregate differentiated labor services provided by a specific household type defined by the ability to access asset markets. In effect, assume there is a continuum of uniformly distributed employment agencies. This setting results in the following demands for labor type i for each household, mapping N_t^O to each optimizing household in the interval $[\varpi_{RT}, 1]$ and N_t^{RT} for all RT agents in the interval $[0, \varpi_{RT}]$:

$$N_t^O(i) = \left(\frac{W_t^{O,n}(i)}{W_t^{O,n}}\right)^{-\epsilon^W} N_t^O, \quad \forall i \in [0,1] \tag{7}$$

$$N_t^{RT}(i) = \left(\frac{W_t^{RT,n}(i)}{W_t^{RT,n}}\right)^{-\epsilon^W} N_t^{RT}, \quad \forall i \in [0,1] \tag{8}$$

Indeed, each optimizing household in the interval $[\varpi_{RT}, 1]$ has its own employment agency, which produces N_t^O. Analogously, each rule-of-thumb household in the interval $[0, \varpi_{RT}]$ also has its specific employment agency that generates as output N_t^{RT}. These agencies, in the interval $[0, 1]$, then supply N_t^O and N_t^{RT} to a third employment agency in a competitive market to produce an aggregate labor N_t according to the following technology:

$$N_t = Min\left\{\int_0^{\varpi_{RT}} \frac{1}{\varpi_{RT}} N_t^{RT} ds, \int_{\varpi_{RT}}^1 \frac{1}{1-\varpi_{RT}} N_t^O ds\right\}$$

where the index s refers to a given household-specific agency in the unit interval.

Since optimizing households are identical, their agencies produce the same amount of labor. This reasoning also applies to rule-of-thumb households. The first integral emerges because each employment agency for optimizing household is uniformly distributed and produces the same amount of labor N_t^O. For the same reason, the second integral is associated with specific agencies producing the amount of labor N_t^{RT} from rule-of-thumb households. Each integral can be interpreted as the mean labor for a given type of household defined by the ability to access asset markets.

Because agencies within the two classes of households produce the same amount of labor, these means collapse to these two values. The Leontief technology indicates that the types of labor are complementary in fixed proportions to produce the aggregate labor used as input in the first stage of production. The specification of a third employment agency is a necessary step because, in contrast to the original version of the model that assumes a single employment agency that acts on behalf of all households, we consider a continuum of employment agencies for optimizers and another continuum set for the rule-of-thumb households.

The optimal input choice implies $N_t = N_t^O = N_t^{RT}$. Unemployment is defined as:

$$U_t^L = \frac{L_t - N_t}{L_t} \tag{9}$$

As the technology exhibits constant returns to scale, the aggregate wage compatible with strictly positive aggregate labor supply leads to zero profits for the employment agency. Hence:

$$W_t^n = \int_0^{\varpi_{RT}} W_t^{RT,n} ds + \int_{\varpi_{RT}}^1 W_t^{O,n} ds = \varpi_{RT} W_t^{RT,n} + (1 - \varpi_{RT}) W_t^{O,n}$$

According to the wage-setting rule for RT households, $W_t^{RT,n} = W_t^{O,n}$. As a consequence, $W_t^n = W_t^{RT,n} = W_t^{O,n}$.

2.1.3 Nominal Wage Setting for Optimizing Households

Monopoly unions representing optimizing households can sign forward-looking wage contracts with the household-specific agency for those who can access asset markets. A representative optimizing household faces a Calvo lottery for wages. In each period and with probability $1-\theta_W$, on behalf of a given optimizing household, the monopoly union may renegotiate its nominal wage contract with the employment agency, in which case the optimal wage chosen is $W_t^{O,n,\star}$. With probability θ_W, the household's union cannot optimally change its nominal wage, but update the wage contract according to the indexation rule:

$$W_t^{O,n}(i) = \Upsilon_{t-1}^W W_{t-1}^n(i)$$

where Υ_t^W is a weighted geometric average of wage inflation, CPI inflation, and labor productivity growth:

$$\Upsilon_t^W = \left(\Pi_{t-1}^W\right)^{\omega_W} \left(\left(Z_{t-1}^{ZC}\right)^4 \Pi_{t-1}^{4C}\right)^{\frac{1-\omega_W}{4}} \left(\frac{1}{Z_t^Z}\right)$$

where $\omega_W \in [0,1]$ is an indexation parameter, $\Pi_t^W \equiv W_t^n / W_{t-1}^n$ is the gross nominal wage inflation, $Z_t^Z = Z_t / Z_{t-1}$ is the stochastic and time-varying gross growth rate of permanent technology shocks, and Z_t^{ZC} is the cyclical component of Z_t^Z, as described in section 2.2. In this expression for Υ_t^W, we have $\Pi_t^{4C} = \Pi_t^C \Pi_{t-1}^C \Pi_{t-2}^C \Pi_{t-3}^C$.

Because $W_t^{O,n} = W_t^n$, we write the indexation rule and wage inflation for a representative optimizing household as functions of the aggregate wage W_t^n. Notation is simplified by writing

$W_t^{O,n}(i) = W_t^n(i)$ and $W_t^{O,n,\star} = W_t^{n,\star}$. Finally, setting $N_t^O = N_t$ reflects the optimal choice of inputs for the aggregate employment agency.

On behalf of a representative optimizing household that provides labor services of type i, the monopoly union can optimally choose its wage, gauging changes in the disutility of labor relative to changes in the real labor income. The optimal wage-setting problem is the following:

$$\max_{W_t^n(i)} E_t \sum_{h=0}^{\infty} (\theta_W \beta)^j \left\{ -Z_{t+h}^C Z_{t+h}^{1-\sigma} Z_{t+j}^L \varphi_{t+h}^S \psi \int_0^1 \frac{N_{t+h}^O(i)^{1+\eta}}{1+\eta} di + \Lambda_{t+h} \left[(1-\tau_t^N) \frac{W_t^n(i)}{P_{t+h}^C} N_{t+h}(i) \right] \right\}$$

subject to: $N_t^O(i) = \left(\frac{W_t^n(i)}{W_t^n} \right)^{-\epsilon^W} N_t$, given the simplified notation for wages in equation (7). In the problem, Λ_{t+h} stands for the Lagrange multiplier from the consumer's problem of a representative optimizing household.

Since each union solves the same wage-setting problem for a specific type of labor service i provided by an optimizing household, there is a $W_t^{n,\star}$ unique for all optimizing households in the interval $[\varpi_{RT}, 1]$. Moreover, $W_t^{n,\star}$ is the same across types of labor services i, because the first-order condition for the wage-setting problem associated with the variety i is such that the optimal wage depends only on aggregate variables.

In short, given W_t^n, $W_t^{n,\star}$, the indexation rule and the wage index derived from the Dixit-Stiglitz aggregator of labor, the first-order condition regarding the optimal wage choice leads to a traditional wage Phillips curve describing the dynamic evolution of wage inflation Π_t^W.

2.1.4 Demand for Imported Goods and Consumer Price Index

In a departure from the original version of the model, households are allowed to buy imported goods directly from the rest of the world. In the previous version of the model, imported goods only mattered for price setting as an input to produce the final good. A consequence of this assumption is the crucial role of real exchange rate level in explaining domestic inflation, as the level of the real exchange rate sets the level of marginal costs in the Phillips curve. With the possibility of replacing domestically produced with imported goods, changes in real exchange rates also plays a role in domestic inflation for consumption goods.

Formally, total consumption C_t^i, $i = \{O, RT\}$, combines domestic consumption $(C_t^{D,i})$ with imported consumption $(C_t^{M,i})$ according to a CES (constant elasticity of substitution) aggregator, with parameters o_C and v_C, the elasticity of substitution, describing how households bundle domestic and imported consumption:

$$C_t^i = \left[(o_C)^{\frac{1}{v_C}} \left(C_t^{D,i} \right)^{\frac{v_C-1}{v_C}} + (1-o_C)^{\frac{1}{v_C}} \left(C_t^{M,i} \right)^{\frac{v_C-1}{v_C}} \right]^{\frac{v_C}{v_C-1}}$$

Prices for these goods are $P_t^{C^D}$ and P_t^M, respectively. Given C_t^i, the choice of $C_t^{D,i}$ and $C_t^{M,i}$ results from the minimization of total expenditure $P_t^{C^D} C_t^{D,i} + P_t^M C_t^{M,i}$ subject to the

CES aggregator constraint. Hence, the first order conditions are:

$$C_t^{D,i} = o_C \left(Q_t^{C^D}\right)^{-v_C} C_t^i$$
$$C_t^{M,i} = (1 - o_C) \left(Q_t^M\right)^{-v_C} C_t^i$$

where $Q_t^{C^D} = P_t^{C^D}/P_t^C$ and $Q_t^M = P_t^M/P_t^C$ are relative prices. Variable P_t^C defines the consumer price index (CPI), given by the following expression:

$$P_t^C = \left[o_C(P_t^{C^D})^{1-v_C} + (1 - o_C)(P_t^M)^{1-v_C}\right]^{\frac{1}{1-v_C}}$$

Using the relative prices defined above, the CPI inflation rate (Π_t^C) is:

$$\Pi_t^C = \left[o_C(\Pi_t^{C^D} Q_{t-1}^{C^D})^{1-v_C} + (1 - o_C)(\Pi_t^M Q_{t-1}^M)^{1-v_C}\right]^{\frac{1}{1-v_C}} \tag{10}$$

where $\Pi_t^C = P_t^C/P_{t-1}^C$, $\Pi_t^{C^D} = P_t^{C^D}/P_{t-1}^{C^D}$ and $\Pi_t^M = P_t^M/P_{t-1}^M$. Notice that the change in real exchange rates is important in setting the inflation of imported goods measured in domestic currency. The presence of nominal rigidities in price-setting of imported goods avoids an immediate complete pass-through from changes in real exchange rate to CPI inflation, as it will be discussed in section 2.2.2.

2.2 Firms

The production of sectoral goods comprises three stages. The first stage involves a domestic input producer and importers. The domestic producer blends capital and labor services in its production process. Importers purchase differentiated raw materials from the rest of the world to sell to an import-specific assembler. Next, this assembler bundles the differentiated commodities into a homogeneous imported good. The output of the assembler and the domestic input producer is used as intermediate inputs in the next stage.

In the second stage, a continuum of intermediate good producers converts imported and domestic inputs into sectoral differentiated goods. These intermediate good producers require the use of financial services since they finance a fraction of their imported raw material with working capital borrowed abroad.

Finally, in the third stage, sectoral assemblers transform the differentiated goods into four sectoral homogeneous goods. The first three sectoral goods are non-tradable goods aimed at the domestic market, corresponding to private consumption, government consumption and investment. The last good is a tradable commodity that is exported to the rest of the world.

2.2.1 First Stage: Domestic Input Producers

A representative domestic producer supplies an input, Y_t^D, to sectoral intermediate good producers. This firm operates a constant returns-to-scale technology, based in a CES technology, blending capital and labor services in its production process in a perfectly competitive market:

$$Y_t^D = Z_t^D \left[\alpha K_t^{\frac{\epsilon_D - 1}{\epsilon_D}} + (1-\alpha)\left(Z_t \left(N_t - \overline{N} \right) \right)^{\frac{\epsilon_D - 1}{\epsilon_D}} \right]^{\frac{\epsilon_D}{\epsilon_D - 1}}, \ \epsilon_D > 0 \tag{11}$$

where K_t is physical capital, N_t is the total labor input, $\overline{N}$ is overhead labor, which we assume constant over time. Parameters α and ϵ_D, the elasticity of substitution, describe this technology. In the context of a Cobb-Douglas production function, Castro et al. (2011)[17] show the presence of overhead labor reduces the coefficient relating labor demand and production of the domestic input. Overhead labor generates a convexity in the production function similar, when the model is linearized, to the assumption of a fixed cost in production, as described in Rotemberg and Woodford (1999)[38].

There are two technology shocks in the production function. Variable Z_t^D is a domestic *transitory* technology shock, and Z_t is a labor-augmenting stochastic trend, representing *permanent* shifts in technology. The temporary technology shock evolves according to a first-order autoregressive process:

$$\log\left(Z_t^D\right) = \rho_D \log\left(Z_{t-1}^D\right) + \varepsilon_t^D \tag{12}$$

The evolution of the stochastic trend follows two components, trying to accommodate the different degrees of wealth effects generated when changes in the level of labor productivity can be at least partially anticipated. Similar decompositions of labor productivity are adopted in the recent version of the DSGE model of the Federal Reserve of New York and the NAWN II model of the European Central Bank[5]. To be clear, both components alter the labor productivity *level*, but the evolution of the *growth rate* of productivity is different in each of the two cases:

$$\log\left(\frac{Z_t}{Z_{t-1}}\right) = \log(Z_t^Z) = \log Z_{ss}^Z + \log(Z_t^{ZC}) + \log(Z_t^{ZT}) \tag{13}$$

$$\log\left(\frac{Z_t^{ZC}}{Z_{ss}^Z}\right) = \rho_Z \log\left(\frac{Z_{t-1}^{ZC}}{Z_{ss}^Z}\right) + \varepsilon_t^{ZC} \tag{14}$$

$$\log(Z_t^{ZT}) = \varepsilon_t^{ZT} \tag{15}$$

In the equations above, Z_{ss}^Z is the steady state for the long-run growth rate of technology, measured in gross terms, Z_t^{ZC} is the partially anticipated, or cyclical, component of the stochastic trend, and Z_t^{ZT} is the unanticipated, or transitory, movement of the stochastic trend.

The domestic input producer takes input prices as given and chooses capital and labor services

[5]See the FRBNY DSGE Model Documentation, version 1002 (December, 2021)[1] and Coenen et al. (2018)[20]. The model in Coenen et al. (2018)[20] follows closely the solution in Edge, Laubach and Williams (2007)[26], assuming that agents follow a Kalman Filter process of learning about the evolution of productivity in the model.

to minimize total input costs, subject to the technology constraint (11):

$$\min_{K_{t-1},N_t} \left\{ \begin{array}{c} R_t^{K,n} K_{t-1} + W_t^n N_t - T_t^{D,n} \\ +P_t^D \left\{ Y_t^D - Z_t^D \left[\alpha K_{t-1}^{\frac{\epsilon_D - 1}{\epsilon_D}} + (1-\alpha) \left(Z_t \left(N_t - \overline{N} \right) \right)^{\frac{\epsilon_D - 1}{\epsilon_D}} \right]^{\frac{\epsilon_D}{\epsilon_D - 1}} \right\} \end{array} \right\}$$

where $T_t^{D,n}$ is a lump-sum transfer from optimizing households which is equal to the overhead labor cost. The transfer ensures a well-defined solution to the optimization problem above. P_t^D is the Lagrange multiplier associated with the technology constraint, which is equal to the nominal marginal cost of changing the use of the domestic inputs. Cost minimization yields the demands for capital and labor. Substituting equations defining factors demands into the technology constraint (11) results in an expression for the marginal cost as function of factor prices for capital and labor.

2.2.2 First Stage: Importers

Consistent with the evidence on deviations of the Law of One Price, the model has nominal rigidities expressed in terms of local currency pricing of imported and exported goods. The inclusion of local currency pricing helps matching relevant moments of real exchange rates and foreign prices. In fact, the literature has shown nominal rigidities are responsible for a significant share of exchange rate fluctuations. In Adolfson et al. (2007)[4], foreign shocks, including those in export and import markups, explain almost all exchange rate fluctuations in Europe. Furthermore, the model with correlated import and export markup shocks has a significantly better match with unconditional second moments of the real exchange rate, compared with a version of the model with iid shocks. In this subsection, the problem of importing firms is detailed, with the problem of exporting firms presented later.

A continuum of importing firms indexed by $j \in [0,1]$ choose first the amount of imported varieties they need to produce a homogeneous good sold in a monopolistically competitive market to imports assembler in the second stage. Importing firms set their prices according to a Calvo scheme. The homogeneous good produced by the assembler is used as an input to produce sectoral differentiated goods. The assembler converts the differentiated inputs into a homogenous good, bundling the differentiated varieties according to a Dixit-Stiglitz aggregator:

$$M_t = \left(\int_0^1 \left(M_{j,t} \right)^{\frac{\epsilon_M - 1}{\epsilon_M}} dj \right)^{\frac{\epsilon_M}{\epsilon_M - 1}}, \ \epsilon_M > 1 \qquad (16)$$

where M_t is the homogenous imported good, $M_{j,t}$ is the imported variety j, and ϵ_M is the elasticity of substitution across varieties. $P_{j,t}^M$ is the *local-currency* price charged by importing firm j and P_t^M is the *local-currency* aggregate import price. The choice problem of the import goods assembler, subject to equation (16), yields the following demand for the jth imported commodity and the following import price index:

$$M_{j,t} = \left(\frac{P^M_{j,t}}{P^M_t} \right)^{-\epsilon_M} M_t \tag{17}$$

$$P^M_t = \left(\int_0^1 \left(P^M_{j,t} \right)^{1-\epsilon_M} dj \right)^{\frac{1}{1-\epsilon_M}} \tag{18}$$

Importing firms are price makers in domestic markets and operate under price rigidity according to the Calvo scheme. Define the sectoral inflation $\Pi^M_t = P^M_t / P^M_{t-1}$. In each period, an importing firm j faces a constant probability $1 - \theta_M$ of choosing its prices according to current market conditions. On the other hand, with probability θ_M, it updates the price according to past sectoral inflation:

$$P^M_{j,t} = \Upsilon^M_t P^M_{j,t-1} \tag{19}$$

$$\Upsilon^M_t = \Pi^M_{t-1} \tag{20}$$

Define $P^{M^*}_t$ as the *foreign-currency* price of imported varieties. The value of profits accrued to firm j is given by:

$$P^M_{j,t} M_{j,t} - \left(S_t P^{M^*}_t \right) M_{j,t}$$

At period t, importing firms allowed to optimally set prices maximize the real value of the expected discounted flow of their profits along the paths over which their own price is not adjusted optimally. From the definition of the value of profits and the demand constraint (17), the firm solves:

$$\max_{P^M_{j,t}} E_t \sum_{i=0}^{\infty} \left(\theta_M \beta \right)^i \Lambda_{t,t+i} \left(\left(\frac{\Upsilon^M_{t,t+i} P^M_{j,t}}{P^M_{t+i}} \right)^{1-\epsilon_M} P^M_{t+i} - \left(\frac{\Upsilon^M_{t,t+i} P^M_{j,t}}{P^M_{t+i}} \right)^{-\epsilon_M} S_{t+i} P^{M^*}_{t+i} \right) M_{t+i}$$

Since all firms able to change their prices face the same costs, they choose the same optimal price. The first-order condition implies the following expression for optimal price $P^{M^\star}_t$:

$$P^{M^\star}_t = \frac{\epsilon_M}{\epsilon_M - 1} \frac{E_t \sum_{i=0}^{\infty} \left(\theta_M \beta \right)^i \Lambda_{t,t+i} \left(\frac{\Upsilon^M_{t,t+i}}{P^M_{t+i}} \right)^{-\epsilon_M} S_{t+i} P^{M^*}_{t+i} M_{t+i}}{E_t \sum_{i=0}^{\infty} \left(\theta_M \beta \right)^i \Lambda_{t,t+i} \left(\frac{\Upsilon^M_{t,t+i}}{P^M_{t+i}} \right)^{1-\epsilon_M} P^M_{t+i} M_{t+i}} \tag{21}$$

According to this expression, the firm sets its optimal price as the ratio of their expected discounted sum of marginal cost to their expected discounted sum of marginal revenues. In the absence of nominal rigidity, the optimal price is the usual markup over the marginal cost, with the markup being $\epsilon_M / (\epsilon_M - 1)$. From expression (18), the following equation characterizes the imported good price index:

$$P_t^M = \left[\theta_M \left(\Upsilon_t^M P_{t-1}^M\right)^{1-\epsilon_M} + (1-\theta_M)\left(P_t^{M^\star}\right)^{1-\epsilon_M}\right]^{\frac{1}{1-\epsilon_M}} \tag{22}$$

One can write the expression above in terms of relative prices $Q_t^M = P_t^M/P_t^C$ and $Q_t^{M^\star} = P_t^{M^\star}/P_t^C$, where P_t^C is the consumer price index (CPI) defined in the household problem.

$$Q_t^M = \left[\theta_M \left(\Upsilon_t^M \frac{Q_{t-1}^M}{\Pi_t^C}\right)^{1-\epsilon_M} + (1-\theta_M)\left(Q_t^{M^\star}\right)^{1-\epsilon_M}\right]^{\frac{1}{1-\epsilon_M}} \tag{23}$$

Lastly, the gross inflation rate of imported goods can also be expressed in terms of relative prices:

$$\Pi_t^M = \frac{P_t^M}{P_{t-1}^M} = \frac{Q_t^M}{Q_{t-1}^M}\Pi_t^C \tag{24}$$

Conditions (19)-(24) lead to the formulation of a New Keynesian Phillips curve describing the evolution of Π_t^M.

2.2.3 Second Stage: Sectoral Intermediate Producers

In the second stage, four sectors in the economy are specified, based on the components of the domestic demand of goods: domestic private consumption (C^D), government consumption (G), investment (I) and exports (X). Each sector is characterized by a particular price-setting behavior, based on variations of the Calvo setup. In each sector, there is a continuum of intermediate good producers indexed by $j \in [0,1]$. Each intermediate good producer operates a constant returns-to-scale technology that converts imported and domestic inputs into sectoral differentiated goods. Moreover, intermediate good producers require the use of financial services since they must finance a fraction of their imported input with working capital borrowed abroad.

Intermediate good producers combine the domestic input with imported goods according to a CES production function to manufacture differentiated intermediate goods that sectoral assemblers use in the third stage:

$$Y_{j,t}^H = \left((\varpi_H)^{\frac{1}{\epsilon_H}}\left(Y_{H,j,t}^D\right)^{\frac{\epsilon_H-1}{\epsilon_H}} + (1-\varpi_H)^{\frac{1}{\epsilon_H}}\left[\left(1-\Gamma_H^M\left(\frac{M_{j,t}^H}{M_{t-1}^H}\right)\right)M_{j,t}^H\right]^{\frac{\epsilon_H-1}{\epsilon_H}}\right)^{\frac{\epsilon_H}{\epsilon_H-1}} \tag{25}$$

where $H = \{C^D, G, I, X\}$, $M_{j,t}^H$ is the imported good, $\Gamma_H^M(.)$ is an adjustment associated with the imported input as a function of the sectoral level of imports, $M_{j,t}^H/M_{t-1}^H$, $\varpi_H \in [0,1]$ is the weight of the domestic input in the production, and $\epsilon_H > 0$ is the elasticity of substitution between the two inputs. The following functional form for the adjustment cost satisfies in the steady state $\Gamma^{M\prime}_{H,t} = \Gamma^{M}_{H} = 0$:

$$\Gamma^M_H\left(\frac{M^H_{j,t}}{M^H_{t-1}}\right) = \frac{\vartheta^M_H}{2}\left(\left(Z^M_{H,t}\right)^{-\frac{1}{\vartheta^M_H}}\frac{M^H_{j,t}}{M^H_{t-1}} - 1\right)^2$$

where $\vartheta^M_H > 0$ and $Z^M_{H,t}$ is an import demand shock.

Each sectoral intermediate good producer takes as given input prices and decides the combination of inputs that minimizes its total cost:

$$\min_{Y^D_{j,t}, M^H_{j,t}} \left\{P^D_t Y^D_{H,j,t} + P^M_t\left[1 + \iota_H\left(R^*_t S^{B^*}_t - 1\right)\right]M^H_{j,t}\right\}, \quad \forall j \in H$$

subject to the technology constraint (25). Parameter ι_H defines the fraction of the imported good that must be financed abroad, by signing intra-period loans contracts at the net interest rate $R^*_t S^{B^*}_t - 1$. The working capital constraint captures some of the observed trade credit frictions in small-open economies.

Define $MC^{H,n}_{j,t}$ as the Lagrange multiplier (in nominal terms) related to the technology constraint and let $P^M_{H,t} \equiv \left[1 + \iota_H\left(R^*_t S^{B^*}_t - 1\right)\right]P^M_t$ be the effective import cost in sector H and $\Gamma^{M\dagger}_H\left(M^H_{j,t}/M^H_{t-1}\right) = \Gamma^{M\prime}_H\left(M^H_{j,t}/M^H_{t-1}\right)M^H_{j,t}$. Given that all firms face the same technology, same input prices and have adjustment costs that do not depend on history – thus, in equilibrium $M^H_{j,t}/M^H_{t-1}$ is the same across firms – and same real marginal cost, the first-order conditions describing the optimal choices of inputs $Y^D_{j,t}$ and $M^H_{j,t}$ result in the following two conditions:

$$Y^D_{H,j,t} = \varpi_H\left(\frac{P^D_t}{MC^H_t}\right)^{-\epsilon_H} Y^H_{j,t} \tag{26}$$

$$M^H_{j,t} = \left(\frac{1-\varpi_H}{1-\Gamma^M_{H,t}}\right)\left(\frac{P^M_{H,t}}{\left(1-\Gamma^M_{H,t}-\Gamma^{M\dagger}_{H,t}\right)MC^{H,n}_t}\right)^{-\epsilon_H} Y^H_{j,t} \tag{27}$$

After substituting the optimal input-output ratio into the technology constraint, the following equation describes the nominal marginal cost in sector H:

$$MC^{H,n}_t = \left(\varpi_H\left(P^D_t\right)^{1-\epsilon_H} + (1-\varpi_H)\left(\frac{P^M_{H,t}}{1-\Gamma^M_{H,t}-\Gamma^{M\dagger}_{H,t}}\right)^{1-\epsilon_H}\right)^{\frac{1}{1-\epsilon_H}} \tag{28}$$

which is a weighted average of the cost of the domestic and imported inputs. For the purpose of the estimated model, we assume the production of government goods does not use imported inputs, i.e., $\varpi_G = 1$. Hence, government consumption is a non-tradable good whose real marginal cost of production is equal to P^D_t, that is, $MC^{G,n}_t = P^D_t$.

2.2.4 Second Stage: Domestic Consumption Sector

Two types of intermediate producers compose the domestic consumption good sector. Firms use the same technology, face the same cost minimization problem and take into account similar downward-sloping demand curves for their products. However, they differ regarding their pricing strategies: the set of *administered prices* identifies firms that are less sensitive to demand and supply conditions or prices that are in some way regulated by public agencies. A firm indexed by $j \in [0, \varpi_A]$ is not free to choose its own price and must follow an exogenous pricing rule, specified by the government. The letter A denotes the set of firms that have their prices monitored or administered by the government, while the letter F denotes the set of *freely-set price* firms, where $A \cup F = C^D$.

In this respect, a fraction $1 - \varpi_A$ of firms in the domestic consumption good sector choose their prices according to the Calvo setup: whenever possible, these firms set their prices freely, responding to current market conditions. In each period, a firm $j \in F$ faces a constant probability θ_F of adjusting its price according to the indexation rule:

$$P^F_{j,t} = \Upsilon^F_t P^F_{j,t-1}, \quad \forall j \in F \tag{29}$$

$$\Upsilon^F_t = \Pi^C_{t-1} \tag{30}$$

The expression for Υ^F_t includes the overall CPI inflation (Π^C_t) with a lag, instead of the freely-set price inflation. In this way, as usual in private contracts, the firms that update prices take into account overall inflation in the economy. On the other hand, if the firm is allowed to choose its prices optimally, it solves the following price optimization problem:

$$\max_{P^F_{j,t}} E_t \sum_{i=0}^{\infty} (\theta_F \beta)^i \Lambda_{t,t+i} \left(\Upsilon^F_{t,t+i} P^F_{j,t} - MC^{C^D,n}_{t+i} \right) Y^F_{j,t+i}, \quad \forall\, j \in F$$

subject to the demand faced by the firm, given by:

$$Y^F_{j,t+i} = \left(\frac{\Upsilon^F_{t,t+i} P^F_{j,t}}{P^{C^D}_{t+i}} \right)^{-\epsilon^P_{C^D}} Y^{C^D}_{t+i}, \forall i \geq 0$$

The variable $MC^{C^D,n}_{t+i}$ denotes the nominal marginal cost incurred in the production of the domestic consumption good is similar to (28). After substituting the demand constraint into the objective function, the optimal freely-set price $P^{F^\star}_t$ is determined by the pricing condition:

$$P^{F^\star}_t = \frac{\epsilon^P_{C^D}}{\epsilon^P_{C^D} - 1} \frac{E_t \sum_{i=0}^{\infty} (\theta_F \beta)^i \Lambda_{t,t+i} \left(\frac{\Upsilon^F_{t,t+i}}{P^{C^D}_{t+i}} \right)^{-\epsilon^P_{C^D}} MC^{C^D,n}_{t+i} Y^{C^D}_{t+i}}{E_t \sum_{i=0}^{\infty} (\theta_F \beta)^i \Lambda_{t,t+i} \left(\frac{\Upsilon^F_{t,t+i}}{P^{C^D}_{t+i}} \right)^{1-\epsilon^P_{C^D}} P^{C^D}_{t+i} Y^{C^D}_{t+i}} \tag{31}$$

We define the freely-set price index as follows:

$$P_t^F \equiv \left(\frac{1}{1-\varpi_A} \int_{j\in F} (P_{j,t}^{C^D})^{1-\epsilon_{CD}^P} dj \right)^{\frac{1}{1-\epsilon_{CD}^P}}$$
$$= \left(\theta_F \left(\Upsilon_t^F P_{t-1}^F \right)^{1-\epsilon_{CD}^P} + (1-\theta_F) \left(P_t^{F^\star} \right)^{1-\epsilon_{CD}^P} \right)^{\frac{1}{1-\epsilon_{CD}^P}} \tag{32}$$

One can write the expression above in terms of relative prices $Q_t^F = P_t^F/P_t^C$ and $Q_t^{F^\star} = P_t^{F^\star}/P_t^C$, where P_t^C is the consumer price index (CPI) defined in the household problem:

$$Q_t^F = \left[\theta_F \left(\Upsilon_t^F \frac{Q_{t-1}^F}{\Pi_t^C} \right)^{1-\epsilon_{CD}^P} + (1-\theta_F) \left(Q_t^{F^\star} \right)^{1-\epsilon_{CD}^P} \right]^{\frac{1}{1-\epsilon_{CD}^P}} \tag{33}$$

Also, by definition, the gross free price inflation rate is:

$$\Pi_t^F = \frac{P_t^F}{P_{t-1}^F} = \frac{Q_t^F}{Q_{t-1}^F} \Pi_t^C \tag{34}$$

Conditions (29)-(34) lead to the formulation of a New Keynesian Phillips curve describing the evolution of Π_t^F.

We now discuss the set of *administered prices* in domestic consumption-good producers. A firm indexed by $j \in [0, \varpi_A]$ is not free to choose its own price and must follow an exogenous pricing rule. Let $P_{j,t}^A$ be the price of firm $j \in A$. In each period t, a fraction θ_A of firms are randomly selected from the set A and are allowed to set their prices following the rule Υ_t^A, which is a function of overall CPI inflation, real exchange rate, commodity-price inflation and a price-specific shock Z_t^A. The remaining fraction $1-\theta_A$ of firms update their prices by choosing $P_t^{A^\star}$, according to a conventional Calvo scheme:

$$P_{j,t}^A = \begin{cases} \Upsilon_t^A P_{j,t-1}^A, & \text{with probability } \theta_A \\ P_t^{A^\star}, & \text{with probability } 1-\theta_A \end{cases}$$

The novelty in the formulation is that, for estimation purposes, parameter θ_A assumes a value arbitrarily close to one. Thus, almost all firms set prices based on a rule capturing changes in domestic factors through CPI inflation as well as through movements in the real exchange rate and commodity price inflation. Define $P_t^{co,*}$ as commodity price index and $\Pi_t^{co,*} = P_t^{co,*}/P_{t-1}^{co,*}$ as commodity price inflation. The functional form defining indexation factor Υ_t^A is given by:

$$\Upsilon_t^A = \left\{ (\Pi_{t-1}^{4C})^{\frac{1}{4}} \left(\frac{Q_t}{Q_{t-1}} \right)^{\upsilon_A^1} \left(\frac{Q_t}{Q_{t-1}} \Pi_t^{co,*} \right)^{\upsilon_A^2} \right\} Z_t^A \tag{35}$$

In the expression above, $\Pi_t^{4C} = \Pi_t^C \Pi_{t-1}^C \Pi_{t-2}^C \Pi_{t-3}^C$. Parameters υ_A^1 and υ_A^2 are positive weights and Q_t denotes the real exchange rate. The term Z_t^A is a stochastic exogenous process capturing shifts in these prices that are not explicitly modeled.

The overall administered price index is defined as follows:

$$P_t^A \equiv \left(\frac{1}{\varpi_A}\int_{j\in A}(P_{j,t}^{C^D})^{1-\epsilon_{C^D}^P}\,dj\right)^{\frac{1}{1-\epsilon_{C^D}^P}}$$

We rewrite the index as:

$$P_t^A = \left(\theta_A\left(\Upsilon_t^A P_{t-1}^A\right)^{1-\epsilon_{C^D}^P} + (1-\theta_A)\left(P_t^{A^\star}\right)^{1-\epsilon_{C^D}^P}\right)^{\frac{1}{1-\epsilon_{C^D}^P}} \tag{36}$$

The relative price $Q_t^A = P_t^A/P_t^C$ is:

$$Q_t^A = \left(\theta_A\left(\Upsilon_t^A \frac{Q_{t-1}^A}{\Pi_t^C}\right)^{1-\epsilon_{C^D}^P} + (1-\theta_A)\left(Q_t^{A^\star}\right)^{1-\epsilon_{C^D}^P}\right)^{\frac{1}{1-\epsilon_{C^D}^P}} \tag{37}$$

The following identity describes the law of motion for the administered price index:

$$\Pi_t^A = \frac{Q_t^A}{Q_{t-1}^A}\Pi_t^C \tag{38}$$

Even with the arbitrary indexation rule, it is still valid that all administered price firms will still converge their price changes to the steady-state inflation, $\Pi^A = \overline{\Pi}^C$. Domestic consumption price index is expressed as a weighted average of freely-set and administered price indices:

$$P_t^{C^D} = \left(\int_0^1 (P_{j,t}^{C^D})^{1-\epsilon_{C^D}^P}\,dj\right)^{\frac{1}{1-\epsilon_{C^D}^P}} = \left(\int_{j\in A}(P_{j,t}^{C^D})^{1-\epsilon_{C^D}^P}\,dj + \int_{j\in F}(P_{j,t}^{C^D})^{1-\epsilon_{C^D}^P}\,dj\right)^{\frac{1}{1-\epsilon_{C^D}^P}}$$

Combining the definition of P_t^F and P_t^A, an expression for domestic consumption price index follows:

$$P_t^{C^D} = \left[\varpi_A\left(P_t^A\right)^{1-\epsilon_{C^D}^P} + (1-\varpi_A)\left(P_t^F\right)^{1-\epsilon_{C^D}^P}\right]^{\frac{1}{1-\epsilon_{C^D}^P}} \tag{39}$$

Finally, the following equation describes the relative domestic consumption price $Q_t^{C^D} = P_t^{C^D}/P_t^C$:

$$Q_t^{C^D} = \left[\varpi_A\left(Q_t^A\right)^{1-\epsilon_{C^D}^P} + (1-\varpi_A)\left(Q_t^F\right)^{1-\epsilon_{C^D}^P}\right]^{\frac{1}{1-\epsilon_{C^D}^P}} \tag{40}$$

Inflation rate for domestic consumption is:

$$\Pi_t^{C^D} = \frac{Q_t^{C^D}}{Q_{t-1}^{C^D}}\Pi_t^C$$

2.2.5 Second Stage: Government Consumption and Investment Sectors

Each sector intermediate good producer in the government consumption and investment sectors faces a downward-sloping demand curve for its good and set prices following a conventional Calvo scheme. In each period t, with probability $1-\theta_H$, each intermediate good firm $j \in H$, for $H = \{G, I\}$, sets its price optimally. Define sectoral inflation as $\Pi_t^H = P_t^H / P_{t-1}^H$. With probability θ_H intermediate good firm j follows the indexation rule:

$$P_{j,t}^H = \Upsilon_t^H P_{j,t-1}^H, \quad j \in H = \{G, I\} \tag{41}$$

$$\Upsilon_t^H = \Pi_{t-1}^H \tag{42}$$

Unlike the previous version of the model in Castro et al. (2011, 2015)[17][18], the indexation mechanism of these prices, and also for freely-set prices, is strictly conventional in the Calvo scheme, without the current value of the inflation target playing a role in inflation indexation. Estimation of the model with the inflation target as an explicit piece of the indexation mechanism generated very similar results compared with the model with conventional Calvo scheme of pricing. Thus, the option for a model with a smaller number of parameters to be estimated was preferred. Intermediate good producer setting prices maximizes expected discounted profits' flow:

$$\max_{P_{j,t}^H} E_t \sum_{i=0}^{\infty} (\theta_H \beta)^i \Lambda_{t,t+i} \left(\Upsilon_{t,t+i}^H P_{j,t}^H - MC_{t+i}^{H,n} \right) Y_{j,t+i}^H, \quad \forall j \in H = \{G, I\}$$

subject to the respective demand constraint:

$$Y_{j,t+i}^H = \left(\frac{\Upsilon_{t,t+i}^H P_{j,t}^H}{P_{t+i}^H} \right)^{-\epsilon_H^P} Y_{t+i}^H, \quad \forall j \in H = \{G, I\}$$

The first-order condition with respect to $P_{j,t}^H$, considering that marginal cost follows equation (28), and that $P_{j,t}^H = P_t^{H^\star}$ for $H = \{G, I\}$, is:

$$P_t^{H^\star} = \frac{\epsilon_H^P}{\epsilon_H^P - 1} \frac{E_t \sum_{i=0}^{\infty} (\theta_H \beta)^i \Lambda_{t,t+i} \left(\frac{\Upsilon_{t,t+i}^H}{P_{t+i}^H} \right)^{-\epsilon_H^P} MC_{t+i}^{H,n} Y_{t+i}^H}{E_t \sum_{i=0}^{\infty} (\theta_H \beta)^i \Lambda_{t,t+i} \left(\frac{\Upsilon_{t,t+i}^H}{P_{t+i}^H} \right)^{1-\epsilon_H^P} P_{t+i}^H Y_{t+i}^H} \tag{43}$$

The sectoral price index P_t^H in sector $H = \{G, I\}$:

$$P_t^H = \left(\theta_H \left(\Upsilon_t^H P_{t-1}^H \right)^{1-\epsilon_H^P} + (1-\theta_H) \left(P_t^{H^\star} \right)^{1-\epsilon_H^P} \right)^{\frac{1}{1-\epsilon_H^P}} \tag{44}$$

The price index above leads to an expression for the relative prices $Q_t^H = P_t^H / P_t^C$ that depends on $Q_t^{H^\star} = P_t^{H^\star} / P_t^C$, where P_t^C is the consumer price index (CPI) defined in the household problem.

$$Q_t^H = \left(\theta_H \left(\Upsilon_t^H \frac{Q_{t-1}^H}{\Pi_t^C}\right)^{1-\epsilon_H^P} + (1-\theta_H)\left(Q_t^{H^\star}\right)^{1-\epsilon_H^P}\right)^{\frac{1}{1-\epsilon_H^P}} \tag{45}$$

Lastly, we define the gross inflation rate as:

$$\Pi_t^H = \frac{P_t^H}{P_{t-1}^H} = \frac{Q_t^H}{Q_{t-1}^H}\Pi_t^C \tag{46}$$

Summing up, prices that are adjusted without considering current market conditions and firm forward-looking behavior are set according to (41), whereas optimal sectoral prices follow equation (43). Lastly, expression (44) determines the sectoral price index. These equations, combined with the definition of sectoral gross inflation rate in equations (45) and (46), considered jointly define New Keynesian Phillips curves for sectors G and I[6].

2.2.6 Second Stage: Export Sector

The price setting in the export good sector follows the Calvo framework, with exporting firms setting their prices in foreign currency. Let $P_{j,t}^X$ be the foreign-currency price of the variety sold by the jth intermediate producer of sector X. In each period t, with probability $1-\theta_X$, each intermediate good firm $j \in X$ sets its price optimally, and with probability θ_X it adjusts the price according to an indexation rule that combines past sectoral inflation in foreign currency and commodity-price inflation[7], with indexation parameter $\omega_X \in [0,1]$:

$$P_{j,t}^X = \Upsilon_t^X P_{j,t-1}^X \tag{47}$$

$$\Upsilon_t^X = \left(\Pi_{t-1}^X\right)^{\omega_X} \left(\Pi_{ss}^* \Pi_t^{co,*}\right)^{1-\omega_X} \tag{48}$$

Compared to the model in Castro et al. (2011, 2015)[17][18], the inclusion of commodity-price inflation on the indexation mechanism for export prices is another simple change to allow for the effect of such prices in the economy.

The producer of the intermediate export good who sets its price optimally maximizes the real value of the expected discounted flow of profits in local currency as follows:

$$\max_{P_{j,t}^X} E_t \sum_{i=0}^{\infty} (\theta_X \beta)^i \Lambda_{t,t+i} \left(\Upsilon_{t,t+i}^X S_{t+i} P_{j,t}^X - MC_{t+i}^{X,n}\right) Y_{j,t+i}^X$$

[6]Appendix A shows the recursive formulation of the Phillips curve for sectors G and I used in the model.

[7]Commodity-price inflation is expressed here as a gap with respect to its mean. Thus, the indexation mechanism must be normalized by foreign inflation, as described in equation (47).

subject to the demand constraint:

$$Y_{j,t+i}^{X} = \left(\frac{\Upsilon_{t,t+i}^{X} P_{j,t}^{X}}{P_{t+i}^{X}} \right)^{-\epsilon_X^P} Y_{t+i}^{X}$$

The first-order condition with respect to $P_{j,t}^{X}$, considering the nominal marginal cost defined in (28) and that, in equilibrium, $P_{j,t}^{X} = P_t^{X^\star}$, is given by:

$$P_t^{X^\star} = \frac{\epsilon_X^P}{\epsilon_X^P - 1} \frac{E_t \sum_{i=0}^{\infty} (\theta_X \beta)^i \Lambda_{t,t+i} \left(\frac{\Upsilon_{t,t+i}^{X}}{P_{t+i}^{X}} \right)^{-\epsilon_X^P} MC_{t+i}^{X,n} Y_{t+i}^{X}}{E_t \sum_{i=0}^{\infty} (\theta_X \beta)^i \Lambda_{t,t+i} \left(\frac{\Upsilon_{t,t+i}^{X}}{P_{t+i}^{X}} \right)^{1-\epsilon_X^P} S_{t+1} P_{t+i}^{X} Y_{t+i}^{X}} \tag{49}$$

The law of motion for the export price index P_t^X is:

$$P_t^X = \left(\theta_X \left(\Upsilon_t^X P_{t-1}^X \right)^{1-\epsilon_X^P} + (1-\theta_X) \left(P_t^{X^\star} \right)^{1-\epsilon_X^P} \right)^{\frac{1}{1-\epsilon_X^P}} \tag{50}$$

An expression that relates the relative prices $Q_t^X = P_t^X / P_t^C$ and $Q_t^{X^\star} = P_t^{X^\star} / P_t^C$ obtains from the export price index above. The variable P_t^C is the consumer price index (CPI) defined in the household problem.

$$Q_t^X = \left(\theta_X \left(\Upsilon_t^X \frac{Q_{t-1}^X}{\Pi_t^C} \right)^{1-\epsilon_X^P} + (1-\theta_X) \left(Q_t^{X^\star} \right)^{1-\epsilon_X^P} \right)^{\frac{1}{1-\epsilon_X^P}} \tag{51}$$

Finally, the expression for the gross inflation rate of the export good sector is given by:

$$\Pi_t^X = \frac{P_t^X}{P_{t-1}^X} = \frac{Q_t^X}{Q_{t-1}^X} \Pi_t^C \tag{52}$$

Equations (47)-(52) describe a New Keynesian Phillips curve governing the dynamics of Π_t^X[8].

2.2.7 Third Stage:

Firms operating in the third stage of production transform sectoral intermediate varieties into a sectoral homogenous good in a perfectly competitive market. Aggregation is based in a Dixit-Stiglitz function:

$$Y_t^H = \left(\int_0^1 \left(Y_{j,t}^H \right)^{\frac{\epsilon_H^P - 1}{\epsilon_H^P}} dj \right)^{\frac{\epsilon_H^P}{\epsilon_H^P - 1}} \tag{53}$$

[8]Appendix A shows the recursive formulation of the Phillips curve for export prices used in the model.

where $H = \{C^D, I, G, X\}$ denotes the four corresponding sectors (private consumption, investment, government consumption, and exports), Y_t^H is the sectoral final output and $Y_{j,t}^H$ is the input associated with intermediate firm j. The variable $\epsilon_H^P > 1$ is the elasticity of substitution between the differentiated intermediate varieties.

In each sector H, sectoral goods assemblers choose the optimal quantities of each variety by solving the following problem:

$$\max_{Y_{j,t}^H} \left\{ P_t^H Y_t^H - \int_0^1 P_{j,t}^H Y_{j,t}^H dj \right\}, \ \forall H, \forall j \in H$$

subject to (53). P_t^H denotes the aggregate price of home good H, and $P_{j,t}^H$ stands for the price of a specific intermediate variety j. In particular, for $H = X$, P_t^H and $P_{j,t}^H$ are expressed in foreign currency because it is assumed the presence of pricing to market. After substituting the Dixit-Stiglitz aggregator into the objective function, the first-order condition with respect to $Y_{j,t}^H$ engenders the following demand for the jth variety:

$$Y_{j,t}^H = \left(\frac{P_{j,t}^H}{P_t^H} \right)^{-\epsilon_H^P} Y_t^H, \ \forall H, \forall j \in H \tag{54}$$

The zero-profit condition for sectoral good assembler yields the sectoral price index:

$$P_t^H = \left(\int_0^1 \left(P_{j,t}^H\right)^{1-\epsilon_H^P} dj \right)^{\frac{1}{1-\epsilon_H^P}}, \forall H \tag{55}$$

2.3 Monetary and Fiscal Policies

Monetary policy follows a simple Taylor rule in which monetary authority sets nominal interest rates based on expected inflation and the growth rate of output. It also considers interest rate inertia and a time-varying target, based on changes of the inflation target and the expected value of the cyclical component of non-stationary productivity[9]. Monetary policy shocks, described by Z_t^R follow an MA(1) process, in order to capture the short-run dynamics in monetary policy decision process. The Taylor rule and the evolution of the inflation target are given by:

$$R_t = R_{t-1}^{\gamma_r} \left[\overline{\Pi}_t^C R_t^{nat} \left(\frac{E_t \Pi_{t+1}^C}{E_t \overline{\Pi}_{t+1}^C} \right)^{\gamma_\pi} \left(\frac{Y_t Z_t^Z}{Y_{t-1} Z_{ss}^Z} \right)^{\gamma_y} \right]^{(1-\gamma_r)} e^{Z_t^R} \tag{56}$$

$$\overline{\Pi}_t^C = \left(\overline{\Pi}_{ss}^C \right)^{1-\rho_{\overline{\Pi}^C}} \left(\overline{\Pi}_{t-1}^C \right)^{\rho_{\overline{\Pi}^C}} + \exp(\epsilon_t^{\overline{\Pi},0} + \epsilon_{t-2}^{\overline{\Pi},2} + \epsilon_{t-6}^{\overline{\Pi},6} + \epsilon_{t-10}^{\overline{\Pi},10}) \tag{57}$$

$$R_t^{nat} = \frac{1}{\beta} \left(E_t Z_{t+1}^{ZC} \right)^{\sigma} \tag{58}$$

[9]Note that the steady state of nominal interest rates, derived from the consumption Euler equation, is given by $R_{ss} = \frac{\Pi_{ss}^C}{\beta} Z_{ss}^Z$.

The structure of the equation describing the inflation target has a few distinctive features. First, we assume the inflation target follows a very persistent process that evolves according to the parameter $\rho_{\overline{\Pi}^C}$ arbitrarily close to one. Second, there is a set of exogenous and independent shocks describing anticipated changes in the inflation target over two, six and ten quarters. The timing of the anticipated shocks is consistent with the Brazilian experience, matching the timing of new target implementations over time.

Compared to the previous version of the model in Castro et al. (2011, 2015)[17][18], fiscal policy is more detailed, now characterized by two rules describing labor taxes and the primary surplus target, completed with an exogenous process for lump sum taxation. One of the taxes is distortionary, affecting labor income, as described in section 2.1. Government chooses the primary surplus target based on past primary results and the expected debt level with respect to its magnitude in the steady state of the model. The rule, thus, generates a smooth transition of the current values of the primary result to the one required to stabilize debt. In Castro et al. (2011, 2015)[17][18], the target for the primary result is set as a function of the contemporaneous level of government debt, which generates significant volatility in a model with distortionary taxation. Labor taxes are set based on the deviation between the current and the target level of the primary result.

$$S_t^y = T_t - \frac{P_t^G G_t}{P_t^Y Y_t} = \frac{\tau_t^N W_t N_t + T_t^{Lump}}{P_t^Y Y_t} - \frac{P_t^G G_t}{P_t^Y Y_t} \tag{59}$$

$$B_{t+1}^y = R_t \left(\frac{B_t^y}{\Pi_t^Y} \frac{Y_{t-1}}{Y_t} - S_t^y \right) \tag{60}$$

$$\overline{S}_t^y = \overline{S}_{ss}^y + \rho_{\overline{S}} \left(\overline{S}_{t-1}^y - \overline{S}_{ss}^y \right) + (1 - \rho_{\overline{S}}) \gamma_B \left(E_t B_{t+1}^y - B_{ss}^y \right) + \epsilon_t^S \tag{61}$$

$$\tau_t^N = \tau_{ss}^N + \gamma_T \left(\tau_{t-1}^N - \tau_{ss}^N \right) + \gamma_S \left(S_t^{4,y} - \overline{S}_t^y \right) + \epsilon_t^T \tag{62}$$

$$\frac{T_t^{Lump}}{P_t^C Z_t} = \widetilde{T}_t^{Lump} = \widetilde{T}_{ss}^{Lump} + \gamma_{TL} \left(\widetilde{T}_{t-1}^{Lump} - \widetilde{T}_{ss}^{Lump} \right) + \epsilon_t^{TL} \tag{63}$$

$$S_t^{4,y} = 0.25 (S_t^y + S_{t-1}^y + S_{t-2}^y + S_{t-3}^y) \tag{64}$$

In the equations above, B_t^y is the debt-to-GDP ratio, S_t^y and $\overline{S}_t^y$ are the primary surplus and its target (both as a proportion of GDP), G_t is exogenous government spending, τ_t^N are labor taxes, as mentioned in the section on households, and T_t^{Lump} is nominal lump sum taxation. Equation (59) also defines an overall tax rate over nominal GDP and equation (63) describes the evolution of T_t^{Lump}. Finally, the last equation specifies a smoothed version for the primary surplus (S_t^y).

2.4 Demand for Brazilian Exports and Rest of the World

Export goods X_t from Brazil are homogeneous goods before they leave the dock. Nevertheless, they become differentiated commodities in international markets. Analogously to the domestic economy, the rest of the world (ROW) uses the Brazilian exported good as a productive input. The exports assembler ships the Brazilian good abroad at the price $P_t^{X^*}$, in foreign currency. Next, foreign producers combine the Brazilian good with the goods produced elsewhere, which we label $Y_t^{D^*}$. In parallel with the domestic economy, the foreign output is a composite between $Y_t^{D^*}$ and imported inputs from Brazil. The following expression summarizes this technology:

$$Y_t^* = \left((\varpi^*)^{\frac{1}{\epsilon^*}} \left[\left(1 - \Gamma_t^{M^*}\right) M_t^* \right]^{\frac{\epsilon^*-1}{\epsilon^*}} + (1-\varpi^*)^{\frac{1}{\epsilon^*}} \left(Y_t^{D^*}\right)^{\frac{\epsilon^*-1}{\epsilon^*}} \right)^{\frac{\epsilon^*}{\epsilon^*-1}}$$

where $M_t^* = X_t$ holds, with $\epsilon^* > 0$ being the elasticity of substitution between Brazilian exports and the goods produced elsewhere. The parameter ϖ^* is the share of Brazilian exports in the ROW output bundle and $\Gamma_t^{M^*}$ is an import adjustment cost with $\vartheta^{M^*} \geq 0$:

$$\Gamma_t^{M^*} = \frac{\vartheta^{M^*}}{2} \left(\left(Z_t^{M^*}\right)^{-\frac{1}{\vartheta^{M^*}}} \frac{X_t/Y_t^*}{X_{t-1}/Y_{t-1}^*} - 1 \right)^2 \tag{65}$$

The foreign good producer chooses the optimal combination of the Brazilian input and the inputs from elsewhere, so as to minimize its total cost:

$$\min_{X_t, Y_t^{D^*}} \left\{ P_t^{D^*} Y_t^{D^*} + P_t^X X_t \right\}$$

subject to the production function for Y_t^*. Denoting P_t^* as the Lagrange multiplier related to the technology constraint, $\Gamma_t^{M^*\dagger}$ as the derivative of the adjustment cost function, and $Q_t^{X^*} \equiv P_t^X / P_t^*$ as the relative price of Brazilian exports in foreign currency, the first-order condition with respect to X_t results in the following expression for the world demand for Brazilian exports:

$$X_t = \left(\frac{\varpi^*}{1 - \Gamma_t^{M^*}} \right) \left(\frac{Q_t^{X^*}}{\left(1 - \Gamma_t^{M^*} - \Gamma_t^{M^*\dagger}\right)} \right)^{-\epsilon^*} Y_t^* \tag{66}$$

One of the main changes of this version of the model is related to the world's economy and the evolution of imported prices in foreign currency. Instead of using AR(1) shocks to characterize these variables, the world's economy is described by a vector autoregressive (VAR) model including five variables: GDP, interest rates, inflation, risk aversion and commodity prices. An additional auxiliary equation links changes in import prices, denominated in foreign currency, with changes in commodity prices. The VAR, combined with the auxiliary equation for import prices, allows for not only an improvement in terms of policy analysis, but also provides a simple approach to include commodity prices in the model without a complete structural description of a commodity sector in the economy. The estimation of the VAR is made apart from the structural model for Brazil, using a Gibbs-sampler with sign restrictions and two lags. From a theoretical perspective, estimating the VAR apart from the rest of the structural model is possible because of the small-open economy hypothesis for Brazil, postulating that shocks in the domestic economy do not influence the rest of the world[10]. From a practical perspective, the procedure also significantly reduces the complexity of the SMC procedure due to the smaller number of parameters.

Define $Z_t^* = [Y_t^*; R_t^*; \Pi_t^*; V_t^*; P_t^{co,*}]$ as the vector stacking foreign variables, A_i as the matrix of coefficients for lag i and ϵ_t^* as the vector of structural shocks with covariance matrix B. The VAR is presented in the following form:

$$Z_t^* = A_1 Z_{t-1}^* + (...) + A_n Z_{t-n}^* + \epsilon_t^*, \quad \epsilon_t^* \sim N(0, B) \tag{67}$$

Sign restrictions identify four structural foreign shocks in the VAR: demand, supply, financial and monetary policy. The sign restrictions imposed are summarized in table 1.

Table 1: Sign Restrictions for VAR Identification

	Interest Rates	GDP	Inflation	Risk	Commodities
Monetary Policy	+		-		
Supply Shock	-	+	-		
Demand Shock	+	+	+		
Financial Shock		-		+	

As mentioned before, a new equation provides a link between the relative import prices in foreign currency and commodity prices. The equation augments the AR(1) process used in Castro et al. (2011, 2015)[17][18] with a lag of the commodity price index as an additional term. Thus, commodity prices have an indirect effect to the domestic economy, through the transmission of changes in these prices to the variables of the rest of the world. It also has a direct link from commodity prices to the domestic economy. The equation is defined as:

$$log(Q_t^{M*}/Q_{ss}^{M*}) = \rho_{QM} log(Q_{t-1}^{M*}/Q_{ss}^{M*}) + \gamma_C log(P_{t-1}^{co*}/P_{ss}^{co*}) + \epsilon_t^{QM*} \tag{68}$$

2.5 Market Clearing Conditions and GDP

Starting at the first stage of production, firms operate under flexible prices. Thus, the total domestic input supplied by the domestic producer to sector $H = \{C^D, I, G, X\}$ matches the total input demand by all firms j operating in sector H. Also, the supply of the imported input in sector $H = \{C^D, I, X\}$ equals the total demand for imports by all firms j operating in sector H. Aggregating across all sectors, the market clearing condition for domestic and imported

[10]See, for instance, Canova (2005)[15] with a discussion and an application of such procedure for VAR models.

inputs is:

$$Y_t^D = Y_{C^D,t}^D + Y_{I,t}^D + Y_{G,t}^D + Y_{X,t}^D \tag{69}$$

$$M_t = M_t^{C^D} + M_t^I + M_t^X + C_t^M \tag{70}$$

Total sectoral inputs result from the aggregation of equations (26) and (27):

$$Y_{H,t}^D = \varpi_H \left(\frac{P_t^D}{MC_t^{H,n}} \right)^{-\epsilon_H} \hat{Y}_t^H \tag{71}$$

$$M_t^H = \left(\frac{1-\varpi_H}{1-\Gamma_{H,t}^M} \right) \left(\frac{P_{H,t}^M}{\left(1-\Gamma_{H,t}^M - \Gamma_{H,t}^{M\dagger}\right) MC_t^{H,n}} \right)^{-\epsilon_H} \hat{Y}_t^H \tag{72}$$

The aggregation of differentiated goods in the context of a Calvo pricing mechanism and a Dixit-Stiglitz technology generates a distortion associated with the price dispersion across goods' varieties. However, in the context of linearization of the equilibrium conditions, the distortion resulting from price dispersion disappears from the model. Formally, aggregating across all firms j in each sector $H = C^D, I, G, X$ yields:

$$\hat{Y}_t^H = \int_0^1 Y_{j,t}^H dj = Y_t^H \int_0^1 \left(\frac{P_{j,t}^H}{P_t^H} \right)^{-\epsilon_H^P} dj = Y_t^H v_t^H$$

The integral $v_t^H = \int_0^1 \left(P_{j,t}^H / P_t^H \right)^{-\epsilon_H^P} dj$ represents the price dispersion effect. In a linear approximation around the steady state, $v_t^H = 1$. Also, the following equalities hold:

$$Y_t^{C^D} = C_t^D,\ Y_t^I = I_t,\ Y_t^G = G_t \text{ and } Y_t^X = X_t$$

The Calvo mechanism also generates a distortion in labor supply, as a consequence of wage dispersion across labor types. However, again, in a first-order approximation around the steady state of the model, labor supply from households will be the same as the labor effectively provided by the aggregate employment agency. Market clearing in labor markets, combined with the adopted hypothesis about wage-setting for rule-of-thumb consumers, results in the following equation:

$$\begin{aligned}
\widehat{N}_t &= \int_{s\in RT} \int_0^1 N_t^{RT}(i) di ds + \int_{s\in O} \int_0^1 N_t^O(i) di ds \\
&= N_t^{RT} \int_{s\in RT} \int_0^1 \left(\frac{W_t^{RT,n}(i)}{W_t^{RT,n}} \right)^{-\epsilon^W} di ds + N_t^O \int_{s\in O} \int_0^1 \left(\frac{W_t^{O,n}(i)}{W_t^{O,n}} \right)^{-\epsilon^W} di ds \\
&= \varpi_{RT} N_t^{RT} + (1-\varpi_{RT}) N_t^O \int_0^1 \left(\frac{W_t^{O,n}(i)}{W_t^{O,n}} \right)^{-\epsilon^W} di \\
&= N_t \left[\varpi_{RT} + (1-\varpi_{RT}) v_t^O \right]
\end{aligned}$$

Again, after defining $v_t^O = \int_0^1 \left(\frac{W_t^{O,n}(i)}{W_t^{O,n}}\right)^{-\epsilon^W} di$, the wage dispersion effect is given by: $v_t^W = \varpi_{RT} + (1-\varpi_{RT})v_t^O$, which equals one if the equilibrium conditions of the model are linearized around the steady state.

With respect to households, equation (6) describes aggregate consumption. Because each household member has full consumption insurance, asset accumulation is a household decision, not an individual one. Therefore:

$$\int_{s\in O} I_{s,t}ds = I_t; \int_{s\in O} K_{s,t}ds = K_t; \int_{s\in O} B_{s,t}ds = B_t \text{ and } \int_{s\in O} B^*_{s,t}ds = B^*_t$$

where I_t, K_t, B_t and B^*_t are the corresponding economy-wide counterparts.

2.5.1 GDP Definition and Law of Motion for Net Foreign Assets

It is important to discuss the definition of nominal and real GDP for market clearing and aggregation purposes. In conventional models with a single sector, real GDP is defined during aggregation, clearing markets with the equations defining supply and demand of goods. In the model described above, where every component of aggregate demand is defined as a unique sector, with a unique price-setting behavior, it is not possible to pin down real GDP. Denote real GDP as Y_t and GDP deflator as P_t^Y. Using the nominal value-added concept, nominal GDP $P_t^Y Y_t$ satisfies the following National Accounts identity:

$$P_t^Y Y_t = P_t^C C_t + P_t^I I_t + P_t^G G_t + S_t P_t^X X_t - S_t P_t^{M^*} M_t v_t^M \tag{73}$$

The last expression v_t^M defined the price dispersion in imports, and, as it was the case with other sectors of the economy adopting the Calvo pricing mechanism, a linear approximation around the steady state of the equilibrium conditions results in $v_t^M = 1$.

Because the model contains only sectoral outputs, there is no equilibrium conditions to pin down real GDP: it is only possible to characterize nominal GDP. Thus, it is necessary to introduce an additional equation to determine this variable. Define the Laspeyres quantity index as follows, with the subscript ss identifying steady state variables:

$$Y_t = \frac{P_{ss}^C C_t + P_{ss}^I I_t + P_{ss}^G G_t + S_{ss} P_{ss}^X X_t - S_{ss} P_{ss}^{M^*} M_t v_t^M}{P_{ss}^C C_{ss} + P_{ss}^I I_{ss} + P_{ss}^G G_{ss} + S_{ss} P_{ss}^X X_{ss} - S_{ss} P_{ss}^{M^*} M_{ss} v_{ss}^M} \tag{74}$$

In the numerator of the expression above, weights on quantities are the base-year prices, considered equal to steady state prices. The denominator displays the nominal value added in steady state. After normalizing real GDP in steady state, $Y_{ss} = 1$, the denominator of the Laspeyres index is $P_{ss}^Y Y_{ss} = P_{ss}^Y$. Using P_{ss}^Y in the denominator of the quantity index, and the fact that $v_t^M = 1$, divide the numerator and the denominator of equation (74) by P_{ss}^C to obtain:

$$Q_{ss}^Y Y_t = C_t + Q_{ss}^I I_t + Q_{ss}^G G_t + Q_{ss} Q_{ss}^X X_t - Q_{ss} Q_{ss}^{M*} M_t v_t^M \tag{75}$$

where $Q_{ss} = S_{ss}P^*_{ss}/P^C_{ss}$ is the real exchange rate in the steady state, $Q^Y_{ss} = P^Y_{ss}/P^C_{ss}$, $Q^I_{ss} = P^I_{ss}/P^C_{ss}$, $Q^G_{ss} = P^G_{ss}/P^C_{ss}$, $Q^X_{ss} = P^X_{ss}/P^*_{ss}$, $Q^{M*}_{ss} = P^{M^*}_{ss}/P^*_{ss}$ and P^*_{ss} denotes the steady-state value for the general price index in the rest of the world.

Under the strategy described above, real GDP is determined in equation (75) and the model pin down GDP deflator according to equation (73). An alternative strategy, used in the original version of the model, is to define a geometric mean price index for P^Y_t and use the expression (73) to determine real GDP. Both strategies are equivalent in a linear approximation around the steady state.

Finally, equation describing the evolution of net foreign assets is obtained by combining the budget constraints of all households in the economy with the previous aggregations results, as well as the government budget constraint. The final equation for net foreign assets is given by:

$$\frac{B^{*y}_{t+1}}{R^*_t S^{B^*}_t} = \left(\frac{Y_{t-1}}{\Pi^Y_t Y_t}\frac{Q_t}{Q_{t-1}}\frac{\Pi^C_t}{\Pi^{C^*}_t}\right) B^{*y}_t + NX^y_t - L^{*y}_t$$

where $B^{*y}_{t+1} = S_t B^*_{t+1}/P^Y_t Y_t$, NX^y_t is the net exports-to-GDP ratio and L^{*y}_t is the total payments of interests on external borrowing due to working capital restrictions, measured as a proportion of nominal GDP. Expressions for NX^y_t and L^{*y}_t are:

$$NX^y_t = \frac{S_t P^X_t X_t}{P^Y_t Y_t} - \frac{S_t P^{M^*}_t M_t}{P^Y_t Y_t} \upsilon^M_t$$

$$L^{*y}_t = \sum_H \iota_H \left(R^*_t S^{B^*}_t - 1\right) \frac{P^M_t M^H_t}{P^Y_t Y_t}$$

2.6 Shocks

This section presents a brief discussion on the functional form of the structural shocks in the model. The model has four technology shocks: transitory productivity (Z^D_t, presented in equation (11)), cyclical component of permanent technology growth rate (Z^{ZC}_t, equation (14)), temporary component of permanent technology growth rate (Z^{ZT}_t, equation (15)), and an investment-specific one (Z^I_t). The investment-specific shock affects the functional form describing the adjustment cost of investment growth. These shocks follow an AR(1) process, except for the temporary component of permanent technology growth rate, which is a white noise. The evolution of non-stationary productivity is discussed in detail in subsection 3.2.

Households are directly affected by five shocks. The intertemporal Euler equation for domestic consumption, in equilibrium for optimizing households, shows a preference shock (Z^C_t) and domestic risk premium shock (S^B_t). The UIP (uncovered interest rate parity) condition, derived from the first order condition of foreign bonds has a country risk premium shock ($Z^{B^*}_t$) and an *ad hoc* disturbance (Z^Q_t). Finally, the labor-leisure choice described by the intratemporal Euler condition is affected by a labor supply shock (Z^L_t). Again, these shocks follow an AR(1) process.

The model also includes *ad hoc* cost-push shocks associated with New Keynesian Phillips curves. We do not impose time-varying elasticities of substitution for Dixit-Stiglitz aggregators as the source of markup shocks in order to preserve the recursive formulation derived from the non-linear equations describing the first-order conditions with respect to wages and prices[11]. Following Smets and Wouters (2007)[40], cost-push shocks are characterized by an ARMA(1,1) process, in order to accommodate both short- and long-lived movements in prices. Moreover, the sign of the MA(1) component is negative, characterizing the exogenous high-frequency movements of the *price level* as partially mean reverting. The model contains shocks to the wage Phillips curve (Z_t^W) and the same cost-push shocks (Z_t^P) for the Phillips curves in sectors G, I and for the equation describing inflation dynamics associated with firms that choose prices freely in sector C^D[12]. Also concerning sector C^D, there are specific shocks driving the pricing rule followed by firms with administered prices (Z_t^A). Lastly, distinct shocks ($Z_t^{P^X}$), different from Z_t^P, drive the Phillips curve for export sector.

The policy block comprises fiscal and monetary rules. Regarding fiscal policy, government consumption (G_t) and lump sum taxes are characterized as an AR(1) process and specify two white noise disturbances associated with fiscal rules for two variables: targets for government surpluses and labor tax rates. Monetary policy follows a Taylor rule with a monetary policy shock (Z_t^R) described by a MA(1) process. In addition, we model the inflation target ($\overline{\Pi}_t^C$) as a random walk with one unanticipated disturbance and three uncorrelated news shocks associated with the horizons of two, six and ten quarters.

Concerning the dynamics between the domestic and the foreign economy, the VAR relating the rest of the world observed variables and the equation characterizing the dynamics of import price goods measured in foreign currency, described in section 2.4, associate six observed variables with six exogenous shocks. Shocks in these equations are white noise. There are also shocks affecting the Brazilian demand for imported goods[13] ($Z_{H,t}^M$, for $H = \{C^D, G, I, X\}$), and the demand from the rest of the world for Brazilian exports ($Z_t^{M^*}$). These shocks follow an AR(1) specification.

2.7 Transformations to induce stationarity

The model has two stochastic trends, a nominal one whose stochastic growth rate is $\overline{\Pi}_t^C$ and a real one whose stochastic growth rate is Z_t^Z. Along the balanced growth path, all real variables, except for labor, grow at the same rate Z_t^Z, whereas levels of all nominal variables, except for nominal wages, grow at the same rate $\overline{\Pi}_t^C$. Nominal wages incorporate the two stochastic trends.

Regarding notation, we denote all detrended variables dv with a tilde ($\widetilde{dv}$). In order to write down the model in its stationary form, all real variables are divided by the level of technology Z_t. Analogously, all variables describing nominal price levels become stationary by dividing their corresponding levels by the price P_t^C. To cast wages in their stationary form, we divide them by both Z_t and P_t^C. The same approach is used for $T_t^{Lump,n}$. Finally, we detrend the smooth

[11]Writing the non-linear system allows for the use of non-linear solution methods to compute the equilibrium conditions, which is essential to carry out welfare evaluations with the model.

[12]Subsection 3.3 also discusses a few motives for this simplification considering the estimation process.

[13]Subsection 3.3 discusses a simplification imposed in the structure of these shocks for estimation purposes.

trend for consumption (C_t^S) as follows: $\widetilde{C}_t^S = \frac{C_t^S}{Z_t^c}$. Some variables, including labor, ratios to GDP and financial variables such as interest rates and foreign investor's risk aversion are already stationary. Finally, a variable with the subscript ss represents its steady-state value.

3 Estimation

3.1 Data

Compared to the model described in Castro et al. (2011, 2015)[17][18], there are both changes in the number of variables observed and also in the treatment of these series in this version of the model[14]. Most of the changes in data treatment, as it will be later discussed, are related to the extended sample size used for estimation. With respect to the number of variables observed, there is a trend in the literature, especially in models designed for policy analysis, of including data that is considered relevant to characterize new features added to old models, or to better characterize the dynamics of the economy. As examples, Corrigan et al. (2021)[22] expanded the number of observed variables in ToTEM III to 50, as the model now incorporates information on exports of commodity goods and a detailed housing market. Following the same line, the evolution of the original model described for Sweden in Adolfson et al. (2013)[3], "Ramses II", includes three additional observed variables: unemployment, interest rate spreads and the interest rates to non-financial corporations[15]. In Coenen et al. (2018)[20], the new model for the Euro Area includes 6 additional variables to characterize the new financial frictions incorporated in the model. It also includes information on long-term GDP growth and inflation expectations.

The model includes 30 observed variables, incorporating information on prices, economic activity, labor market, fiscal and monetary policy, and the foreign sector. Sample starts in 2001Q4 and finishes in 2019Q4, for a total of 73 observations in the time dimension. However, the first eight observations are used to initialize the Kalman filter. This is particularly important for the current version of SAMBA, where the non-stationary inflation target and the unit root generated from the functional form of monitored prices play a significant role. Information before 2001Q4 was not available for all observed variables, and information after 2019Q4 includes the Covid-19 pandemic period, where inference based on the linear-Gaussian framework of the Kalman filter is inappropriate. Note that the time span includes almost the whole period under an inflation targeting regime, thus avoiding significant breaks in the functional forms characterizing monetary policy.

Data on domestic prices remain the same as in Castro et al. (2011, 2015)[17][18], with CPI inflation, freely-set price inflation, and administered price inflation all detrended by the steady state inflation – annual inflation of 4.5%, consistent with the most frequent value of the inflation

[14]Table 6 in the appendix summarizes data treatment for each time series used for estimation.

[15]A new model for policy analysis in Sweden, "MAJA", described in Corbo and Strid (2020)[21], incorporates a larger number of observed variables describing the international economy, but reduces the number of observed domestic variables, as the new model includes a detailed structural block describing the foreign economy, but simplifies the financial sector described in "Ramses II".

target in the sample. Data on inflation expectations, from the survey conducted by the Banco Central do Brasil, is included to provide model discipline in terms of short-term responses of prices to exogenous shocks. With respect to National Accounts data, the demographic transition observed in Brazil justifies the use of information measured in per capita terms. Combined with the information on GDP growth, a measure of the Brazilian output gap is informed to the model. Details on the computation of the output gap are provided in subsection 3.2.

With respect to labor market, the growth rate of occupied population used in Castro et al. (2011, 2015)[17][18] is replaced by the unemployment rate, detrended by the sample mean. Information on the participation rate is also included. Due to significant methodological changes in the employment survey for data starting in 2012, time series on unemployment, participation rate, and the growth rate of wages were simulated for the period between 1999 and 2011 using mixed-frequency VAR models based on the procedure in Alves and Fasolo (2015)[5]. Simulations combine information from other surveys and the General Registry of Employed and Unemployed Persons (CAGED, in Portuguese), of the Ministry of Labor and Social Security (MTP)[16].

For the structural VAR describing the rest of the world, the Fed Funds rate and the CPI still measure world's interest rates and inflation, respectively. Output gap is the cyclical component of weighted real GDP measure of the main trade partners detrended by the one-sided HP filter. The last two elements of the VAR, risk aversion and commodity prices, are measured as the principal component of a set of normalized variables related to each element. For risk aversion, variables include the VIX (S&P 500, CBOE), MOVE (Treasuries, Merrill Lynch), VXY (currencies, JP Morgan) and the spread between the Moody's corporate yields for BAA bonds and US Treasury 10-year yield. For the commodity prices, variables include Banco Central do Brasil's Commodity Index (IC-BR), Brent oil prices, and CRB and CRY Indexes.

Concerning other external variables, the relative price of imports is still measured as the gap between index of imported good and foreign inflation. However, the observed variable now is the demeaned growth rate of the relative price of imports. The larger sample now includes the reversion of the commodity boom observed in the early 2000's, allowing for the growth rate of the relative price to be representative of the evolution of such variable[17]. Also, in terms of country risk premium, the credit default swap (CDS) for Brazilian debt of 5 years replaces the JP Morgan's Emerging Market Bond Index (EMBI) Brazil.

With respect to monetary policy, nominal interest rates are demeaned from an exogenous target of 8.5%. Combined with the inflation steady state of 4.5% and an international real interest rate of 1% (nominal rates of 3% and inflation of 2%), these assumptions imply a country risk premium of 300 basis points. Data also includes the announced changes in the inflation target, consistent with the historical Brazilian experience and the description of shocks in equation (57).

Combining all the details described above, there is a total of 30 observed variables informed

[16]Data from CAGED can not be used as a standard indicator for labor market conditions, as it only considers information from the formal labor market.

[17]In Castro et al. (2011, 2015)[17][18], the relative price of imports is detrended using a linear trend. The sample covers the period between 1999Q3 and 2010Q2.

to the model. In order to improve the model's fit to the data and reduce the estimated volatility of structural shocks, measurement errors were included in equations describing a few observed variables. Specifically, the output gap and the growth rates of nominal wages and GDP components have measurement errors characterized by an MA(1) process. The volatility of the MA(1) process is estimated in the model under specific restrictions discussed in section 3.4. Define X_t as a generic observed variable of the model where measurement errors are included, x_t as the model (endogenous) representation of such observed variable, and $\epsilon_t^{me,x}$ as the measurement error in period t for the observed variable. Equations characterizing X_t have the following form:

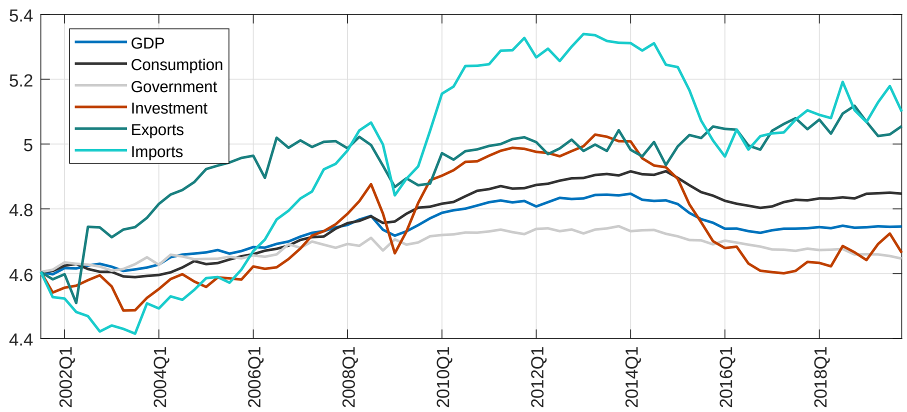
$$X_t = x_t + \sigma_x \overline{\sigma}_x (\epsilon_t^{me,x} + \mu_{me,x} \epsilon_{t-1}^{me,x})/((1 + \mu_{me,x}^2)^{0.5}) \tag{76}$$

with the following parameters: σ_x is the volatility of the observed variable; $\overline{\sigma}_x$ is a cap for the maximum value of the volatility of the measurement error; and $\mu_{me,x}$ is a parameter defining the MA(1) process.

3.2 Estimating the output gap

Inference with respect to the output gap, used as an observable in the model, is made outside the estimation procedure. There are a few reasons to adopt this approach. First, as described in figure 1, it is hard to consider the existence of a clear balanced growth path in the Brazilian economy, as imports and exports per capita seem to grow at a faster pace compared to aggregate GDP per capita and its domestic components[18]. The increase in the degree of openness, especially considering the short samples available for Emerging Economies, and the consequences for macroeconomic modeling are discussed in Brazdik et al. (2020)[13] in the context of a review of the main structural model of the Czech Republic National Bank.

Figure 1: Log per capita GDP and components – Index 2001Q3 = 100



[18]Other approaches try to make inference about joint trend in data. For Brazil, Costa (2016)[23] provides an unified framework using Brazilian data, but does not include exports and imports in his model.

A second reason to use the output gap as an observable in the model is the lack of data on productivity for the Brazilian economy available in real time. It has become common practice, especially among models designed for policy analysis, to incorporate information about the productivity growth, either by a direct or proxy measurements of this variable[19]. The approach for estimation here is closer to the model used by the Bank of Canada, described in Corrigan et al. (2021)[22]. Specifically for the model described here, the central role played by non-stationary productivity shocks forced the inclusion of an indirect measurement in order to discipline the dynamics of the economy, especially during crisis periods[20]. Historical decompositions in versions of the model that did not include the output gap usually consider extreme events, as the 2008 Global Financial Crisis, as a permanent change in productivity, resulting in an implied output gap close to zero during that period.

One final reason to use the output gap as an observable is the ability to compare the inputs of the model with other available estimates of the output gap calculated by other filters or produced by the Banco Central do Brasil staff. The procedure also does not preclude, as a policy exercise, to inform the model with these other estimates, thus generating alternative paths for productivity growth in Brazil.

The estimation of the output gap is based on the procedure described in Andrle (2013)[6] as a modified HP filter, where it is assumed that the growth rate of the trend has a steady state. The modified equations describing the filter are specified in state-space form to perform Bayesian evaluation of the posterior distribution of parameters. The hypothesis of a steady state for the growth rate of the trend makes the evolution of potential output coincide with the one described in equations (13)-(15). Define Y_t^{obs} as the demeaned log-level of real GDP per capita. The filter is characterized by the following set of equations:

$$
\begin{aligned}
Y_t^{obs} &= y_t + \log(Z_t) \\
y_t &= \alpha_1 y_{t-1} + (\alpha_2 - \alpha_1)\, y_{t-2} + \sigma_y \epsilon_t^y \\
\log\left(\frac{Z_t}{Z_{t-1}}\right) &= \log(Z_t^Z) = \log Z_{ss}^Z + \log(Z_t^{ZC}) + \log(Z_t^{ZT}) \\
\log\left(\frac{Z_t^{ZC}}{Z_{ss}^Z}\right) &= \rho_Z \log\left(\frac{Z_{t-1}^{ZC}}{Z_{ss}^Z}\right) + \left(1-\rho_Z^2\right)^{0.5} \sigma_{ZC}\varepsilon_t^{ZC} \\
\log(Z_t^{ZT}) &= \left(1-\rho_Z^2\right)^{0.5} \sigma_{ZT}\varepsilon_t^{ZT} \\
\epsilon_t^y \sim N(0,1), &\quad \varepsilon_t^{ZT} \sim N(0,1), \quad \varepsilon_t^{ZC} \sim N(0,1)
\end{aligned}
$$

Note that the model normalizes the standard deviation of shocks in productivity by the

[19]See the documentation of the latest version of the FED-NY DSGE model[1], which incorporates total factor productivity for the US economy as an observable for the model. The estimation in Coenen et al. (2018)[20] includes long term growth expectations for the Euro Area as a proxy for productivity growth.

[20]We describe the inclusion of the output gap as an indirect measurement of productivity because the combination of GDP per capita growth and the output gap as observed variables in the model provides an inference about the growth rate of per capita non-stationary productivity, adjusted by measurement errors included in the output gap equation described in (76).

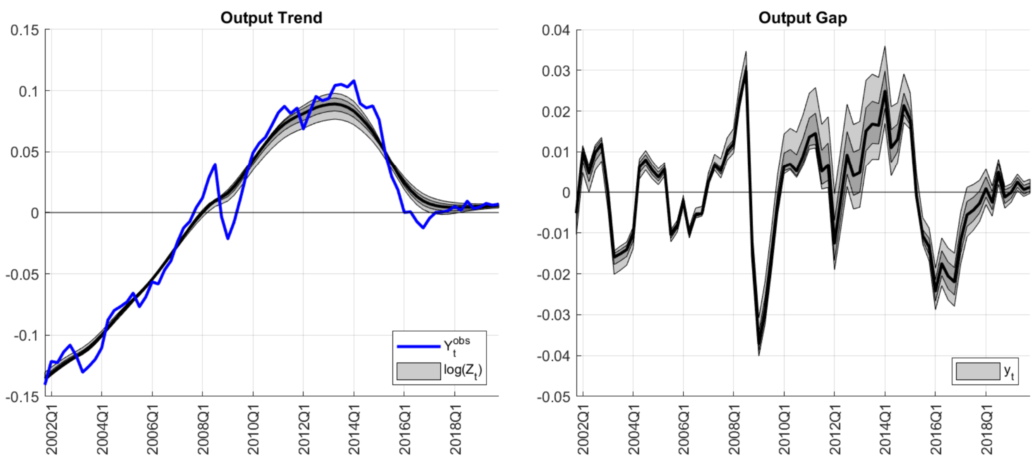
autoregressive parameter of the cyclical component of productivity. Also, the combination of parameters in the cyclical component of output, y_t, combined with a proper choice of prior distribution, ensures that this term is stationary. In order to estimate the model, it is necessary to specify prior distributions for α_1, α_2, ρ_Z, the standard deviation σ_{ZT}, and the ratio of the standard deviations σ_{ZC}/σ_{ZT} and $(\sigma_{ZT}+\sigma_{ZC})/\sigma_y$. The use of priors on the ratio of the volatility of shocks simulates a similar role played in the HP filter by the parameter characterizing the signal-to-noise ratio of the trend. It is also important to take a stance on the steady state growth of potential output, characterized by $\log Z_{ss}^Z$, given the significant decrease in average GDP per capita growth after 2014. It is assumed here that the decrease in GDP growth after 2014 is the result of a very persistent change in productivity growth, with a slow convergence towards the long run growth average ($\log Z_{ss}^Z$) observed between the end of 2001 and 2013Q4. Table 4 shows the definition of the priors and the posterior evaluated with the last half of 200,000 draws using the Metropolis-Hastings algorithm.

Table 2: Estimated Parameters

Param.	Prior Distribution	Prior Mean	Prior StD	Posterior Mean	Posterior StD	90% HPD Inf. Limit	90% HPD Sup. Limit
α_1	Beta	0.8	0.1	0.836	0.085	0.708	0.967
α_2	Beta	0.5	0.2	0.841	0.103	0.693	0.981
ρ_Z	Beta	0.9	0.01	0.931	0.009	0.917	0.945
σ^{ZT}	Inv.Gamma	5	inf	0.495	0.043	0.421	0.560
σ_{ZC}/σ_{ZT}	Gamma	20	1	16.903	0.914	15.337	18.320
$(\sigma_{ZT}+\sigma_{ZC})/\sigma_y$	Gamma	10	1	4.308	0.602	3.305	5.286

Figure 2 shows the evolution of potential output with confidence sets based on moments of the posterior distribution and the estimated output gap. Notice that the confidence sets around the potential output and the output gap are quite small.

Figure 2: Log per capita output trend and gap – Median and confidence sets (68% and 95%)



Additional exercises based on recursive estimation of the output gap, available upon request, shows that the volatility of the output gap at the end of the sample used for estimation is significantly smaller than the one observed when using the HP filter. This is a very important property of the estimated output gap considering the use of the model for forecasting exercises.

3.3 Additional restrictions for the estimated model

The model, as described in subsection 2.2, is characterized by firms in different sectors manufacturing goods in order to satisfy specific demand for goods. A complete characterization of these sectors would require the existence of data, both in cross-section and in time series dimensions, to pin down all the sector-specific parameters. The lack of sector-specific data resulted in parameter identification problems during the model estimation. In order to minimize these problems, a few simplifications were made imposing additional restrictions on parameters of the model.

First, the lack of information on the share of imports to produce specific goods resulted in a simplification of the demand for imports in firms in the second stage of production. It is assumed that the functional form characterizing the aggregation of domestic and imported goods is the same across sectors. Consequently, the proportion of domestic goods, the elasticity of substitution across imported and domestic goods, and the function characterizing the adjustment cost related to imports, are all the same across sectors:

$$\varpi_I = \varpi_{C^D} = \varpi_X, \quad \epsilon_I = \epsilon_{C^D} = \epsilon_X, \quad \vartheta_I^M = \vartheta_{C^D}^M = \vartheta_X^M$$

Also, with respect to the demand of imported goods, the sectoral adjustment costs for imports is completed with a single common shock Z_t^M, resulting in the following restrictions:

$$Z_{I,t}^M = Z_{C^D,t}^M = Z_{X,t}^M = Z_t^M$$

The lack of references with respect to working capital constraints faced by firms in Brazil operating in international markets, combined with parameter identification problems in estimation, suggests another simplification of the model, setting the constraint the same across all sectors:

$$\iota_I = \iota_{C^D} = \iota_X$$

With respect to price-setting, in order to ensure convergence, the elasticity of substitution across differentiated goods for sectors supplying goods for domestic economy is the same. This simplification results in the same markup over prices in steady state across sectors. For the sake of convenience, assume prices for firms trading goods with the rest of the world also face the same markup over prices:

$$\epsilon_{C^D}^P = \epsilon_I^P = \epsilon_G^P = \epsilon_X^P = \epsilon^M$$

Also related to the price setting process, it is assumed that price-setting in sectors supplying goods for domestic economy faces the same exogenous markup shifts. This assumption avoids an excessive number of *ad-hoc* cost-push shocks affecting the model, especially considering that only one of the sectoral prices is observed during estimation. Therefore, Z_t^P is the same for sectors C^D, I and G.

3.4 Priors, posteriors and SMC estimation

The model uses SMC methods to infer about the posterior distribution of the parameters. The estimation is based on Cai et al. (2021)[14]. The use of the MH algorithm proved difficult given the size of the model, both in terms of the number of state variables and the number of observed variables. The posterior distribution showed significant irregularities, leading to very sensitive results after small changes in the priors under the MH algorithm. The ability of the SMC method proposed in Cai et al. (2021)[14] to slowly incorporate information from the dataset, resulting in a more complete evaluation of the posterior distribution of parameters, helped in stabilizing the results, even in the presence of a very irregular posterior distribution.

The main idea of the method described in Cai et al. (2021)[14] is to slowly incorporate information to the estimation, starting from a known distribution, using bridge distributions to reach a complete characterization of the objective function – here, the posterior distribution of parameters. The estimation starts with 30,000 independent draws from the prior distribution, adding more weight to the likelihood as the independent draws move towards the relevant regions of the tempered posterior distribution. The independent draws move on the tempered likelihood based on MH draws, but the move does not depend on the computation of modes of the distribution. Once the weight of the likelihood equals the prior weight, the resulting draws characterize the posterior distribution. Note that, since each draw is independent of each other, a significant degree of parallelization of the algorithm can be achieved[21].

In order to discuss the prior distributions used in estimation, it is useful to split the set of parameters in the model in four groups: calibrated parameters and ratios – not estimated in the model; steady state parameters, related to average moments on data that parameters should match; endogenous parameters, related to the dynamics of the model; and exogenous parameters, associated with the volatility and persistence of exogenous shocks. This classification and the procedure for estimation is closely related to Del Negro and Schorfheide (2008)[25]. The prior distributions are completed by a set of the so-called "system priors", discussed in Andrle and Benes (2013)[7] and Andrle and Plašil (2018)[8].

Calibrated parameters not estimated in the model include the inflation target, the long run nominal interest rate and productivity growth. It also includes long run shares of GDP components and the average unemployment and participation rates in sample. The inclusion of labor taxes in the model demands an additional restriction, calibrating the share of labor taxes revenues on total taxation. This number is available in Azevedo and Fasolo (2015)[10]. Also, the share of monitored prices on the CPI, $weight_A$, is not the same as parameter ϖ_A, which is the

[21]For this paper, likelihood computation of the draws was performed with significant gains on a GPU, while the model solution was distributed across the different CPU cores available.

share of monitored firms on the domestic production of the consumption good. Parameter ϖ_A is computed given the share of monitored prices on the CPI, $weight_A$, and the share of domestic input in production, ϖ_H. The share of firms in sector A not allowed to optimize prices, θ_A, is set during estimation to a value arbitrarily close to 1, as the calibrated value creates a unit root in the structural model. Table 3 summarizes the calibrated parameters.

Table 3: Calibrated Parameters and Steady State Values

Parameter	Value	Description	Parameter	Value	Description
$\overline{\Pi}_{ss}^{C}$	$1.045^{0.25}$	Inflation	U_{ss}^{L}	0.095	Unemployment
R_{ss}	$1.085^{0.25}$	Interest rates	L_{ss}	0.62	Participation rate
Π_{ss}^{*}	$1.020^{0.25}$	Foreign inflation	$\overline{N}$	0.3	Overhead labor
R_{ss}^{*}	$1.030^{0.25}$	Foreign interest rate	ϖ_{RT}	0.2	Share of RT households
Z_{ss}^{Z}	$1.020^{0.25}$	Productivity growth	ϵ^{P}	11	EoS across goods
σ	1.0001	Intertemporal substitution	$weight_A$	0.27	Share monitored prices on CPI
s_I	0.183	Investment/GDP	$\overline{S}_{ss}^{y}$	0.016	Primary Result/GDP
s_G	0.171	Government spending/GDP	$\sigma(\epsilon_t^{\overline{\Pi},0})$	0.0005	Vol. Inflation Target, t
s_M	0.107	Imports/GDP	$\sigma(\epsilon_{t-2}^{\overline{\Pi},2})$	0.0524	Vol. Inflation Target, $t-2$
ι	0.50	Working capital constraint	$\sigma(\epsilon_{t-6}^{\overline{\Pi},6})$	0.0240	Vol. Inflation Target, $t-6$
ϖ_H	0.883	Share of domestic input	$\sigma(\epsilon_{t-10}^{\overline{\Pi},10})$	0.0114	Vol. Inflation Target, $t-10$
$\frac{\tau_{ss}^{N}\tilde{W}_{ss}N_{ss}}{T_{ss}}$	0.335	Share of labor taxes			

A few extra parameters are calibrated in the model, related to inflation target process and the calibration of measurement errors in some observed variables equations. First, given that the inflation target is completely exogenous to the rest of the model, the volatility of the the shocks describing the anticipated announcement of the inflation target is estimated apart from the DSGE model. This procedure allows us to estimate the volatility of the shocks assuming the non-stationary structure of the inflation target ($\rho_{\overline{\Pi}^C} = 1$), which is not possible during the SMC procedure as the second moments of the distribution of endogenous variables are necessary to implement the "system priors".

With respect to measurement errors in observed variables, presented in equation (76), it is assumed the output gap, nominal wages, and the GDP components are measured with errors following an MA(1) process. The volatility of the measurement error for the output gap, nominal wages, and domestic components of GDP (consumption, investment, and government spending) is estimated with a restriction that its maximum value ($\overline{\sigma}_x$ in equation (76)) is equal to 50% of the volatility of the observed variable. Measurement errors for imports and exports are estimated with the restriction that its maximum value is equal to the volatility of the observed variable.

Steady state parameters are parameters of the structural model closely associated with relevant moments of the economy in steady state. The usual strategy is to use long run averages to compute statistics like the labor share in the production function, the government debt-to-output ratio or the foreign debt-to-output ratio – the "great ratios" of the economy. The procedure here is to impose a distribution around these moments, with flat priors on the structural parameters, allowing for the structural parameters to match these moments. In the case of the estimation of

SAMBA, a prior is imposed for the labor share of the economy and the nominal trade balance-to-output ratio. For the first prior, note that the production function in equation (11) is now characterized by a CES aggregator, instead of the special case of the Cobb-Douglas function used in Castro et al. (2011, 2015)[17][18]. Combined with a loose prior for the elasticity of substitution between labor and capital, the prior on the labor share helps to pin down the value of parameter α during estimation. For the second prior, the nominal trade balance-to-output ratio helps to pin down the foreign debt-to-output ratio in steady state, which is a critical component of the equation describing the risk premium in foreign bonds, presented in equation (1).

Priors for endogenous parameters are fairly standard in the literature. Among the significant differences, parameters characterizing adjustment costs in investment, Brazilian exports, and Brazilian imports (ϑ_I , ϑ^M and ϑ^{M^*}) have a higher prior mean, mostly due to the significant gaps generated after detrending data in this observed variables. However, the estimation allows for significant fluctuation of these parameters, as the prior standard deviation is quite large. Prior for the inverse of the Frisch elasticity of labor supply, η, is based on the survey of micro results in Chetty (2012)[19] Also, the parameter describing the effects of net foreign assets as a proportion to exports on the country risk premium has a very low mean, in order to avoid excessive fluctuations of the country risk premium due to shocks affecting the domestic economy.

Priors for exogenous parameters follow closely the procedure in Del Negro and Schorfheide (2008)[25], with the use of a matrix characterizing volatility and first-order correlation of a subset of observed variables. Priors for the volatility of exogenous shocks are not informative, while the priors for autoregressive (AR) and moving average (MA) processes for shocks are constrained in a tight interval[22]. The matrix with information on volatility and first-order correlation of observed variables provides targets for the volatility of shocks. At the same time, the matrix creates a tension for the estimation of the persistence of exogenous processes: on the one hand, the tight priors for AR and MA parameters restricts the persistence of shocks; on the other hand, the matrix provides information about observed variables with potentially high autocorrelation. It is up to the information provided in the data to choose a highly persistent exogenous shock or to fit the model with an endogenous dynamics that generates the correlation in data while keeping the exogenous process with low autocorrelation. The matrix contains information on interest rates, real exchange rates, participation and unemployment rates, consumption, investment and government consumption growth, inflation of freely-set and monitored prices and inflation expectations.

Finally, the "system priors" used in estimation draw on the impulse response functions (IRFs) of a monetary policy shock from other models of Banco Central do Brasil. Specifically, IRFs of

[22]The exceptions for this procedure are the volatility of the measurement errors for CPI and GDP aggregation, the process for the non-stationary productivity, and the volatility of imports' adjustment shocks. On the non-stationary productivity, the prior for the AR process for the shock has a higher mean and standard deviation, while the volatility of the cyclical and temporary components are tied in a single prior, specifying a prior for the volatility of the cyclical component and the ratio between the cyclical and temporary volatility. The volatility of imports' adjustment shocks was constrained in order to avoid an exaggerated role of this shock in historical decompositions.

monetary policy shocks of the semi-structural models used for forecasting purposes and published in the Inflation Report[23] are used as a benchmark to set priors for the impact of monetary policy shocks on inflation in different periods. The priors impose independent Gaussian distributions for three periods of the IRFs: the immediate impact of a monetary policy shock on inflation and the impact of a monetary policy shock on inflation after one and five years. While the first two priors are relatively loose, providing only a benchmark for the path of monetary policy shocks, the prior for the impact of monetary policy after five years is very tight, consistent with the idea of policy neutrality in the long run. This configuration for "system priors" is less restrictive than the proposal from Lombardi and Nicoletti (2012)[33], who summarize the IRF in a single quadratic function and impose a single prior on the whole profile of the IRF. Here, each of the three periods selected is associated with a single, independent prior. The set of "system priors" here is also related to the use in Coenen et al. (2018)[20] for the ECB NAWN II model. The authors impose a prior on the combination of parameters representing the slope of the Phillips curve, in order to avoid very small responses from prices after a monetary policy shock. Here, the prior is imposed directly on the IRF, allowing for a larger set of parameters to be influenced to match these moments.

Table 4 presents the main results of estimation, in terms of the prior and posterior distributions of parameters. Figures 13 and 14 in Appendix D show the histogram of the distribution for each parameter and the density of the marginal prior distribution used for estimation[24]. The rest of this section details a few results highlighted by the choice of priors and the estimation method.

Table 4: Estimated Parameters – Priors and Posterior Moments

Param.	Description	Prior Distribution	Posterior Mean	Posterior StD	90% HPD Inf.Limit	90% HPD Sup.Limit
		Endogenous parameters				
ϵ_D	EoS between capital and labor	$G(1.00, 0.40)$	0.954	0.106	0.784	1.138
δ	Depreciation rate of capital	$G(0.02, 0.005)$	0.027	0.003	0.023	0.032
κ	Habit persistence	$B(0.60, 0.05)$	0.784	0.022	0.749	0.819
η	(Inverse) labor supply elasticity	$N(1.852, 0.90)$	4.481	0.304	4.006	5.001
υ	Weight labor supply wealth effect	$B(0.50, 0.20)$	0.093	0.009	0.079	0.108
ϵ_{CD}	EoS: domestic input and imported goods	$G(1.00, 0.40)$	1.575	0.314	1.067	2.114
φ^*_V	Country risk: Risk aversion	$G(0.10, 0.01)$	0.035	0.004	0.028	0.042
φ^*_B	Country risk: NFA	$G(0.002, 0.001)$	0.001	0.000	0.000	0.001
ϵ_M	EoS: across import varieties	$G(1.00, 0.40)$	1.911	0.241	1.528	2.324
ϵ_{M*}	EoS: exports and foreign inputs	$G(1.00, 0.40)$	0.762	0.165	0.504	1.045
ϑ^I	Investment adjustment cost	$G(12.0, 1.00)$	11.88	0.594	10.91	12.86
ϑ^M	Import adjustment cost	$G(25.0, 2.00)$	24.48	1.393	22.18	26.79
ϑ^{M^*}	Foreign import adjustment cost	$G(25.0, 2.00)$	25.15	1.223	23.12	27.19
θ_F	Calvo: freely-set price	$B(0.60, 0.20)$	0.573	0.021	0.533	0.603
θ_G	Calvo: government goods price	$B(0.60, 0.20)$	0.665	0.087	0.514	0.803
θ_I	Calvo: investment goods price	$B(0.60, 0.20)$	0.580	0.211	0.265	1.000
θ_M	Calvo: imports prices	$B(0.70, 0.20)$	0.632	0.048	0.552	0.709
θ_X	Calvo: exports prices	$B(0.70, 0.20)$	0.819	0.019	0.787	0.848
ω_X	Export price indexation	$B(0.50, 0.20)$	0.442	0.035	0.384	0.500

(Continued on next page)

[23]See, for instance, the September-2020 Inflation Report box *"New small-scale aggregate model with Bayesian estimation"*, the March-2021 Inflation Report box *"New small-scale disaggregate model"*, and, notably, the December-2021 Inflation Report box *"Revision of the small-scale aggregate model"*.

[24]We refer to "marginal priors" as the marginal densities of the prior distribution of individual parameters in the absence of "system priors". It is not the same as the priors effectively used in the model, as the presence of "system priors" breaks the usual independence assumed for prior distributions across parameters

Table 4 (continued from previous page)

Param.	Description	Prior Distribution	Posterior Mean	Posterior StD	90% HPD Inf.Limit	90% HPD Sup.Limit
$\upsilon_{1,A}$	Elast: monitored prices to RER	$G(0.20, 0.10)$	0.034	0.008	0.021	0.048
$\upsilon_{2,A}$	Elast: monitored prices to commodities	$G(0.20, 0.10)$	0.021	0.005	0.012	0.03
θ_W	Calvo: wages	$B(0.60, 0.20)$	0.784	0.012	0.764	0.802
ω_W	Wage indexation	$B(0.60, 0.20)$	0.815	0.044	0.744	0.888
γ_R	Taylor rule: inertial parameter	$B(0.50, 0.25)$	0.804	0.010	0.788	0.820
γ_π	Taylor rule: inflation	$G(1.50, 0.75)$	2.211	0.204	1.875	2.552
γ_Y	Taylor rule: output growth	$G(0.50, 0.25)$	0.120	0.034	0.066	0.178
γ_T	AR labor taxes	$B(0.30, 0.10)$	0.404	0.042	0.334	0.474
γ_S	Elast.: labor taxes to prim. surplus	$G(0.50, 0.20)$	0.474	0.055	0.389	0.571
γ_B	Elast.: prim. surplus target to debt	$G(0.02, 0.01)$	0.012	0.001	0.010	0.014
γ_C	Elast.: import prices to commodity	$G(10.0, 4.00)$	15.641	1.453	13.25	18.069
“Big Ratios” – Steady state values						
LS	Labor share from CES function	$N(0.42, 0.02)$	0.417	0.013	0.396	0.438
TB-GDP	(Minus) Trade Balance-to-GDP	$G(0.01, 0.003)$	0.007	0.001	0.005	0.009
ARMA structure of exogenous shocks						
ρ_C	AR Household preference	$B(0.30, 0.05)$	0.581	0.043	0.509	0.651
ρ_I	AR Investment adj. cost	$B(0.30, 0.05)$	0.274	0.027	0.229	0.318
ρ_L	AR Labor supply	$B(0.30, 0.05)$	0.374	0.052	0.291	0.465
ρ_M	AR Import adj. cost	$B(0.30, 0.05)$	0.298	0.033	0.242	0.354
ρ_Q	AR UIP shocks	$B(0.30, 0.05)$	0.728	0.021	0.694	0.764
ρ_P	AR Price markup	$B(0.30, 0.05)$	0.283	0.026	0.241	0.326
μ_P	MA Price markup	$B(0.30, 0.05)$	0.310	0.024	0.269	0.350
ρ_W	AR Wage markup	$B(0.30, 0.05)$	0.301	0.030	0.252	0.350
μ_W	MA Wage markup	$B(0.30, 0.05)$	0.280	0.028	0.233	0.326
ρ_A	AR Monitored prices	$B(0.30, 0.05)$	0.269	0.025	0.229	0.310
μ_A	MA Monitored prices	$B(0.30, 0.05)$	0.338	0.024	0.297	0.378
ρ_D	AR Stationary productivity	$B(0.30, 0.05)$	0.428	0.030	0.378	0.478
ρ_Z	AR Perm. tech. - cyclical	$B(0.95, 0.02)$	0.956	0.005	0.947	0.965
μ_R	MA Monetary policy shocks	$B(0.30, 0.05)$	0.421	0.035	0.363	0.479
ρ_B	AR Domestic risk premium	$B(0.30, 0.05)$	0.293	0.034	0.237	0.350
ρ_{B*}	AR Country risk premium	$B(0.30, 0.05)$	0.535	0.040	0.470	0.603
ρ_{TL}	AR Lump sum taxation	$B(0.30, 0.05)$	0.405	0.041	0.341	0.478
ρ_G	AR Government spending	$B(0.30, 0.05)$	0.262	0.029	0.215	0.310
$\rho_{\overline{S}}$	AR Primary surplus target	$B(0.30, 0.05)$	0.684	0.041	0.618	0.753
ρ_{P*}	AR Export price markup	$B(0.30, 0.05)$	0.299	0.032	0.248	0.352
μ_{P*}	MA Export price markup	$B(0.30, 0.05)$	0.297	0.029	0.251	0.346
ρ_{QM}	AR Import price	$B(0.30, 0.05)$	0.538	0.034	0.484	0.596
ρ_{M*}	AR World import demand	$B(0.30, 0.05)$	0.295	0.032	0.241	0.349
Volatility of exogenous shocks						
σ_C	St.D. Household preference	$Uniform$	5.393	0.428	4.745	6.071
σ_I	St.D. Investment adj. cost	$Uniform$	3.244	0.177	2.957	3.537
σ_L	St.D. Labor supply	$Uniform$	0.962	0.296	0.477	1.457
σ_M	St.D. Import adj. cost	$IG(0.20, 0.10)$	0.173	0.039	0.116	0.243
σ_Q	St.D. UIP shocks	$Uniform$	2.419	0.158	2.159	2.678
σ_P	St.D. Price markup	$Uniform$	3.625	0.424	2.869	4.308
σ_W	St.D. Wage markup	$Uniform$	74.46	8.562	60.60	89.56
σ_A	St.D. Monitored prices	$Uniform$	1.495	0.062	1.395	1.598
σ_D	St.D. Stationary productivity	$Uniform$	0.662	0.040	0.597	0.728
σ_{ZC}/σ_{ZT}	Ratio Perm. tech. shocks	$G(1.00, 0.20)$	0.481	0.061	0.380	0.584
σ_{ZC}	St.D. Perm. tech. - cyclical	$IG(0.20, Inf)$	0.296	0.021	0.262	0.332
σ_R	St.D. Monetary policy shocks	$Uniform$	0.225	0.011	0.207	0.244
σ_B	St.D. Domestic risk premium	$Uniform$	0.321	0.179	0.054	0.636
σ_{B*}	St.D. Country risk premium	$Uniform$	0.982	0.151	0.729	1.232
σ_G	St.D. Government spending	$Uniform$	0.922	0.052	0.838	1.009
σ_T	St.D. Labor taxation	$Uniform$	1.093	0.144	0.856	1.319
σ_{TL}	St.D. Lump sum taxation	$Uniform$	16.51	1.340	14.34	18.75
$\sigma_{\overline{S}}$	St.D. Primary surplus target	$Uniform$	1.714	0.140	1.479	1.945
σ_{P*}	St.D. Export price markup	$Uniform$	93.51	18.35	65.08	127.5
σ_{QM}	St.D. Import price	$Uniform$	5.156	0.369	4.556	5.790
σ_{M*}	St.D. World import demand	$Uniform$	60.17	7.697	47.00	73.30
σ_{CPI}	St.D. CPI aggregation shock	$IG(0.05, Inf)$	0.113	0.007	0.102	0.124
$\sigma_{agg,Y}$	St.D. GDP aggregation shock	$IG(0.05, Inf)$	0.480	0.052	0.396	0.568
Observed variables: Measurement error						
$\mu_{me,Y}$	MA(1) - M.E. Output gap	$B(0.50, 0.25)$	0.722	0.129	0.495	0.920
$\mu_{me,C}$	MA(1) - M.E. Consumption	$B(0.50, 0.25)$	0.453	0.046	0.376	0.528
$\mu_{me,G}$	MA(1) - M.E. Gov. spending	$B(0.50, 0.25)$	0.543	0.089	0.410	0.702
$\mu_{me,I}$	MA(1) - M.E. Investment	$B(0.50, 0.25)$	0.713	0.067	0.607	0.830
$\mu_{me,X}$	MA(1) - M.E. Exports	$B(0.50, 0.25)$	0.316	0.055	0.227	0.408
$\mu_{me,M}$	MA(1) - M.E. Imports	$B(0.50, 0.25)$	0.514	0.034	0.456	0.571
$\mu_{me,W}$	MA(1) - M.E. Nominal wages	$B(0.50, 0.25)$	0.558	0.017	0.530	0.586
$\overline{\sigma}_Y$	St.D. M.E. Output gap	$B(0.50, 0.25)$	0.014	0.006	0.004	0.025
$\overline{\sigma}_C$	St.D. M.E. Consumption	$B(0.50, 0.25)$	0.938	0.032	0.882	0.985
$\overline{\sigma}_G$	St.D. M.E. Gov. spending	$B(0.50, 0.25)$	0.731	0.095	0.569	0.892

(Continued on next page)

Table 4 (continued from previous page)

Param.	Description	Prior Distribution	Posterior Mean	Posterior StD	90% HPD Inf.Limit	90% HPD Sup.Limit
$\overline{\sigma}_I$	St.D. M.E. Investment	$B(0.50, 0.25)$	0.975	0.014	0.950	0.995
$\overline{\sigma}_X$	St.D. M.E. Exports	$B(0.50, 0.25)$	0.650	0.056	0.558	0.744
$\overline{\sigma}_M$	St.D. M.E. Imports	$B(0.50, 0.25)$	0.899	0.037	0.836	0.961
$\overline{\sigma}_W$	St.D. M.E. Nominal wages	$B(0.50, 0.25)$	0.986	0.008	0.972	0.997

Note: Prior distribution described as *"Prior(mean, std)"*, where *"Prior"* is the distribution used, *"(mean, std)"* are the first two moments. List of distributions: *B* – Beta distribution; *G* – Gamma distribution; *IG* – Inverse-Gamma distribution; *N* – Gaussian distribution; *Unif* – Uniform (positive values);

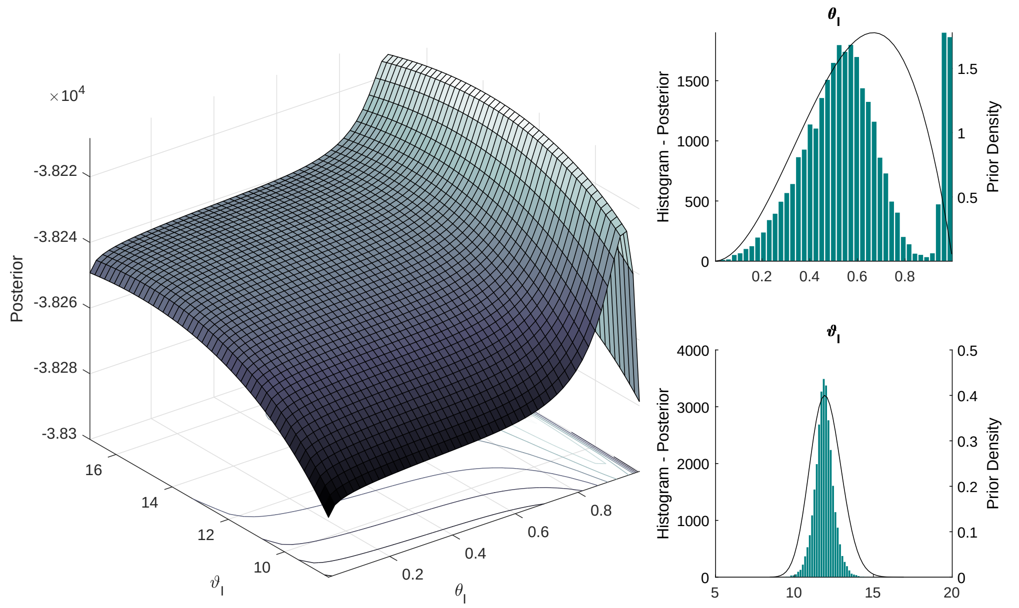
As it is usual in the estimation of DSGE models, a few parameters presented identification issues, where the surface of the likelihood around these parameters is essentially flat, resulting in significant weight to the prior definition in order to compute the posterior distribution. Notably, some of the parameters characterizing the AR(1) process of exogenous shocks and the parameters characterizing adjustment costs in investment, Brazilian exports, and Brazilian imports (ϑ_I , ϑ^M and ϑ^{M^*}) seem to be highly influenced by the prior choice, as the central moment of each distribution is close to the mode of the marginal prior distribution. However, in all these cases, measures of dispersion of the posterior distribution seem to be more concentrated, compared to the marginal prior, showing the likelihood still is capable to provide some information about structural parameters.

Compared to Castro et al. (2011, 2015)[17][18], estimates for the mean of the posterior of habit persistence (κ) is slightly higher in this version of the model. On the other hand, parameters on interest rate inertia (γ_R) and the sensitivity to inflation in the Taylor rule (γ_π) are quite similar. These are processes quite comparable across the different versions of the model, using the same dataset with updated samples. Parameters characterizing the wage Phillips curve – Calvo wages (θ_W) and wage indexation (ω_W) – have a higher posterior. However, not only the labor market structure differs, but also data on labor market is now based on a new survey provided by IBGE, as discussed before. It is also hard to make inference about Calvo parameters characterizing prices, as the model here adopts a conventional indexation mechanism, based on past inflation and not on the inflation target, as in the original model.

The strategy of choosing tight priors for the autoregressive process, and combining it with a matrix with information on moments of observed variables, resulted in only five exogenous processes with AR(1) coefficient above 0.5, not counting the estimate for the cyclical component of non-stationary productivity, whose prior has a mean of 0.95. Two of those processes were expected to have a high AR(1) process, as they are associated with observed financial variables: UIP shocks (Z_t^Q) and country risk premium shocks ($Z_t^{B^*}$). The other shocks with relatively high persistence are household preference shocks, import price shocks, and the primary surplus target shock. It is a stark contrast to Castro et al. (2011, 2015)[17][18], where eleven shocks (not counting the shock to the inflation target, which is calibrated here) have estimated AR(1) coefficient above 0.5. Thus, compared to the original model, it is expected that the match of persistence observed in data becomes more based on the endogenous dynamic of the model, and not on exogenous disturbances.

One critical feature for the dynamics of the model exposed by the estimation procedure is related to the evolution of investment prices. The SMC procedure captured a bi-modal distribution for the Calvo parameter of the Phillips curve of investment prices: one peak of the distribution associated with a low value of θ_I combined with another peak of θ_I very close to one. As it will be described in section 4, this posterior distribution affects the evaluation of the transmission of monetary policy and exchange rate shocks to investment. Figure 3 presents the surface figure relating θ_I and the parameter characterizing the adjustment cost of investment, ϑ_I, together with the marginal densities of each parameter. It highlights the importance of the SMC method to bring stability to the estimation, as the conventional Metropolis-Hastings algorithm could potentially target only one of the modes, instead of showing the entire posterior distribution.

Figure 3: Estimation: Prior and Joint Posterior Distribution of Parameters



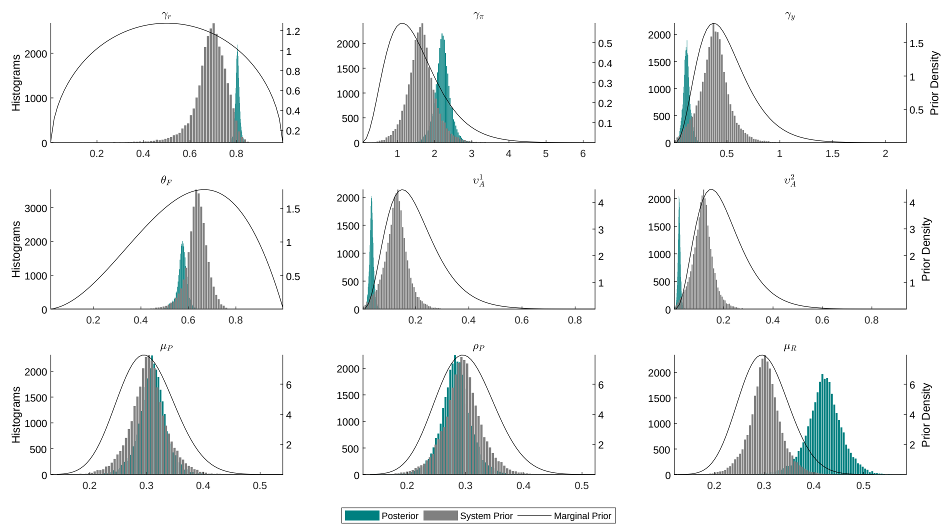
The figure shows the joint posterior distribution of parameters θ_I and ϑ_I in the left panel, together with the marginal histogram and the prior density for each parameter in the right panel.

It is important also to understand the role of "system priors" in model estimation. Figure 4 shows, for a few parameters, the histogram of the simulated marginal prior distribution, the histogram of the prior distribution assuming the presence of "system priors"[25], and the histogram of the posterior distribution. The first row of plots presents parameters characterizing the Taylor rule. While the marginal prior suggests a large variance for the distribution of the parameter

[25]In order to simulate the parameter distribution under "system priors", the SMC method used in estimation is applied again, but without the computation of the likelihood. Thus, the SMC integrates the set of independent marginal priors for each parameter with the "system priors" imposed on large ratios, exogenous processes and IRFs of monetary policy.

describing interest rate inertia (γ_r), the use of "system priors" provides a significant support for high interest rate inertia in the model. "System priors" also provide more weight to higher interest rate sensitivity to inflation expectations (γ_π), while they are neutral with respect to the response of interest rates to output growth (γ_y). The second row of plots presents parameters associated with the pricing mechanism of the CPI, both for freely-set and monitored prices. For all cases, the use of "system priors" concentrated the support of the prior distribution closer to the mode of the marginal density, reducing the probability of extreme values allowed by marginal density. The third row of plots shows that the effects of "system priors" over the prior distribution of parameters governing the ARMA process of exogenous shocks for freely-set prices and interest rates are quite small. The simulated distribution is very similar to the density of the marginal prior, both in terms of the mode and the dispersion. Thus, the use of "system priors" helped shaping the posterior distribution of endogenous parameters of the model in terms of CPI pricing and monetary policy.

Figure 4: Estimation: Marginal Priors, "System Priors" and Posterior Distribution



The figure shows the marginal prior density and the simulated posterior distribution and "system prior" distribution of selected parameters.

4 Moments, IRFs and Shock Decomposition

This section presents the main properties of the model in terms of relevant moments, impulse response functions, and shock decompositions, given the estimated parameters from the previous section. It is worth noting that the use of Sequential Monte Carlo methods, as described before, provides a significant improvement in the assessment of uncertainty with respect to results. As it will become clear ahead, SMC methods helped highlighting how the irregularities of the posterior distribution translated in higher uncertainty with respect to some characteristics of the model.

4.1 Moments

Table 5 compares the second moments (standard deviation and first-order autocorrelation) of the model with the data used in estimation. Considering the number of observed variables included in the model, results can be considered good in terms of second moments. The table highlights the information on domestic variables (thus, the fit of the structural VAR and the exogenous equation for the inflation target are not included in the table) and the data associated with nominal variables is detrended by the inflation target. In a general perspective, the model does a good job matching second moments of GDP and its components and also the volatility of prices. The model has mixed results in terms of labor market variables but misses the volatility of fiscal variables.

Table 5: Moments of Observed Variables

Observed Variable	Standard Dev.			Autocorrelation		
	Data	Model	95% CI	Data	Model	95% CI
CPI inflation	0.90	0.60	[0.56 0.65]	0.62	0.39	[0.34 0.44]
Monitored prices inflation	1.58	1.45	[1.35 1.57]	0.41	0.03	[-0.02 0.10]
Free prices inflation	0.88	0.72	[0.67 0.79]	0.56	0.26	[0.22 0.32]
Inflation expectations	1.18	0.87	[0.78 0.99]	0.77	0.78	[0.75 0.81]
Output gap	1.32	5.18	[4.15 15.7]	0.73	0.98	[0.96 1.00]
Output growth	1.17	1.13	[1.04 1.24]	0.34	0.01	[-0.08 0.10]
Consumption growth	1.20	1.15	[1.05 1.28]	0.30	0.27	[0.21 0.32]
Investment growth	3.89	3.36	[3.13 3.63]	0.30	0.27	[0.21 0.32]
Government spending growth	1.25	1.25	[1.14 1.37]	-0.42	-0.24	[-0.29 -0.17]
Imports growth	5.17	4.80	[4.40 5.17]	0.19	0.08	[-0.05 0.21]
Exports growth	4.60	3.96	[3.50 4.49]	-0.28	-0.09	[-0.21 0.06]
Export prices	4.94	4.85	[4.48 5.30]	0.43	0.48	[0.42 0.53]
Import prices inflation	3.03	5.14	[4.32 6.06]	0.45	-0.16	[-0.21 -0.10]
Unemployment rate	1.68	1.76	[1.60 1.94]	0.97	0.83	[0.80 0.85]
Participation rate	0.41	2.08	[1.60 2.82]	0.89	0.99	[0.99 1.00]
Nominal wage growth	0.84	0.77	[0.74 0.80]	0.10	0.41	[0.36 0.46]
Real exchange rate	18.7	31.9	[28.5 35.7]	0.94	0.98	[0.97 0.98]
Interest rates	4.24	2.56	[2.38 2.80]	0.94	0.91	[0.90 0.92]
Risk premium	0.95	1.67	[1.23 2.06]	0.91	0.78	[0.73 0.83]
Primary result target	2.37	22.0	[10.4 59.4]	0.96	1.00	[0.99 1.00]
Primary result	2.36	8.31	[4.03 23.1]	0.82	0.96	[0.81 0.99]

Note: nominal variables presented as deviation from the inflation target. Interest rates are annualized.

Observing the first four lines of the table, describing the second moments of prices, the model underestimates the autocorrelation of domestic prices. The volatility of CPI and its components is overall well matched, despite a small underestimation of the volatility of freely-set prices. Consistent with the underestimation of the volatility of CPI inflation, nominal interest rates and inflation expectations also have lower volatility in the model compared to data, but a very good match in terms of autocorrelation.

The model does a good job matching second moments of GDP and its components. Notably, it does a very good job approximating the negative autocorrelation of exports and government spending growth. It should be noted that the model representation of these observed variables includes measurement errors. However, for both exports and government spending growth, measurement errors do not have the same volatility of the observed variable. Thus, measurement errors do not seem to heavily influence the matching of the negative autocorrelation. With respect to the output gap, it is curious that the model misses on both the volatility and the autocorrelation, but it also did not rely on the measurement error to improve model fit, as seen in section 3.4.

In terms of labor market variables, the model has a good fit on the second moments of unemployment. The autocorrelation of wages in the model is overestimated. Given that wage markups have the same ARMA(1,1) process used in the markup of domestic prices, it was expected a better fit of the sample autocorrelation. However, given that wage markups play a key role in explaining unemployment in the Galí, Smets and Wouters (2011)[28] framework, it is possible that the estimation process faced some tension between matching the autocorrelation of wages and unemployment. The model misses on moments of participation rate. The near unit root observed in simulated moments is a direct consequence of the two non-stationary processes included in the inflation target equation and the monitored prices' Calvo mechanism. The non-stationary mechanism in monitored prices, specifically, results in a very slow convergence of the relative price of these goods, altering households' aggregate consumption decisions and, by consequence, the labor supply[26].

4.2 IRFs: Monetary Policy and Exchange Rate Pass-Through

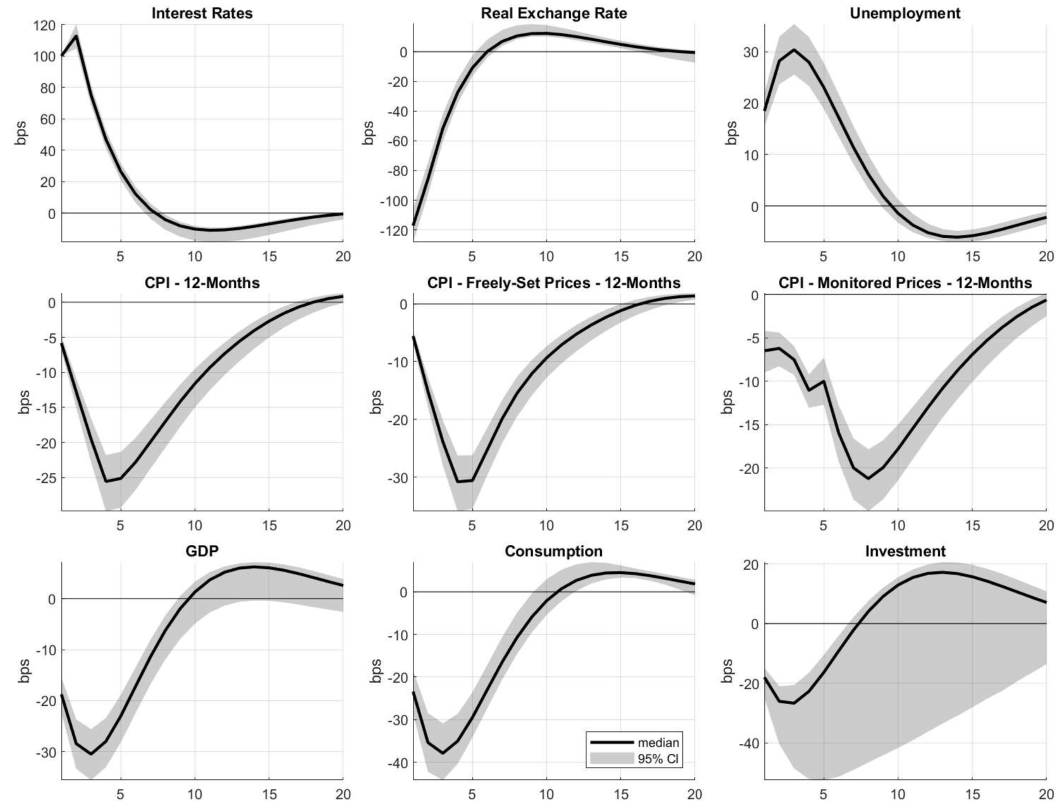
Figure 5 shows the impulse response functions (IRFs) of a monetary policy shock. As expected, prices fall immediately, with maximum effects on CPI inflation reached after one year. Also, consistent with the price sectoral dynamics proposed in the model, effects on freely-set prices are more significant and react faster to the monetary policy shock, when compared to monitored prices. Domestic GDP falls and unemployment increases almost at the same proportion. Worth also noting that, despite a significant share of RT households in the model, consumption falls proportionally less than output after the shock.

Quantitatively, there is a small impact of monetary policy on inflation, which is apparently in line with the so-called "flat Phillips Curve" hypothesis. The hypothesis states that inflation in the last 30 years has become less responsive to measures of economic activity usually included in the New Keynesian Phillips Curve. See empirical evidence in Del Negro et al. (2020)[24], Hazell et al. (2022)[30], among others. In Coenen et al. (2018)[20], there is a "system prior" imposed on the non-linear combination of structural parameters representing the slope of the Phillips Curve. The posterior implies a coefficient around 0.007, slightly higher than the value obtained (0.005) in the previous version of the ECB model[27].

[26]Preliminary tests using the same Calvo parameter estimated for freely-set prices in monitored prices resulted in autocorrelation for participation rate closer to the observed in data.

[27]Obviously, other structural changes in the model might also have affected the estimate of this parameter,

Figure 5: IRF: Monetary Policy Shock



The figure shows the median impulse response functions (black line), the 95% credible intervals (CI, shaded area) for the DSGE model.

There is, however, a disconnection between the estimated parameters of the model for Brazil and the slope of the Phillips Curve for freely-set price firms in Brazil. Linearization of conditions (29)-(33) results in the following expression for the slope of the Phillips Curve (SPC^F):

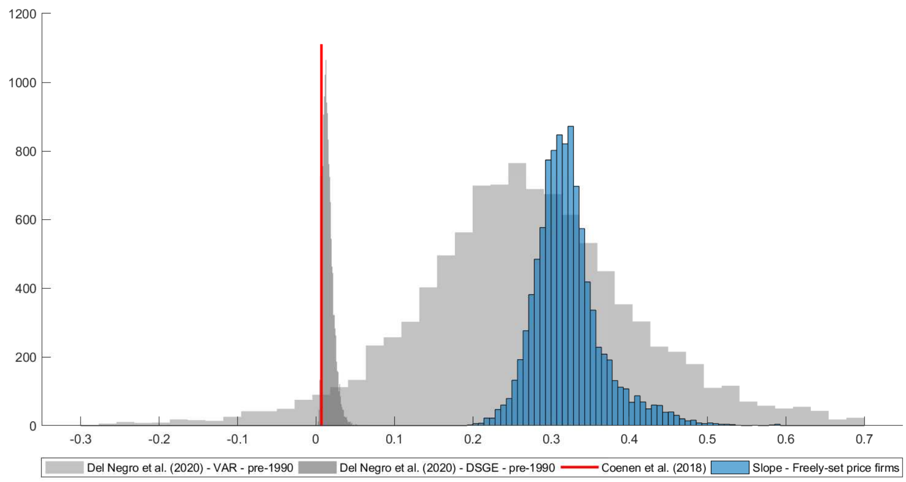
$$SPC^F = (1 - \theta_F \beta (Z_{ss}^Z)^{1-\sigma})(1 - \theta_F)/\theta_F$$

Figure 6 shows the histogram of the slope of the Phillips curve, based on the estimation of the model, comparing with the central value of the estimation in Coenen et al. (2018)[20] and the histogram of the estimate of both the VAR and the DSGE models in Del Negro et al. (2020)[24] for the period before 1990[28]. It is clear that the Phillips Curve for freely-set price firms is much more responsive to changes in economic activity, but the effects of a monetary policy shock on inflation are in line with the literature.

compared to NAWN I.

[28]Histograms for both VAR and DSGE models for the period after 1990 showed very little variance around zero, making difficult any comparison.

Figure 6: Histogram: Slope of Phillips Curve – Freely-Set Price Firms

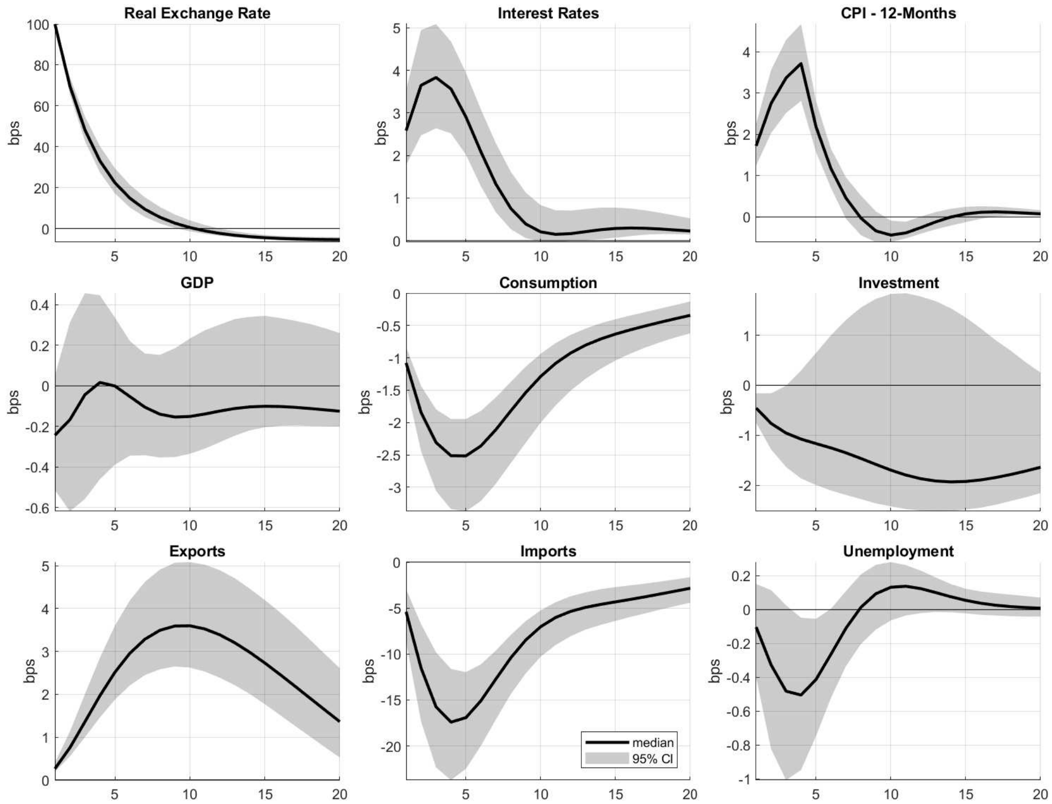


The key to understand this apparent disconnection is the impact of monetary policy shock on economic activity. Domestic GDP, household consumption, and specially investment, show significantly smaller responses to monetary policy shocks, compared to other papers. It is well known from the literature that credit and financial markets in the Brazilian economy are distorted by the existence of earmarked credit, where the government provides loans to firms based on criteria that are almost totally insensitive to the short-run interest rates set by the Central Bank. Indeed, Bonomo and Martins (2016)[12] provided empirical evidence using firm-level data. They documented that changes in the policy rate have smaller effects on the level of employment for firms with more access to earmarked and government-owned banks loans.

Exchange rate pass-through is evaluated based on the IRFs from the shock included in the uncovered interest parity (UIP) condition of the model, derived from the first order condition of optimizing households with respect to foreign bonds. The UIP shock should be interpreted as an exogenous intervention that temporarily moves the real exchange rate away from its fundamental value based on the states of the economy. Figure 7 shows the impact of a deviation of 100bps of the real exchange rate from the UIP condition and the impacts on the domestic economy. The estimated pass-through to CPI inflation is immediate, but quite low. The propagation of the shock results in higher inflation, reaching 3.5bps over 12 months. The impact on economic activity is also quite small, and quite uneven across different components of aggregate demand: considering the 95% credible intervals, impact on GDP is not significant. However, household consumption reduces in the short run, while exports grow almost in the same proportion. Imports are quite sensitive to the shock, with a strong fall over one year, almost four times the size of the change in exports. Unemployment shows a small reduction over the first year after the shock.

Compared to Castro et al. (2011, 2015)[17][18], the effects of real exchange rate shocks as

Figure 7: IRF: Real Exchange Rate Shock



The figure shows the median impulse response functions (black line), the 95% credible intervals (CI, shaded area) for the DSGE model.

discussed here are more muted, both in terms of prices and economic activity. The same shock would result in an increase in CPI around 7bps over 12 months. The response of output is muted here over the whole horizon, while in Castro et al. (2011, 2015)[17][18] GDP falls in the long term.

Related to the response of investment to a monetary policy shock, Castro et al. (2011, 2015)[17][18] find a slightly higher response for output, consumption and inflation, compared to results here, but a significantly higher response for investment. The effect on economic activity is amplified in other estimated DSGE models for Brazil with financial frictions[29]. However, all these models use data for the period before the 2014 crisis. Indeed, Bonomo et al. (2015)[11] showed that government-driven credit had an important role in countervailing the private credit crunch in Brazil during the financial crisis and that government credit concessions continued to expand after the economy recovered. This behavior has the potential to reduce the effect

[29]See, for instance, Carvalho, Castro and Costa (2014)[16], among others.

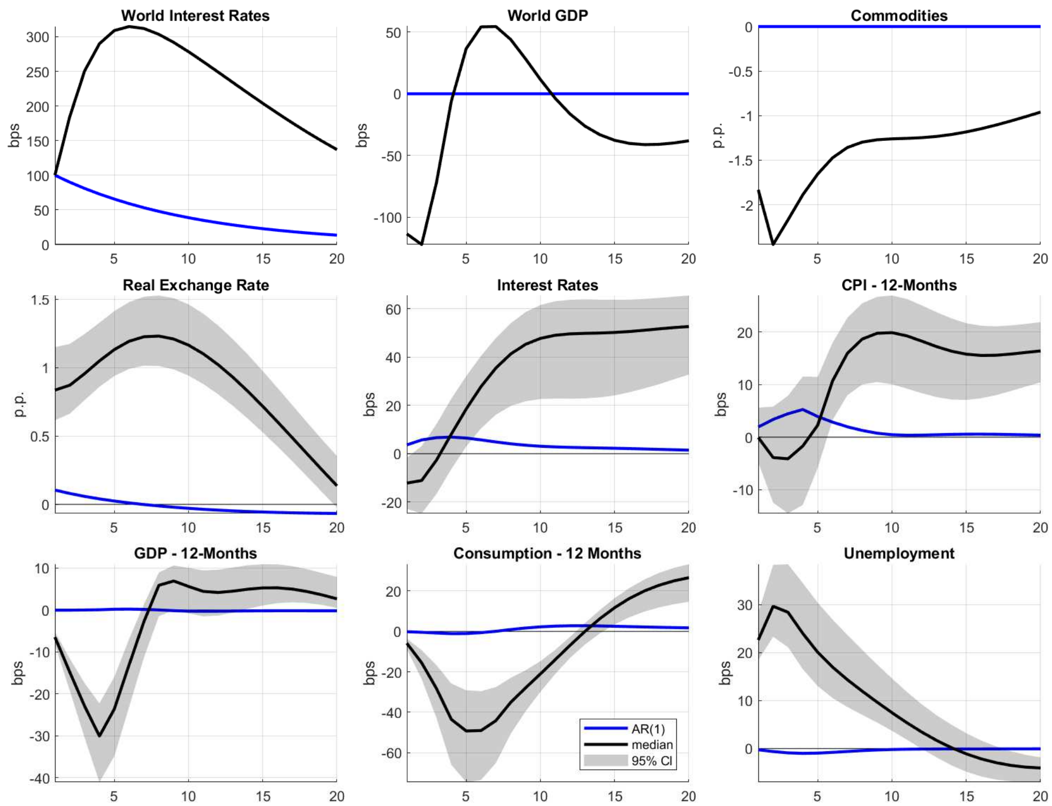
of monetary policy shocks on economic activity in recent years because interest rates on earmarked loans are lower than market interest rates, which tend to closely follow the policy rate. Ramos-Tallada (2015)[36] reached a similar conclusion concerning supply-driven credit responses to monetary shocks. The relationship between changes in the economic environment and the inflation sensitivity to monetary policy is not a feature of emerging markets since Pancrazi and Vukotić (2019)[35] showed similar behavior for the US economy.

4.3 IRFs: Foreign Shocks

One of the significant changes of this version of the model, compared with Castro et al. (2011, 2015)[17][18], is the use of a structural VAR to characterize shocks from the rest of the world to the domestic economy. This section compares the IRFs for domestic variables of the model using the structural VAR with IRFs using an exogenous AR(1) process calibrated to match the persistence of shocks estimated in Castro et al. (2011, 2015)[17][18]. Considering the different samples used in the estimation here and in Castro et al. (2011, 2015)[17][18], the calibration adopted for the exercise keeps for each observed variable the variance estimated in the structural VAR, but eliminates the covariance and the cross-terms for other variables across equations. The objective of the exercise is to understand the gains in using a structural VAR to estimate the transmission process of foreign shocks to the domestic economy. It is well documented in Justiniano and Preston (2010)[31] the difficulties of small open-economy models to generate significant impact from foreign shocks to the domestic economy. Though structural models have a hard time in matching international co-movement, a multivariate approach to the foreign block considers the interaction between demand, supply, and monetary policy in the world economy. Moreover, a structural VAR leads to the identification of shocks with a clear economic interpretation compared with simple autoregressions, in which a particular shock does not propagate to other international variables of interest. The exercise here provides some evidence on the importance of second round effects to amplify the impact of foreign shocks on the Brazilian economy. In this context, a particular shock contemporaneously affects all international variables that interact with the domestic economy, enriching the propagation mechanism of that shock.

The first example on the difference of representing the rest of the world as a VAR is presented in figure 8, with the IRF of a monetary policy shock from the rest of the world. A contractionary shock of 100bps generates, by the feedback rule estimated in the structural VAR, a persistent process where nominal interest rates increase 300bps after six periods. Consequently, the output gap of the rest of the world moves to negative values. This is in sharp contrast with the AR(1) process, where, by construction, the increase in nominal interest rates does not affect the world's output gap. The impact in the domestic economy, in both cases, starts with a real exchange rate depreciation, but the expectations of further nominal interest rates increase generate a larger depreciation on impact in the model with the VAR. According to Azad and Serletis (2019)[9], this pattern of exchange rate change after a foreign monetary policy shock is common to other emerging economies with inflation targeting regimes. The exchange rate depreciation combined with a negative output gap in the rest of the world generates a tension for domestic monetary policy: on the one hand, the pass-through of the exchange rate tends to increase inflation; on the other hand, the negative world's output gap combined with the increase in production costs from the exchange rate devaluation reduces domestic output. The estimated Taylor rule suggests an initial reduction in nominal exchange rates. However, with the increase in inflation, real interest rates rise to stabilize prices.

Figure 8: IRF: Foreign Monetary Policy Shock

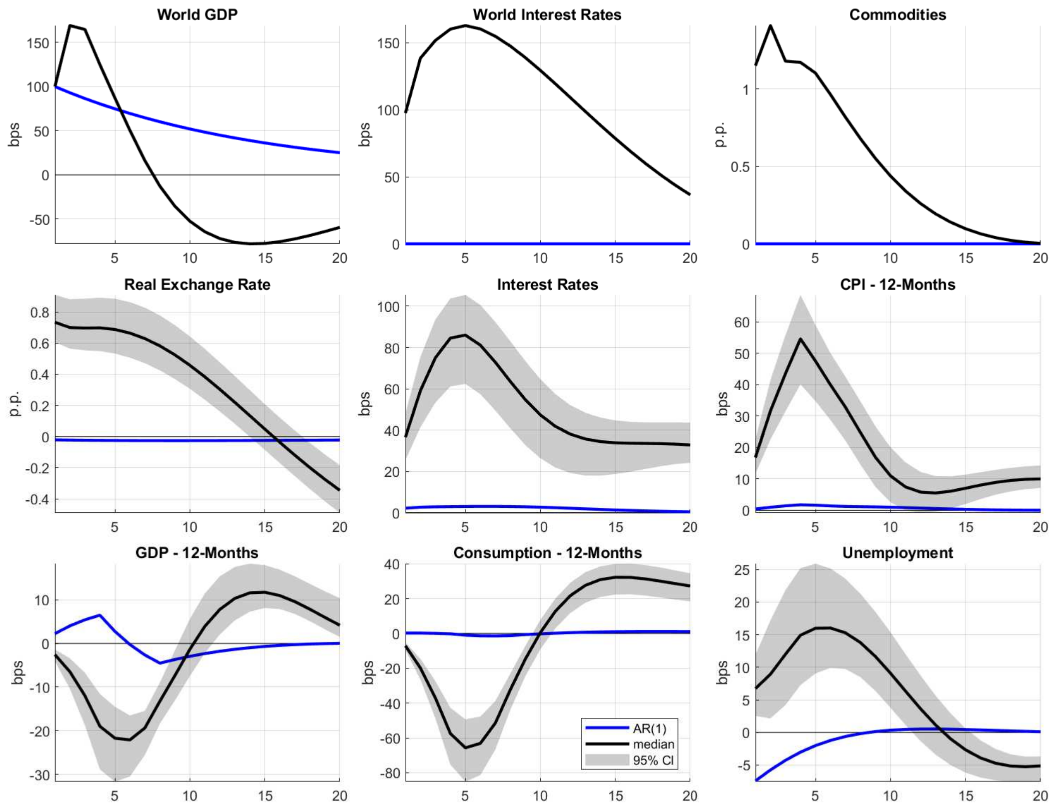


The figure shows the median impulse response functions (black line), the 95% credible intervals (CI, shaded area) for the DSGE model, and the impulse response function for the model without the structural VAR (blue line). Parameters of the structural VAR are kept constant across SMC particles.

Results with the AR(1) process follow the expected profile of first round effects in the model: the exchange rate depreciation increases domestic prices, reducing domestic demand and resulting in an increase of domestic nominal interest rates. However, the magnitude of the responses suggests the second-order effects are relevant to characterize a significant transmission of foreign monetary policy shocks to the Brazilian economy.

The second example, in figure 9, shows the IRF of an increase in world's demand, represented by an exogenous shock that increases the world's output gap by 100bps. In the model with the structural VAR, the increase in demand is followed by an increase in foreign interest rates. The response of foreign interest rates creates a new tension for domestic variables, as the real exchange rates depreciates. It is a stark contrast with the responses from the AR(1) process, where the increase in world's demand generates a trade surplus, followed by real exchange rate appreciation. The exchange rate depreciation in the model with the structural VAR amplifies the impact on domestic inflation, forcing domestic interest rates to increase. The estimated Taylor rule suggests the increase in interest rates is enough to generate a small contraction of domestic real GDP, offsetting the increase in foreign demand for Brazilian exports.

Figure 9: IRF: Foreign Demand Shock



The figure shows the median impulse response functions (black line), the 95% credible intervals (CI, shaded area) for the DSGE model, and the impulse response function for the model without the structural VAR (blue line). Parameters of the structural VAR are kept constant across SMC particles.

Effects of the model with the AR(1) differ not only quantitatively, but also with respect to the sign of responses, especially for domestic economic activity. This is a consequence of the missing tension generated from the real exchange rate depreciation in the model with the structural VAR. Without the exchange rate depreciation, domestic interest rates do not show a significant increase, allowing for the business cycle of the domestic economy closely follow the rest of the world, with increases in GDP and consumption and a fall in unemployment rate.

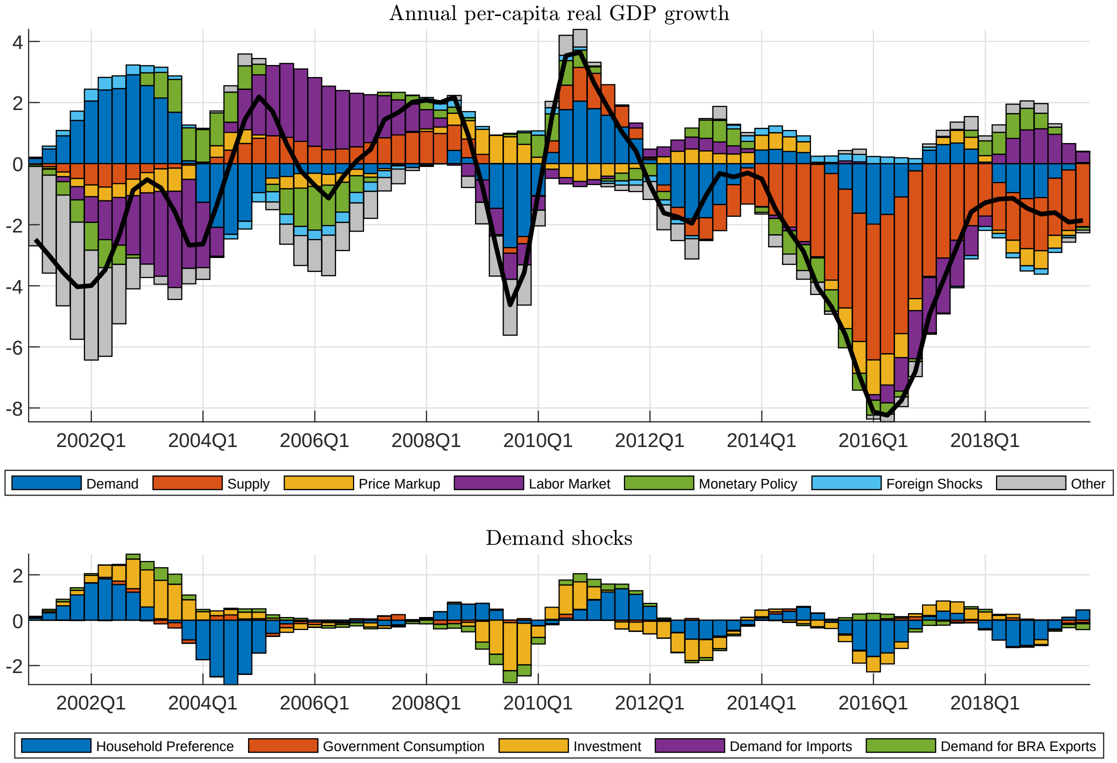
4.4 Shock Decomposition

This subsection presents the shock decomposition of three observed variables of the model: output growth, CPI inflation and unemployment. All variables are measured as deviation with respect to their steady state, except CPI inflation, that is expressed as a deviation from the inflation target at the period. Inflation and output are measured in annual terms. For simplicity, shocks are aggregated in the following groups: demand, supply, price markups, labor market, monetary policy, foreign shocks, and "others". Demand shocks include the preference shock (ε_t^C), investment-specific technology shock (ε_t^I), government spending shock (ε_t^G), import demand shock (ε_t^M), and export demand shock (ε_t^{M*}). Supply shocks include all changes in aggregate productivity: both shocks describing the non-stationary productivity (ε_t^{ZT} and ε_t^{ZC}) and the stationary productivity shock (ε_t^D). Price markups comprise shocks in the sectoral Phillips curves of freely-set (ε_t^P) and administered price firms (ε_t^A), import prices (ε_t^{QM*}) and export prices (ε_t^{P*}). Labor market shocks include the labor supply shock (ε_t^L) and the wage markup shock (ε_t^W). Wage markup shocks are included here due to the importance of such shocks in characterizing the dynamics of unemployment in the Galí, Smets and Wouters (2011)[28] framework. Monetary policy shocks (ε_t^R) characterize the deviations of interest rates from the path projected by the Taylor rule of the model. Foreign shocks comprise all shocks from the structural VAR (ε_t^{Y*}, ε_t^{R*}, ε_t^{P*}, ε_t^{V*}, and ε_t^{CO*}), and the shocks to the foreign risk premium (ε_t^{B*}) and real exchange rate (ε_t^Q).

The first panel of figure 10 characterizes the shock decomposition of annual GDP growth based on the groups described above. From the supply side, it shows the change in productivity growth observed after 2014 and highlighted before in figure 1. Overall, external shocks, monetary policy and markup shocks play a small role in GDP growth. The first panel also shows the increase in GDP volatility during and after the Great Financial Crisis, in 2008. With respect to the crisis, the model describes both the recession and the recovery as mostly demand-driven events. However, the second panel provides more clarity about the drivers of the recession and the recovery: the recession was characterized by negative shocks in exports and investment, while the recovery was based on the recuperation of these components, combined with significant positive shocks on household consumption. With respect to the role of exports, the negative shock is a consequence of a mismatch in data between world GDP and domestic exports: world GDP gap reached a peak around 2003 and was in a sharp reversal even before the crisis, while Brazilian exports was still showing significant growth before 2008Q3. The role of the investment shock during 2008 is an issue for models without financial frictions, as the lack of endogenous propagation mechanisms forces the model to give a significant weight to the shock in order to match data.

Beyond the recovery after the Great Financial Crisis, between 2010 and 2012, the disaggregation of demand shocks in the second panel also captures other periods where exogenous stimuli increased household consumption. In 2014, several government policies tried to stimulate credit

Figure 10: Shock Decomposition: Annual GDP – Deviation from Steady State



The first panel shows the shock decomposition of annual GDP based on the groups of shocks. The second panel shows demand shocks from the first panel disaggregated by their components.

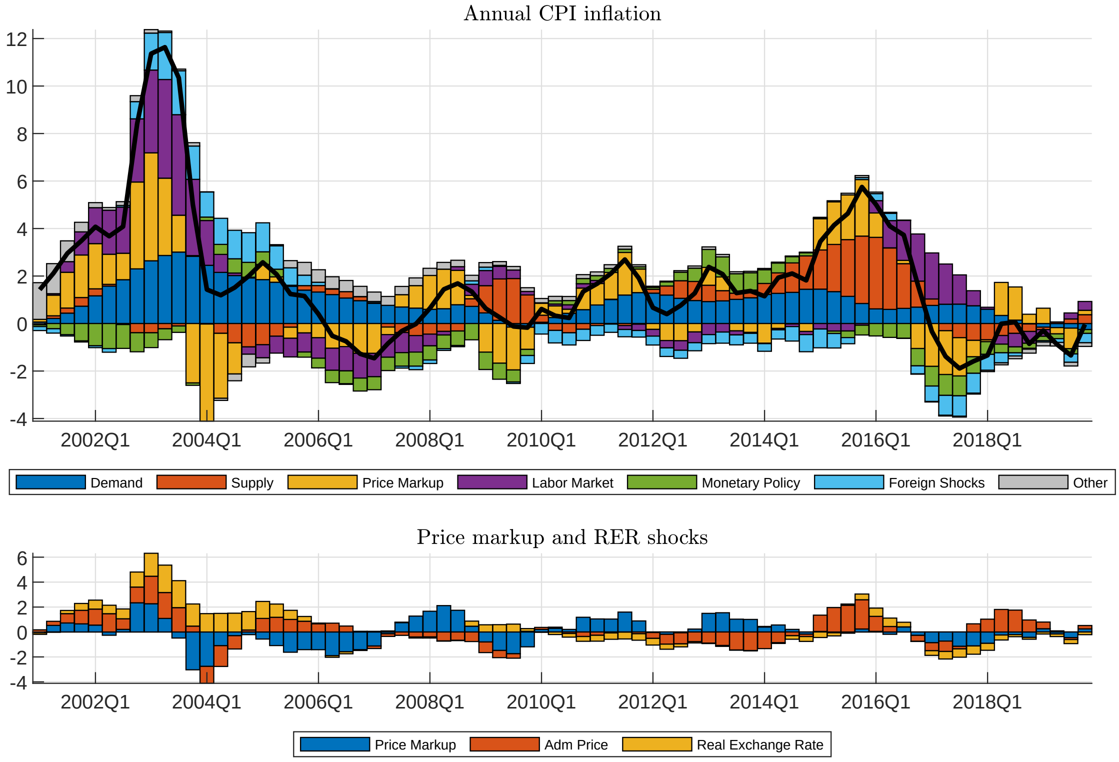
markets, including lowering reserve requirements ratios for deposits that banks must hold at the Banco Central do Brasil, both for firms and households. Another important period is the first semester of 2017, when the federal government allowed workers to make early withdrawals from inactive accounts of a severance fund (*FGTS*, in Portuguese).

Figure 11 shows the shock decomposition of annual CPI inflation. Demand shocks play a significant role, as expected, as they include unexpected changes in both the domestic demand for goods and the external demand for Brazilian goods. Supply shocks have a large role after 2014, just like GDP. However, the effects of the supply shocks on prices move to the usual contribution after 2017, the opposite of GDP growth, where these shocks are still relevant. Monetary policy was clearly in a contractionary stance from 2006 until the end of 2009, which includes the Great Financial Crisis[30]. Monetary policy was expansionary between 2012 and 2014, moving back to the contractionary stance after a significant change in economic policy in 2015, right before the

[30]It is worth noting that the Monetary Policy Committee raised nominal interest rates from 11.25% to 13.75% between March and September 2008, while acknowledging that *"a severe financial crisis in the USA and, to a minor extent, in Europe, must be added to this adverse scenario"*, according to the September 2008 MPC meeting's minutes.

2016 political crisis. Labor market shocks pressure inflation in both periods when unemployment is high, at the beginning and at the very end of the sample, after 2014 (see figure 12), suggesting wages have had problems in clearing labor markets specifically when labor demand is low – assuming a relative stability of labor supply, observed in data on participation rate.

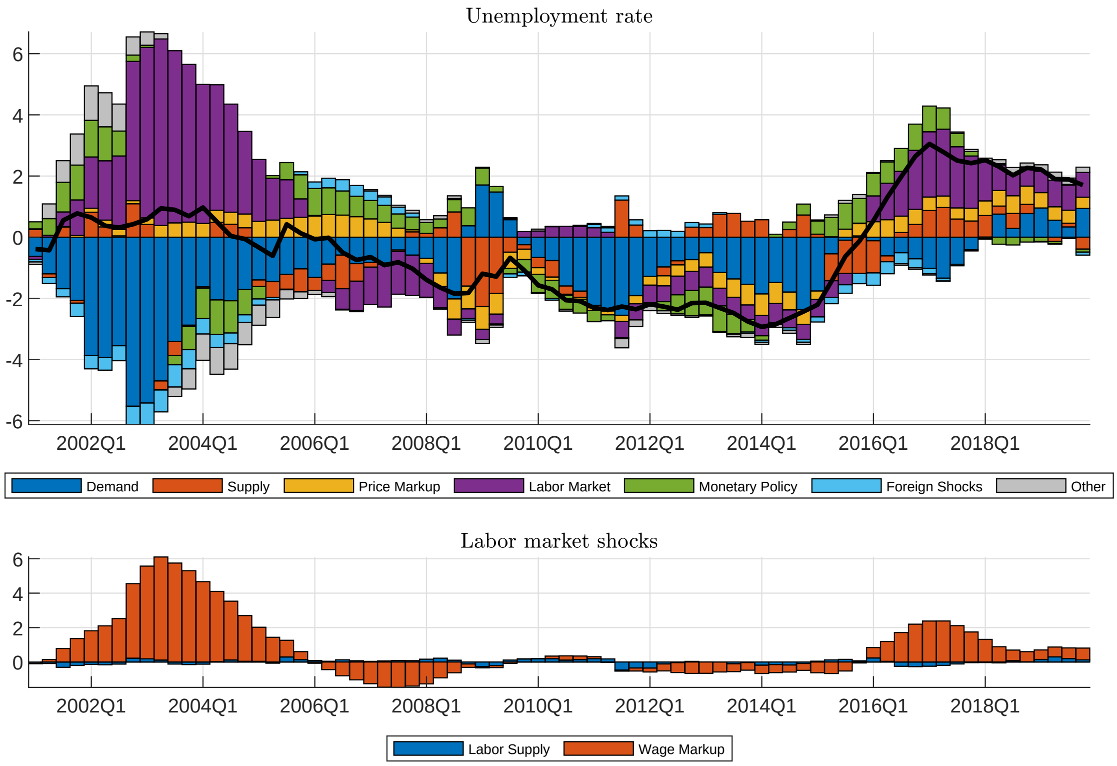
Figure 11: Shock Decomposition: Annual CPI Inflation – Deviation from Inflation Target



The first panel shows the shock decomposition of annual CPI inflation based on the groups of shocks. The second panel shows specific shocks from the first panel.

Price markups move in the way proposed in the description of priors for estimation, characterizing unexpected short-lived movements in prices, usually associated with unique episodes, like unexpected changes in monitored prices, food prices and other events. The lower panel of figure 11 highlights the role of markups of freely-set and administered prices, and exchange rate shocks. Again, the decomposition is consistent with well-documented episodes of temporary price changes discussed in official documents. As an example, the lower panel shows the unexpected rapid fall in inflation during 2003-04, considering the severe crisis that hit the economy before the 2002 elections. The lower panel also shows the increase in administered price markups in 2015Q1, combined with an unexpected real exchange rate devaluation, consistent with the change in economic policy during that period. Markups of administered price remained negative between 2012 and 2014, quickly increasing at the beginning of 2015. Another episode worth noting is the sequence of positive shocks in food prices that reduced freely-set prices between 2017 and 2018.

Figure 12: Shock Decomposition: Unemployment Rate – Deviation from Steady State



The first panel shows the shock decomposition of unemployment rate based on the groups of shocks. The second panel shows labor market shocks from the first panel disaggregated by their components.

Figure 12 shows the decomposition of unemployment rate. It is interesting to observe, in the higher panel, that, as expected, labor market shocks play a significant role in unemployment dynamics. However, the importance of these shocks is highlighted in periods of high unemployment. Other shocks usually dominate the decomposition in periods of low unemployment. Notably, demand shocks in unemployment pretty much mirror the behavior in the decomposition of CPI inflation presented before. Just as expected, periods where demand shocks raise inflation are the same periods where these shocks reduce unemployment. It is remarkable, however, that the relative importance of demand shocks in both observed variables is very similar, even in terms of dynamics. It is interesting that the supply shocks that reduced GDP growth after 2014 took some time in affecting unemployment, with most of the increase in unemployment due to low productivity beeing observed after 2016. Foreign shocks and monetary policy shocks have a small contribution in the decomposition. From the lower panel, of figure 12, it is worth noting that the labor supply shocks are dominated by the wage markup shocks among the labor market shocks. The relative stability of labor supply in the sample supports the small role of labor supply shocks, especially compared to the high variance of unemployment rate.

The shock decomposition of unemployment shows the importance of the Galí, Smets and Wouters (2011)[28] in interpreting periods of high unemployment in the economy. First, in the early part of the sample, the model is forced to reconcile low economic growth, high inflation, and high unemployment. The shock decomposition suggests a combination of positive demand shocks (focused on household consumption and investment, according to the lower panel of figure 10) with negative wage markup shocks. While both sets of shocks generate high inflation, wage markup shocks are set to generate low economic activity and high unemployment between 2001 and 2006. In the second period, between 2016 and 2019 (end of the sample), the model must adjust itself to a situation of high unemployment and low growth again, but now with declining inflation. In this case, the shock decomposition suggests wage markup shocks provide inflation persistence that would not be observed in an environment of low GDP growth. For both episodes, it is worth noting that wage markups play a more prominent, truly structural role instead of simply matching moments of wage growth in the Phillips curve.

5 Conclusion

This paper presented the current version of the SAMBA model, a DSGE model developed at the Banco Central do Brasil. Compared to the original version, the current model presents significant innovations both in terms of modeling choices and estimation procedures. The new framework characterizing the labor market, based on Galí, Smets and Wouters (2011)[28], allowed for a better description of the labor market dynamics, more appropriate when compared to the basic framework for labor supply based on the representative agent offering hours of work to firms. Under the new framework, it is possible to incorporate information on the labor supply, in line with the conventional information on wages and unemployment rate.

The use of a structural VAR to characterize the rest of the world also allows for an amplification of the transmission mechanism of foreign shocks to the domestic economy. It helps in alleviating the concerns raised in Justiniano and Preston (2010)[31], as it amplifies the effects from foreign observed variables in small open economy models like SAMBA. The exercises with IRFs show a clear improvement, compared to a simple AR(1) specification for foreign shocks. However, the total impact of foreign shocks still seems small, especially during stress periods, like the Great Financial Crisis, as shown in the shock decomposition.

Finally, the use of Sequential Monte Carlo methods, together with the adoption of "system priors" not only provided stability for the estimation of the structural parameters in a large model like SAMBA, but also offered the tools for understanding the trade-offs faced when matching data moments in a complex theoretical environment. From a practical perspective, these methods also allowed for a better use of technological resources, like GPU computing, as it improved the possibilities of parallel implementations optimizing the estimation procedure.

The model presented here is continually developed, with several possible improvements for policy analysis. With respect to estimation, it is still a challenge to incorporate information related to the Covid-19 pandemic crisis. The usual linear-Gaussian framework for likelihood inference based on the Kalman Filter does not seem capable of handling the extreme variations of observed variables during 2020. With respect to model fit, the structural VAR characterizing the rest of the world provides a good benchmark for the analysis of the transmission of foreign shocks to the domestic economy. However, the model is muted with respect to possible policy changes associated with the lower bound of nominal interest rates observed in major economies. Models with financial frictions should also be considered in order to improve the model fit during crises episodes.

References

[1] FRBNY DSGE Model Documentation. Technical report, Federal Reserve Bank of New York, Dec. 2021.

[2] S. Adjemian, H. Bastani, M. Juillard, F. Karamé, F. Mihoubi, W. Mutschler, J. Pfeifer, M. Ratto, N. Rion, and S. Villemot. Dynare: Reference manual version 5. Dynare Working Papers 72, CEPREMAP, 2022.

[3] M. Adolfson, S. Laseen, L. J. Christiano, M. Trabandt, and K. Walentin. Ramses II – Model Description. Occasional Paper Series 12, Sveriges Riksbank, 2013.

[4] M. Adolfson, S. Laseen, J. Linde, and M. Villani. Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through. *Journal of International Economics*, 72(2):481–511, 2007.

[5] S. A. L. Alves and A. M. Fasolo. Not Just Another Mixed Frequency Paper. Working Paper Series 400, Banco Central do Brasil, Research Department, 2015.

[6] M. Andrle. What Is in Your Output Gap? Unified Framework & Decomposition into Observables. IMF Working Papers 2013/105, International Monetary Fund, 2013.

[7] M. Andrle and J. Benes. System Priors: Formulating Priors about DSGE Models System Properties. IMF Working Papers 2013/257, International Monetary Fund, 2013.

[8] M. Andrle and M. Plašil. Econometrics with system priors. *Economics Letters*, 172:134–137, 2018.

[9] N. F. Azad and A. Serletis. Monetary Policy Spillovers in Emerging Economies. *International Journal of Finance & Economics*, 25(4):664–683, 2020.

[10] C. F. Azevedo and A. M. Fasolo. Effective Tax Rates on Consumption and Factor Incomes: a Quarterly Frequency Estimation for Brazil. Working Paper Series 398, Banco Central do Brasil, Research Department, 2015.

[11] M. Bonomo, R. D. Brito, and B. Martins. The After Crisis Government-Driven Credit Expansion in Brazil: A Firm Level Analysis. *Journal of International Money and Finance*, 55:111–134, 2015.

[12] M. Bonomo and B. Martins. The Impact of Government-Driven Loans in the Monetary Transmission Mechanism: What Can We Learn from Firm-Level Data? Working Paper Series 419, Banco Central do Brasil, Research Department, 2016.

[13] F. Brazdik, T. Hledik, Z. Humplova, I. Martonosi, K. Musil, J. Rysanek, T. Sestorad, J. Tonner, S. Tvrz, and J. Zacek. The g3+ Model: An Upgrade of the Czech National Bank's Core Forecasting Framework. Technical Report 2020/7, Czech National Bank, 2020.

[14] M. Cai, M. Del Negro, E. Herbst, E. Matlin, R. Sarfati, and F. Schorfheide. Online Estimation of DSGE Models. *The Econometrics Journal*, 24(1):C33–C58, 2021.

[15] F. Canova. The Transmission of US Shocks to Latin America. *Journal of Applied Econometrics*, 20(2):229–251, 2005.

[16] F. A. Carvalho, M. R. Castro, and S. M. A. Costa. Traditional and Matter-of-Fact Financial Frictions in a DSGE Model for Brazil: the Role of Macroprudential Instruments and Monetary Policy. Bank for International Settlements Working Papers 460, 2014.

[17] M. R. Castro, S. N. Gouvea, A. Minella, R. Santos, and N. F. Souza-Sobrinho. SAMBA: Stochastic Analytical Model with a Bayesian Approach. Working Paper Series 239, Banco Central do Brasil, Research Department, 2011.

[18] M. R. Castro, S. N. Gouvea, A. Minella, R. Santos, and N. F. Souza-Sobrinho. SAMBA: Stochastic Analytical Model with a Bayesian Approach. *Brazilian Review of Econometrics*, 35(2):103–170, 2015.

[19] R. Chetty. Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply. *Econometrica*, 80(3):969–1018, 2012.

[20] G. Coenen, P. Karadi, S. Schmidt, and A. Warne. The New Area-Wide Model II: an Extended Version of the ECB's Micro-Founded Model for Forecasting and Policy Analysis with a Financial Sector. Working Paper Series 2200, European Central Bank, 2018.

[21] V. Corbo and I. Strid. MAJA: A Two-Region DSGE Model for Sweden and its Main Trading Partners. Working Paper Series 391, Sveriges Riksbank, 2020.

[22] P. Corrigan, H. Desgagnés, J. Dorich, V. Lepetyuk, W. Miyamoto, and Y. Zhang. ToTEM III: The Bank of Canada's Main DSGE Model for Projection and Policy Analysis. Technical Reports 119, Bank of Canada, 2021.

[23] S. M. A. Costa. Structural Trends and Cycles in a DSGE Model for Brazil. Working Paper Series 434, Banco Central do Brasil, Research Department, 2016.

[24] M. Del Negro, M. Lenza, G. E. Primiceri, and A. Tambalotti. What's Up with the Phillips Curve? *Brookings Papers on Economic Activity*, 2020(1):301–373, 2020.

[25] M. Del Negro and F. Schorfheide. Forming Priors for DSGE Models (and How it Affects the Assessment of Nominal Rigidities). *Journal of Monetary Economics*, 55(7):1191–1208, 2008.

[26] R. M. Edge, T. Laubach, and J. C. Williams. Learning and Shifts in Long-Run Productivity Growth. *Journal of Monetary Economics*, 54(8):2421–2438, 2007.

[27] C. J. Erceg, D. W. Henderson, and A. T. Levin. Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*, 46(2):281–313, 2000.

[28] J. Galí, F. Smets, and R. Wouters. Unemployment in an Estimated New Keynesian Model. In *NBER Chapters*, volume 26 of *NBER Macroeconomics Annual 2011*, pages 329–360. NBER, 2011.

[29] J. Greenwood, Z. Hercowitz, and G. W. Huffman. Investment, Capacity Utilization, and the Real Business Cycle. *American Economic Review*, 78(3):402–17, 1988.

[30] J. Hazell, J. Herreño, E. Nakamura, and J. Steinsson. The Slope of the Phillips Curve: Evidence from U.S. States. *The Quarterly Journal of Economics*, 137(3):1299–1344, 2022.

[31] A. Justiniano and B. Preston. Can Structural Small Open-Economy Models Account for the Influence of Foreign Disturbances? *Journal of International Economics*, 81(1):61–74, 2010.

[32] A. Kraay and V. Nehru. When is External Debt Sustainable? *The World Bank Economic Review*, 20(3):341–365, 2006.

[33] M. J. Lombardi and G. Nicoletti. Bayesian Prior Elicitation in DSGE Models: Macro- vs Micropriors. *Journal of Economic Dynamics and Control*, 36(2):294–313, 2012.

[34] J. P. Medina and C. Soto. The Chilean Business Cycles Through the Lens of a Stochastic General Equilibrium Model. Working Papers 457, Central Bank of Chile, Dec. 2007.

[35] R. Pancrazi and M. Vukotić. Inflation Sensitivity to Monetary Policy: What Has Changed since the Early 1980s? *Oxford Bulletin of Economics and Statistics*, 81(2):412–436, 2019.

[36] J. Ramos-Tallada. Bank Risks, Monetary Shocks and the Credit Channel in Brazil: Identification and Evidence from Panel Data. *Journal of International Money and Finance*, 55:135–161, 2015.

[37] C. M. Reinhart, K. S. Rogoff, and M. A. Savastano. Debt Intolerance. *Brookings Papers on Economic Activity*, 34(1):1–74, 2003.

[38] J. J. Rotemberg and M. Woodford. The Cyclical Behavior of Prices and Costs. In *Handbook of Macroeconomics*, volume 1B, pages 1051–1135. 1999.

[39] S. Schmitt-Grohé and M. Uribe. Closing Small Open Economy Models. *Journal of International Economics*, 61(1):163–185, 2003.

[40] F. Smets and R. Wouters. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *The American Economic Review*, 97(3):586–606, 2007.

Appendices

A The Equilibrium System

Households

1. Aggregate Consumption

$$\tilde{C}_t = \varpi_{RT}\tilde{C}_t^{RT} + (1-\varpi_{RT})\,\tilde{C}_t^{O}$$

2. Aggregate Intratemporal conditions

$$\tilde{C}_t^{D} = o_C\left(Q_t^{C^D}\right)^{-v_C}\tilde{C}_t$$

$$\tilde{C}_t^{M} = (1-o_C)\left(Q_t^{M}\right)^{-v_C}\tilde{C}_t$$

3. CPI Inflation

$$\Pi_t^C = \left[o_C(\Pi_t^{C^D}Q_{t-1}^{C^D})^{1-v_C} + (1-o_C)(\Pi_t^M Q_{t-1}^M)^{1-v_C}\right]^{\frac{1}{1-v_C}}$$

4. Consumption for Rule-of-Thumb Households

$$\tilde{C}_t^{RT} = (1-\tau_t^N)\tilde{W}_t N_t$$

5. Marginal Utility of Consumption for Optimizing Households

$$\tilde{\Lambda}_t = Z_t^C\left(\tilde{C}_t^O - \kappa\frac{\tilde{C}_{t-1}^O}{Z_t^Z}\right)^{-\sigma}$$

6. The Euler Equation for Optimizing Households

$$Z_t^C\left(\tilde{C}_t^O - \kappa\frac{\tilde{C}_{t-1}^O}{Z_t^Z}\right)^{-\sigma} = \beta E_t Z_{t+i}^C\left(Z_{t+1}^Z\right)^{-\sigma}\left(\tilde{C}_{t+1}^O - \kappa\frac{\tilde{C}_t^O}{Z_{t+1}^Z}\right)^{-\sigma}\frac{R_t S_t^B}{\Pi_{t+1}^C}$$

7. Uncovered Interest Rate Parity Condition

$$E_t\left(\frac{R_t S_t^B}{\Pi_{t+1}^C}\left(Z_{t+1}^Z\right)^{-\sigma}\tilde{\Lambda}_{t+1}\right) = E_t\left(\frac{Q_{t+1}}{Q_t}\frac{R_t^* S_t^{B^*}}{\Pi_{t+1}^{C^*}}\left(Z_{t+1}^Z\right)^{-\sigma}\tilde{\Lambda}_{t+1}Z_t^Q\right)$$

8. Capital and Investment Decisions

$$\tilde{\Lambda}_t Q_t^K = \beta E_t\left(Z_{t+1}^Z\right)^{-\sigma}\tilde{\Lambda}_{t+1}\left[R_{t+1}^K + Q_{t+1}^K(1-\delta)\right]$$

$$1 = \frac{Q_t^K}{Q_t^I}\left[1-\frac{\vartheta_I}{2}\left(\frac{Z_t^Z\tilde{I}_t}{Z_t^I\tilde{I}_{t-1}}-Z_{ss}^Z\right)^2-\vartheta_I\left(\frac{Z_t^Z\tilde{I}_t}{Z_t^I\tilde{I}_{t-1}}-Z_{ss}^Z\right)\frac{Z_t^Z\tilde{I}_t}{Z_t^I\tilde{I}_{t-1}}\right]$$
$$+\frac{\beta}{\tilde{\Lambda}_t Q_t^I}E_t\left(Z_{t+1}^Z\right)^{-\sigma}\tilde{\Lambda}_{t+1}Q_{t+1}^K\vartheta_I\left(\frac{Z_{t+1}^Z\tilde{I}_{t+1}}{Z_{t+1}^I\tilde{I}_t}-Z_{ss}^Z\right)\left(\frac{Z_{t+1}^Z\tilde{I}_{t+1}}{Z_{t+1}^I\tilde{I}_t}\right)^2 Z_{t+1}^I$$

$$\tilde{K}_{t+1} = (1-\delta)\left(\tilde{K}_t/Z_t^Z\right)+\left[1-\frac{\vartheta_I}{2}\left(\frac{Z_t^Z\tilde{I}_t}{Z_t^I\tilde{I}_{t-1}}-Z_{ss}^Z\right)^2\right]\tilde{I}_t$$

9. Country Risk Premium ($S_t^{B^*}$)

$$S_t^{B^*} = S^{B^*}\left[\exp\left(-\varphi_B^*\left(B_{t+1}^{*^x}-B^{*^x}\right)+\varphi_V^*\left(V_t^*-V\right)+Z_t^{B^*}\right)\right]$$

10. Non-Linear Wage Phillips Curve

$$SW_t^1 = \tilde{\Lambda}_t(1-\tau_t^N)\tilde{W}_t^{\epsilon^W}N_t(\epsilon^W-1)+\theta_W\beta E_t\left[\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^W}\right)^{\epsilon^W-1}SW_{t+1}^1\right]$$

$$SW_t^2 = Z_t^C Z_t^W Z_t^L\psi\varphi_t^S\tilde{W}_t^{\epsilon^W(1+\eta)}N_t^{(1+\eta)}\epsilon^W+\theta_W\beta E_t\left[\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^W}\right)^{\epsilon^W(1+\eta)}SW_{t+1}^2\right]$$

$$\left(\tilde{W}_t^\star\right)^{1+\eta\epsilon^W}SW_t^1 = SW_t^2$$

11. Wage Indexation Rule

$$\Upsilon_t^W = \left(\Pi_{t-1}^W\right)^{\omega_W}\left(\left(Z_{t-1}^{ZC}\right)^4\Pi_{t-1}^{4C}\right)^{\frac{1-\omega_W}{4}}\left(\frac{1}{Z_t^Z}\right)$$

$$\Pi_t^{4C} = \Pi_t^C\Pi_{t-1}^C\Pi_{t-2}^C\Pi_{t-3}^C$$

12. Wage Index

$$\tilde{W}_t = \left(\theta_W\left(\frac{\Upsilon_t^W\tilde{W}_{t-1}}{\Pi_t^C}\right)^{1-\epsilon^W}+(1-\theta_W)\left(\tilde{W}_t^\star\right)^{1-\epsilon^W}\right)^{\frac{1}{1-\epsilon^W}}$$

13. Wage Inflation

$$\Pi_t^W = \left(\frac{\tilde{W}_t}{\tilde{W}_{t-1}}\right)\Pi_t^C Z_t^Z$$

14. Wage Dispersion and Adjusted Labor

$$v_t^O = (1-\theta_W)\left(\frac{\tilde{W}_t^{\star}}{\tilde{W}_t}\right)^{-\epsilon^W} + \theta_W\left(\frac{\Upsilon_t^W \tilde{W}_{t-1}}{\tilde{W}_t \Pi_t^C}\right)^{-\epsilon^W} v_{t-1}^O$$

$$v_t^W = \varpi_{RT} + (1-\varpi_{RT})v_t^O$$

$$\hat{N}_t = N_t v_t^W$$

15. Additional Equations from the GSW specification for Labor Markets

$$\varphi_t^S = \frac{\widetilde{C}_t^S \tilde{\Lambda}_t}{Z_t^C}$$

$$\widetilde{C}_t^S = \left(Z_t^Z\right)^{-(1-v)\sigma}\left(\widetilde{C}_{t-1}^S\right)^{1-v}\left(\frac{Z_t^C}{\tilde{\Lambda}_t}\right)^{v}$$

$$(1-\tau_t^N)\tilde{W}_t = Z_t^L \psi \widetilde{C}_t^S L_t^{\eta}$$

$$U_t^L = \frac{L_t - N_t}{L_t}$$

Firms - Domestic Input Producers

16. Capital Demand

$$R_t^K = \alpha Q_t^D \left(Z_t^D\right)^{\frac{\epsilon_D - 1}{\epsilon_D}}\left(\frac{Z_t^Z \tilde{Y}_t^D}{\tilde{K}_{t-1}}\right)^{\frac{1}{\epsilon_D}}$$

17. Labor Demand

$$\tilde{W}_t = (1-\alpha)\, Q_t^D \left(Z_t^D\right)^{\frac{\epsilon_D - 1}{\epsilon_D}}\left(\frac{\tilde{Y}_t^D}{N_t - \overline{N}}\right)^{\frac{1}{\epsilon_D}}$$

18. Domestic Input Supply

$$Q_t^D = \left[\alpha\left(\frac{R_t^K}{\alpha}\right)^{1-\epsilon_D} + (1-\alpha)\left(\frac{\tilde{W}_t}{1-\alpha}\right)^{1-\epsilon_D}\right]^{\frac{1}{1-\epsilon_D}} \frac{1}{Z_t^D}$$

Firms - Importers

19. Non-Linear Phillips Curve

$$SM_t^1 = \left(Q_t^M\right)^{\varepsilon^M} \widehat{\widetilde{M}}_t + \theta_M \beta E_t \left[\left(Z_{t+1}^Z\right)^{1-\sigma} \left(\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\right) \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^M}\right)^{\epsilon^M} SM_{t+1}^1 \right]$$

$$SM_t^2 = \left(Q_t^M\right)^{\varepsilon^M} Q_t Q_t^{M*} \widehat{\widetilde{M}}_t = \theta_M \beta E_t \left[\left(Z_{t+1}^Z\right)^{1-\sigma} \left(\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\right) \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^M}\right)^{-(1-\epsilon^M)} SM_{t+1}^2 \right]$$

$$Q_t^{M\star} = \frac{\varepsilon^M}{\varepsilon^M - 1} \left(\frac{SM_t^2}{SM_t^1}\right)$$

20. Imported Input Indexation Rule

$$\Upsilon_t^M = \Pi_{t-1}^M$$

21. Imported Input Price Index

$$Q_t^M = \left(\theta_M \left(\frac{\Upsilon_t^M Q_{t-1}^M}{\Pi_t^C}\right)^{1-\epsilon^M} + (1-\theta_M) \left(Q_t^{M\star}\right)^{1-\epsilon^M} \right)^{\frac{1}{1-\epsilon^M}}$$

22. Imported Input Inflation

$$\Pi_t^M = \frac{Q_t^M}{Q_{t-1}^M} \Pi_t^C$$

23. Price Dispersion and Adjusted Imports

$$v_t^M = (1-\theta_M) \left(\frac{Q_t^{M\star}}{Q_t^M}\right)^{-\epsilon^M} + \theta_M \left(\frac{\Upsilon_t^M}{\Pi_t^M}\right)^{-\epsilon^M} v_{t-1}^M$$

$$\widehat{\widetilde{M}}_t = \widetilde{M}_t v_t^M$$

Firms - Sectoral Intermediate Producers: Investment Good

24. Real Marginal Cost

$$MC_t^I = \left(\varpi_I \left(Q_t^D\right)^{1-\epsilon_I} + (1-\varpi_I)\left(\frac{Q_{I,t}^M}{1-\Gamma_{I,t}^M - \Gamma_{I,t}^{M\dagger}}\right)^{1-\epsilon_I}\right)^{\frac{1}{1-\epsilon_I}}$$

25. Domestic Input Demand

$$\tilde{Y}_{I,t}^D = \varpi_I \left(\frac{Q_t^D}{MC_t^I}\right)^{-\epsilon_I} \tilde{Y}_t^I v_t^I$$

26. Imported Input Demand

$$\tilde{M}_t^I = \left(\frac{1-\varpi_I}{1-\Gamma_{I,t}^M}\right)\left(\frac{Q_{I,t}^M}{\left(1-\Gamma_{I,t}^M - \Gamma_{I,t}^{M\dagger}\right) MC_t^I}\right)^{-\epsilon_I} \tilde{Y}_t^I v_t^I$$

27. Effective Import Cost

$$Q_{I,t}^M = \left[1 + \iota_I \left(R_t^* S_t^{B^*} - 1\right)\right] Q_t^M$$

28. Adjustment Cost for Imports ($\Gamma_{I,t}^M$) and its first derivative ($\Gamma_{I,t}^{M\dagger}$)

$$\Gamma_{I,t}^M = \frac{\vartheta_I^M}{2}\left(\left(Z_{I,t}^M\right)^{-\frac{1}{\vartheta_I^M}} \frac{\tilde{M}_t^I}{\tilde{M}_{t-1}^I} Z_t^Z - Z_{ss}^Z\right)^2$$

$$\Gamma_{I,t}^{M\dagger} = \vartheta_I^M \left(\left(Z_{I,t}^M\right)^{-\frac{1}{\vartheta_I^M}} \frac{\tilde{M}_t^I}{\tilde{M}_{t-1}^I} Z_t^Z - Z_{ss}^Z\right)\left(\left(Z_{I,t}^M\right)^{-\frac{1}{\vartheta_I^M}} \frac{\tilde{M}_t^I}{\tilde{M}_{t-1}^I} Z_t^Z\right)$$

29. Non-Linear Phillips Curve

$$SI_t^1 = \left(Q_t^I\right)^{\varepsilon_I^P} \widetilde{Y}_t^I + (\theta_I \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t}\right)\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^I}\right)^{\varepsilon_I^P - 1} SI_{t+1}^1\right]$$

$$SI_t^2 = \left(Q_t^I\right)^{\varepsilon_I^P} \widetilde{Y}_t^I MC_t^I Z_t^P + (\theta_I \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t}\right)\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^I}\right)^{\varepsilon_I^P} SI_{t+1}^2\right]$$

$$Q_t^{I\star} = \frac{\varepsilon_I^P}{\varepsilon_I^P - 1}\left(\frac{SI_t^2}{SI_t^1}\right)$$

30. Sectoral Indexation Rule

$$\Upsilon_t^I = \Pi_{t-1}^I$$

31. Sectoral Price Index

$$Q_t^I = \left(\theta_I \left(\frac{\Upsilon_t^I Q_{t-1}^I}{\Pi_t^C} \right)^{1-\epsilon_I^P} + (1-\theta_I) \left(Q_t^{I\star} \right)^{1-\epsilon_I^P} \right)^{\frac{1}{1-\epsilon_I^P}}$$

32. Sectoral Inflation

$$\Pi_t^I = \frac{Q_t^I}{Q_{t-1}^I} \Pi_t^C$$

33. Price Dispersion

$$v_t^I = (1-\theta_I) \left(\frac{Q_t^{I\star}}{Q_t^I} \right)^{-\epsilon_I^P} + \theta_I \left(\frac{\Upsilon_t^I}{\Pi_t^I} \right)^{-\epsilon_I^P} v_{t-1}^I$$

Firms - Sectoral Intermediate Producers: Government Consumption Good

34. Real Marginal Cost

$$MC_t^G = Q_t^D$$

35. Domestic Input Demand

$$\tilde{Y}_{G,t}^D = \tilde{Y}_t^G v_t^G$$

36. Non-Linear Phillips Curve

$$SG_t^1 = \left(Q_t^G\right)^{\varepsilon_G^P} \widetilde{Y}_t^G + (\theta_G \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \right) \left(Z_{t+1}^Z\right)^{1-\sigma} \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^G} \right)^{\varepsilon_G^P - 1} SG_{t+1}^1 \right]$$

$$SG_t^2 = \left(Q_t^G\right)^{\varepsilon_G^P} \widetilde{Y}_t^G MC_t^G Z_t^P + (\theta_G \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \right) \left(Z_{t+1}^Z\right)^{1-\sigma} \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^G} \right)^{\varepsilon_G^P} SG_{t+1}^2 \right]$$

$$Q_t^{G\star} = \frac{\varepsilon_G^P}{\varepsilon_G^P - 1} \left(\frac{SG_t^2}{SG_t^1} \right)$$

37. Sectoral Indexation Rule

$$\Upsilon_t^G = \Pi_{t-1}^G$$

38. Sectoral Inflation

$$\Pi_t^G = \frac{Q_t^G}{Q_{t-1}^G} \Pi_t^C$$

39. Sectoral Index

$$Q_t^G = \left(\theta_G \left(\frac{\Upsilon_t^G Q_{t-1}^G}{\Pi_t^C}\right)^{1-\epsilon_G^P} + (1-\theta_G)\left(Q_t^{G\star}\right)^{1-\epsilon_G^P}\right)^{\frac{1}{1-\epsilon_G^P}}$$

40. Price Dispersion

$$v_t^G = (1-\theta_G)\left(\frac{Q_t^{G\star}}{Q_t^G}\right)^{-\epsilon_G^P} + \theta_G \left(\frac{\Upsilon_t^G}{\Pi_t^G}\right)^{-\epsilon_G^P} v_{t-1}^G$$

Firms - Sectoral Intermediate Producers: Consumption Good

41. Real Marginal Cost

$$MC_t^{C^D} = \left(\varpi_{C^D}\left(Q_t^D\right)^{1-\epsilon_{C^D}} + (1-\varpi_{C^D})\left(\frac{Q_{C^D,t}^M}{1-\Gamma_{C^D,t}^M - \Gamma_{C^D,t}^{M\dagger}}\right)^{1-\epsilon_{C^D}}\right)^{\frac{1}{1-\epsilon_{C^D}}}$$

42. Domestic Input Demand

$$\tilde{Y}_{C^D,t}^D = \varpi_{C^D}\left(\frac{Q_t^D}{MC_t^{C^D}}\right)^{-\epsilon_{C^D}} \tilde{Y}_t^{C^D} v_t^{C^D}$$

43. Imported Input Demand

$$\tilde{M}_t^{C^D} = \left(\frac{1-\varpi_{C^D}}{1-\Gamma_{C^D,t}^M}\right)\left(\frac{Q_{C^D,t}^M}{\left(1-\Gamma_{C^D,t}^M - \Gamma_{C^D,t}^{M\dagger}\right) MC_t^{C^D}}\right)^{-\epsilon_{C^D}} \tilde{Y}_t^{C^D} v_t^{C^D}$$

44. Effective Import Cost

$$Q_{C^D,t}^M = \left[1 + \iota_{C^D}\left(R_t^* S_t^{B^*} - 1\right)\right] Q_t^M$$

45. Adjustment Cost for Imports ($\Gamma_{C^D,t}^M$) and its first derivative ($\Gamma_{C^D,t}^{M\dagger}$)

$$\Gamma_{C^D,t}^M = \frac{\vartheta_{C^D}^M}{2}\left(\left(Z_{C^D,t}^M\right)^{-\frac{1}{\vartheta_{C^D}^M}} \frac{\tilde{M}_t^{C^D}}{\tilde{M}_{t-1}^{C^D}} Z_t^Z - Z_{ss}^Z\right)^2$$

$$\Gamma_{C^D,t}^{M\dagger} = \vartheta_{C^D}^M \left(\left(Z_{C^D,t}^M\right)^{-\frac{1}{\vartheta_{C^D}^M}} \frac{\tilde{M}_t^{C^D}}{\tilde{M}_{t-1}^{C^D}} Z_t^Z - Z_{ss}^Z\right)\left(\left(Z_{C^D,t}^M\right)^{-\frac{1}{\vartheta_{C^D}^M}} \frac{\tilde{M}_t^{C^D}}{\tilde{M}_{t-1}^{C^D}} Z_t^Z\right)$$

46. Non-Linear Phillips Curve for goods with freely-set prices

$$SF_t^1 = (Q_t^{C^D})^{\epsilon_{C^D}^P} \tilde{Y}_t^{C^D} + (\theta_F \beta) E_t \left[\left(\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\right)\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^F}\right)^{\varepsilon_{C^D}^P - 1} SF_{t+1}^1\right]$$

$$SF_t^2 = (Q_t^{C^D})^{\epsilon^P_{C^D}} \widetilde{Y}_t^{C^D} MC_t^{C^D} Z_t^P + (\theta_F \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \right) (Z_{t+1}^Z)^{1-\sigma} \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^F} \right)^{\varepsilon^P_{C^D}} SF_{t+1}^2 \right]$$

$$Q_t^{F\star} = \frac{\varepsilon^P_{C^D}}{\varepsilon^P_{C^D} - 1} \left(\frac{SF_t^2}{SF_t^1} \right)$$

47. Indexation Rule for goods with freely set prices

$$\Upsilon_t^F = \Pi_{t-1}^C$$

48. Price Index for goods with freely set prices

$$Q_t^F = \left(\theta_F \left(\frac{\Upsilon_t^F Q_{t-1}^F}{\Pi_t^C} \right)^{1-\epsilon^P_{C^D}} + (1-\theta_F) \left(Q_t^{F\star} \right)^{1-\epsilon^P_{C^D}} \right)^{\frac{1}{1-\epsilon^P_{C^D}}}$$

49. Inflation for goods with freely set prices

$$\Pi_t^F = \frac{Q_t^F}{Q_{t-1}^F} \Pi_t^C$$

50. Non-Linear Phillips Curve for goods with administered prices

$$SA_t^1 = (Q_t^{C^D})^{\epsilon^P_{C^D}} \widetilde{Y}_t^{C^D} + (\theta_A \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \right) (Z_{t+1}^Z)^{1-\sigma} \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^A} \right)^{\varepsilon^P_{C^D}-1} SA_{t+1}^1 \right]$$

$$SA_t^2 = (Q_t^{C^D})^{\epsilon^P_{C^D}} \widetilde{Y}_t^{C^D} MC_t^{C^D} Z_t^P + (\theta_A \beta) E_t \left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t} \right) (Z_{t+1}^Z)^{1-\sigma} \left(\frac{\Pi_{t+1}^C}{\Upsilon_{t+1}^A} \right)^{\varepsilon^P_{C^D}} SA_{t+1}^2 \right]$$

$$Q_t^{A\star} = \frac{\varepsilon^P_{C^D}}{\varepsilon^P_{C^D} - 1} \left(\frac{SA_t^2}{SA_t^1} \right)$$

51. Price Setting for goods with administered prices

$$Q_t^A = \left(\theta_A \left(\Upsilon_t^A \frac{Q_{t-1}^A}{\Pi_t^C} \right)^{1-\epsilon^P_{C^D}} + (1-\theta_A) \left(Q_t^{A\star} \right)^{1-\epsilon^P_{C^D}} \right)^{\frac{1}{1-\epsilon^P_{C^D}}}$$

52. Indexation Rule for goods with administered prices

$$\Upsilon_t^A = \left\{ (\Pi_{t-1}^{4C})^{\frac{1}{4}} \left(\frac{Q_t}{Q_{t-1}} \right)^{\upsilon_A^1} \left(\frac{Q_t}{Q_{t-1}} \Pi_t^{co,*} \right)^{\upsilon_A^2} \right\} Z_t^A$$

53. Inflation for goods with administered prices

$$\Pi_t^A = \frac{Q_t^A}{Q_{t-1}^A}\Pi_t^C$$

54. Relative price for domestic consumption

$$Q_t^{C^D} = \left[\varpi_A \left(Q_t^A\right)^{1-\epsilon_{C^D}^P} + (1-\varpi_A)\left(Q_t^F\right)^{1-\epsilon_{C^D}^P}\right]^{\frac{1}{1-\epsilon_{C^D}^P}}$$

55. Inflation for domestic consumption

$$\Pi_t^{C^D} = \frac{Q_t^{C^D}}{Q_{t-1}^{C^D}}\Pi_t^C$$

56. Price Dispersion

$$v_t^F = (1-\theta_F)\left(\frac{Q_t^{F\star}}{Q_t^F}\right)^{-\epsilon_{C^D}^P} + \theta_F\left(\frac{\Upsilon_t^F}{\Pi_t^F}\right)^{-\epsilon_{C^D}^P} v_{t-1}^F$$

$$v_t^A = (1-\theta_A)\left(\frac{Q_t^{A\star}}{Q_t^A}\right)^{-\epsilon_{C^D}^P} + \theta_A\left(\frac{\Upsilon_t^A}{\Pi_t^A}\right)^{-\epsilon_{C^D}^P} v_{t-1}^A$$

$$v_t^{C^D} = \varpi_A\left(\frac{Q_t^A}{Q_t^{C^D}}\right)^{-\epsilon_{C^D}^P} v_t^A + (1-\varpi_A)\left(\frac{Q_t^F}{Q_t^{C^D}}\right)^{-\epsilon_{C^D}^P} v_t^F$$

Firms - Sectoral Intermediate Producers: Export Good

57. Real Marginal Cost

$$MC_t^X = \left(\varpi_X\left(Q_t^D\right)^{1-\epsilon_X} + (1-\varpi_X)\left(\frac{Q_{X,t}^M}{1-\Gamma_{X,t}^M-\Gamma_{X,t}^{M\dagger}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}}$$

58. Domestic Input Demand

$$\tilde{Y}_{X,t}^D = \varpi_X\left(\frac{Q_t^D}{MC_t^X}\right)^{-\epsilon_X}\tilde{Y}_t^X v_t^X$$

59. Imported Input Demand

$$\tilde{M}_t^X = \left(\frac{1-\varpi_X}{1-\Gamma_{X,t}^M}\right)\left(\frac{Q_{X,t}^M}{\left(1-\Gamma_{X,t}^M-\Gamma_{X,t}^{M\dagger}\right)MC_t^X}\right)^{-\epsilon_X}\tilde{Y}_t^X v_t^X$$

60. Effective Import Cost

$$Q_{X,t}^M = \left[1+\iota_X\left(R_t^* S_t^{B^*}-1\right)\right]Q_t^M$$

61. Adjustment Cost for Imports ($\Gamma_{X,t}^M$) and its first derivative ($\Gamma_{X,t}^{M\dagger}$)

$$\Gamma_{X,t}^{M} = \frac{\vartheta_X^M}{2}\left(\left(Z_{X,t}^{M}\right)^{-\frac{1}{\vartheta_X^M}} \frac{\tilde{M}_t^X}{\tilde{M}_{t-1}^X} Z_t^Z - Z_{ss}^Z\right)^2$$

$$\Gamma_{X,t}^{M\dagger} = \vartheta_X^M \left(\left(Z_{X,t}^{M}\right)^{-\frac{1}{\vartheta_X^M}} \frac{\tilde{M}_t^X}{\tilde{M}_{t-1}^X} Z_t^Z - Z_{ss}^Z\right)\left(\left(Z_{X,t}^{M}\right)^{-\frac{1}{\vartheta_X^M}} \frac{\tilde{M}_t^X}{\tilde{M}_{t-1}^X} Z_t^Z\right)$$

62. Non-Linear Phillips Curve

$$SX_t^1 = \left(Q_t^X\right)^{\varepsilon_X^P} Q_t \widetilde{Y}_t^X + (\theta_X\beta)E_t\left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t}\right)\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^*}{\Upsilon_{t+1}^X}\right)^{\varepsilon_X^P - 1} SX_{t+1}^1\right]$$

$$SX_t^2 = \left(Q_t^X\right)^{\varepsilon_X^P} \widetilde{Y}_t^X MC_t^X Z_t^{P*} + (\theta_X\beta)E_t\left[\left(\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t}\right)\left(Z_{t+1}^Z\right)^{1-\sigma}\left(\frac{\Pi_{t+1}^*}{\Upsilon_{t+1}^X}\right)^{\varepsilon_X^P} SX_{t+1}^2\right]$$

$$Q_t^{X\star} = \frac{\varepsilon_X^P}{\varepsilon_X^P - 1}\left(\frac{SX_t^2}{SX_t^1}\right)$$

63. Sectoral Indexation Rule

$$\Upsilon_t^X = \left(\Pi_{t-1}^X\right)^{\omega_X}\left(\frac{P_t^{co*}}{P_{t-1}^{co*}}\Pi_{ss}^*\right)^{1-\omega_X}$$

64. Sectoral Price Index

$$Q_t^X = \left(\theta_X\left(\frac{\Upsilon_t^X Q_{t-1}^X}{\Pi_t^*}\right)^{1-\epsilon_X^P} + (1-\theta_X)\left(Q_t^{X\star}\right)^{1-\epsilon_X^P}\right)^{\frac{1}{1-\epsilon_X^P}}$$

65. Sectoral Inflation

$$\Pi_t^X = \frac{Q_t^X}{Q_{t-1}^X}\Pi_t^*$$

66. Price Dispersion

$$v_t^X = (1-\theta_X)\left(\frac{Q_t^{X\star}}{Q_t^X}\right)^{-\epsilon_X^P} + \theta_X\left(\frac{\Upsilon_t^X}{\Pi_t^X}\right)^{-\epsilon_X^P} v_{t-1}^X$$

Demand For Brazilian Exports

67. Imported Input Demand concerning foreign production

$$\tilde{X}_t = \left(\frac{\varpi^*}{1-\Gamma_t^{M*}}\right)\left(\frac{Q_t^X}{\left(1-\Gamma_t^{M*} - \Gamma_t^{M*\dagger}\right)}\right)^{-\epsilon^*}\tilde{Y}_t^*$$

68. Adjustment Cost for Imports ($\Gamma_t^{M^*}$) and its first derivative ($\Gamma_t^{M*\dagger}$) concerning foreign

production

$$\Gamma_t^{M^*} = \frac{\vartheta^{M^*}}{2}\left(\left(Z_t^{M^*}\right)^{-\frac{1}{\vartheta^{M^*}}}\frac{\tilde{X}_t/\tilde{Y}_t^*}{\tilde{X}_{t-1}/\tilde{Y}_{t-1}^*}-1\right)^2$$

$$\Gamma_t^{M^*\dagger} = \vartheta^{M^*}\left(\left(Z_t^{M^*}\right)^{-\frac{1}{\vartheta^{M^*}}}\frac{\tilde{X}_t/\tilde{Y}_t^*}{\tilde{X}_{t-1}/\tilde{Y}_{t-1}^*}-1\right)\left(\left(Z_t^{M^*}\right)^{-\frac{1}{\vartheta^{M^*}}}\frac{\tilde{X}_t/\tilde{Y}_t^*}{\tilde{X}_{t-1}/\tilde{Y}_{t-1}^*}\right)$$

Government - Monetary Policy

69. Taylor Rule

$$R_t = R_{t-1}^{\gamma_r}\left[\overline{\Pi}_t^C R_t^{nat}\left(\frac{E_t\Pi_{t+1}^C}{E_t\overline{\Pi}_{t+1}^C}\right)^{\gamma_\pi}\left(\frac{Y_tZ_t^Z}{Y_{t-1}Z_{ss}^Z}\right)^{\gamma_y}\right]^{(1-\gamma_r)} e^{\epsilon_t^R}$$

70. The natural rate of interest

$$R_t^{nat} = \frac{1}{\beta}\left(Z_t^{ZC}\right)^\sigma$$

71. Law of motion for the Inflation Target

$$\overline{\Pi}_t^C = \left(\overline{\Pi}_{ss}^C\right)^{1-\rho_{\overline{\Pi}^C}}\left(\overline{\Pi}_{t-1}^C\right)^{\rho_{\overline{\Pi}^C}} + \exp(\epsilon_t^{\overline{\Pi},0} + \epsilon_{t-2}^{\overline{\Pi},2} + \epsilon_{t-6}^{\overline{\Pi},6} + \epsilon_{t-10}^{\overline{\Pi},10})$$

Government - Fiscal Policy

72. Government Spending to GDP Ratio

$$G_t^Y = \frac{\tilde{G}_t}{\tilde{Y}_t}$$

73. Primary Surplus

$$S_t^y = T_t - \frac{P_t^G G_t}{P_t^Y Y_t} = T_t\frac{Q_t^G}{Q_t^Y}G_t^Y$$

74. Law of motion for Government Debt

$$B_{t+1}^y = R_t\left(\frac{B_t^y}{\Pi_t^Y}\frac{Y_{t-1}}{Y_t} - S_t^y\right)$$

75. Tax Rule

$$\tau_t^N = \tau_{ss}^N + \gamma_T\left(\tau_{t-1}^N - \tau_{ss}^N\right) + \gamma_S\left(S_t^{4,y} - \overline{S}_t^y\right) + \epsilon_t^T$$

76. Tax Revenue

$$\tau_t^N\tilde{W}_tN_t + \widetilde{T}_t^{Lump} = T_tQ_t^Y\widetilde{Y}_t$$

77. Lump-Sum Taxation

$$\frac{T_t^{Lump}}{P_t^C Z_t} = \widetilde{T}_t^{Lump} = \widetilde{T}_{ss}^{Lump} + \gamma_{TL}\left(\widetilde{T}_{t-1}^{Lump} - \widetilde{T}_{ss}^{Lump}\right) + \epsilon_t^{TL}$$

78. Targeted and Smoothed Primary Surplus

$$\overline{S}_t^y = \overline{S}_{ss}^y + \rho_{\overline{S}}\left(\overline{S}_{t-1}^y - \overline{S}_{ss}^y\right) + (1-\rho_{\overline{S}})\gamma_B\left(E_t B_{t+1}^y - B_{ss}^y\right) + \epsilon_t^S$$

$$S_t^{4,y} = 0.25(S_t^y + S_{t-1}^y + S_{t-2}^y + S_{t-3}^y)$$

Resources Constraints and Identities

79. Sectoral Resource Constraints

$$\tilde{Y}_t^{C^D} = \tilde{C}_t^D$$

$$\tilde{Y}_t^I = \tilde{I}_t$$

$$\tilde{Y}_t^G = \tilde{G}_t$$

$$\tilde{Y}_t^X = \tilde{X}_t$$

80. Market Clearing for Domestic Intermediate Inputs

$$\tilde{Y}_t^D = \tilde{Y}_{C^D,t}^D + \tilde{Y}_{I,t}^D + \tilde{G}_t + \tilde{Y}_{X,t}^D$$

81. Market Clearing for Imported Goods

$$\tilde{M}_t = \tilde{M}_t^{C^D} + \tilde{M}_t^I + \tilde{M}_t^X + \tilde{C}_t^M$$

82. Interest on External Borrowing to GDP Ratio

$$L_t^{*y} = \sum_{H\in\{C^D,I,X\}} \iota_H\left(R_t^* S_t^{B^*} - 1\right)\frac{Q_t^M \tilde{M}_t^H}{Q_t^Y \tilde{Y}_t}$$

83. Net Exports to GDP Ratio

$$NX_t^y = \frac{Q_t Q_t^{X^*}\tilde{X}_t}{Q_t^Y \tilde{Y}_t} - \frac{Q_t Q_t^{M^*}\tilde{M}_t}{Q_t^Y \tilde{Y}_t} v_t^M$$

84. Law of Motion for Net Foreign Assets to GDP Ratio

$$\frac{B_{t+1}^{*y}}{R_t^* S_t^{B^*}} = \left(\frac{\tilde{Y}_{t-1}}{\Pi_t^Y Z_t^Z \tilde{Y}_t}\frac{Q_t}{Q_{t-1}}\frac{\Pi_t^C}{\Pi_t^*}\right) B_t^{*y} + NX_t^y - L_t^{*y}$$

85. Nominal GDP Identity

$$Q_t^Y \tilde{Y}_t = \tilde{C}_t + Q_t^I \tilde{I}_t + Q_t^G \tilde{G}_t + Q_t Q_t^{X^*} \tilde{X}_t - Q_t Q_t^{M^*} \tilde{M}_t v_t^M$$

86. Real GDP according to a Laspeyres Quantity Index

$$Q_{ss}^Y \tilde{Y}_t = \tilde{C}_t + Q_{ss}^I \tilde{I}_t + Q_{ss}^G \tilde{G}_t + Q_{ss} Q_{ss}^X \tilde{X}_t - Q_{ss} Q_{ss}^{M*} \tilde{M}_t v_t^M$$

87. Inflation based on the GDP deflator

$$\Pi_t^Y = \frac{Q_t^Y}{Q_{t-1}^Y} \Pi_t^C$$

Rest of the World - VAR System and Import Price dynamics

We specify the VAR as follows:

$$Z_t^* = A_1 Z_{t-1}^* + (...) + A_n Z_{t-n}^* + \epsilon_t^*, \quad \epsilon_t^* \sim N(0, B)$$

The vector $Z_t^* = [Y_t^*; R_t^*; \Pi_t^*; V_t^*; P_t^{co,*}]$ stacks the foreign variables, A_i as the matrix of coefficients for lag i and ϵ_t^* as the vector of structural shocks with covariance matrix B.

Finally, the equation bellow connects import prices (Q_t^{M*}) to commodity prices (P_t^{co*}):

$$log(Q_t^{M*}/Q_{ss}^{M*}) = \rho_{QM} log(Q_{t-1}^{M*}/Q_{ss}^{M*}) + \gamma_C log(P_{t-1}^{co*}/P_{ss}^{co*}) + \epsilon_t^{QM*}$$

Shocks - AR(1) or ARMA(1,1) Specifications

The shocks Z_t^{ZC}, Z_t^M, Z_t^C, $Z_t^{B^*}$, S_t^B, Z_t^D, Z_t^M, $\widetilde{G}_t$, Z_t^I, Z^{M^*}, Z_t^L, Z_t^Q follow an AR(1) process. Z_t^{ZT} is white noise, while Z_t^A, Z_t^R, Z_t^W and Z_t^{P*} follow an ARMA(1,1) process.

B Steady State

Analytical computation of the steady state

Given the parametric restrictions previously discussed, we normalize some variables and then use first order conditions to compute the steady state according to the following steps:

1. Ad hoc Calibration: required values from other data sources

- $Z^Z_{ss}, \quad U^L_{ss}, \quad L_{ss}, \quad \overline{N}, \quad \overline{\Pi}^C_{ss}, \quad \bar{\Upsilon}^A = \overline{\Pi}^C_{ss}, \quad \Pi^*_{ss}, \quad \varpi_{RT}$
- $\overline{S}^y_{ss}, \quad S^y_{ss} = \overline{S}^y_{ss}, \quad R_{ss}, \quad R^{nat}_{ss} = \frac{R_{ss}}{\overline{\Pi}^C_{ss}}, \quad R^*_{ss}$
- $Q^F_{ss} = 1, \quad Q_{ss} = 1, \quad P^{co*}_{ss} = 1, \quad \tilde{Y}_{ss} = 1, \quad \widetilde{Y}^*_{ss} = 1, \quad V^*_{ss} = 0, \quad Q^{C^D}_{ss} = 1, \quad Q^M_{ss} = 1$
- $\delta, \quad \sigma, \quad weight\_A$ (weight of administered prices in CPI)
- $\epsilon^P_{C^D} = \epsilon^P_I = \epsilon^P_G = \epsilon^P_X = \epsilon^M$
- $\epsilon_I = \epsilon_{C^D} = \epsilon_X$
- $\iota_I = \iota_{C^D} = \iota_X$
- $\varpi_I = \varpi_{C^D} = \varpi_X$
- Share of labor tax in total government revenues: $s_{TL} = \frac{\tau^N_{ss}\tilde{W}_{ss}N_{ss}}{T_{ss}}$

2. Shocks

$$Z^M_{ss} = Z^C_{ss} = Z^{B^*}_{ss} = S^B_{ss} = Z^D_{ss} = Z^M_{ss} = Z^I_{ss} = Z^{M^*}_{ss} = Z^L_{ss} = Z^Q_{ss} = Z^A_{ss} = Z^R_{ss} = Z^W_{ss} = Z^{P*}_{ss} = 1$$

3. Adjustment Costs

$$\Gamma^M_{I_{ss}} = \Gamma^M_{C^D_{ss}} = \Gamma^M_{X_{ss}} = \Gamma^{M^*}_{G_{ss}} = \Gamma^{M\dagger}_{I_{ss}} = \Gamma^{M\dagger}_{C^D_{ss}} = \Gamma^{M\dagger}_{X_{ss}} = \Gamma^{M^*\dagger}_{ss} = 0$$

4. Price Dispersion

$$v^O_{ss} = v^W_{ss} = v^M_{ss} = v^I_{ss} = v^G_{ss} = v^F_{ss} = v^A_{ss} = v^{C^D}_{ss} = v^X_{ss} = 1$$

5. Inflation Rates

$$\Pi^C_{ss} = \Pi^{C^D}_{ss} = \Pi^M_{ss} = \Pi^I_{ss} = \Pi^G_{ss} = \Pi^F_{ss} = \Pi^A_{ss} = \Pi^Y_{ss} = \overline{\Pi}^C_{ss}, \Pi^{4C}_{ss} = \left(\overline{\Pi}^C_{ss}\right)^4, \Pi^X_{ss} = \Pi^*_{ss}, \Pi^W_{ss} = \overline{\Pi}^C_{ss} Z^Z_{ss}$$

6. Indexation Factors

$$\Upsilon^W_{ss} = \Upsilon^M_{ss} = \Upsilon^I_{ss} = \Upsilon^G_{ss} = \Upsilon^F_{ss} = \Upsilon^A_{ss} = \overline{\Pi}^C_{ss}, \Upsilon^X_{ss} = \Pi^*_{ss}$$

7. Calibrated GDP shares and sectoral import shares

Based on the sample used for estimation, we compute the following ratios:

$$s_C, \quad s_I, \quad s_G, \quad s_X, \quad s_M$$

8. GDP Components

$$\tilde{C}_{ss} = s_C \tilde{Y}_{ss}, \quad \tilde{I}_{ss} = s_I \tilde{Y}_{ss}, \quad \widetilde{G}_{ss} = s_G \tilde{Y}_{ss}, \quad \tilde{X}_{ss} = s_X \tilde{Y}_{ss}, \quad \tilde{M}_{ss} = s_M \tilde{Y}_{ss}$$

9. Calibration based on Dynamic Equations

$$\beta = \left(Z_{ss}^Z\right)^{\sigma} \frac{\overline{\Pi}_{ss}^C}{R_{ss}}, S_{ss}^{B^*} = \left(\frac{R_{ss}}{\overline{\Pi}_{ss}^C}\right)\left(\frac{\Pi_{ss}^*}{R_{ss}^*}\right), B_{ss}^y = \frac{S_{ss}^y}{-\frac{Z_{ss}^Z}{\overline{\Pi}_{ss}^C} - \frac{1}{R_{ss}}}$$

10. Sector Sectoral Marginal Costs and Relative Prices

$$MC_{ss}^{C^D} = \frac{\epsilon_{C^D}^P - 1}{\epsilon_{C^D}^P} Q_{ss}^F$$

$$MC_{ss}^I = MC_{ss}^{C^D}$$

$$MC_{ss}^X = MC_{ss}^{C^D}$$

$$Q_{ss}^{M*} = \frac{\varepsilon^M}{\varepsilon^M - 1}\left(\frac{Q_{ss}^M}{Q_{ss}}\right)$$

$$Q_{ss}^D = \left[\frac{(MC_{ss}^{C^D})^{1-\epsilon_{C^D}} - (1-\varpi_{C^D})\,(Q_{C_{ss}^D}^M)^{1-\epsilon_{C^D}}}{\varpi_{C^D}}\right]^{\frac{1}{1-\epsilon_{C^D}}}$$

$$Q_{C_{ss}^D}^M = 1 + \iota_C\left(R_{ss}^* S_{ss}^{B^*} - 1\right) Q_{ss}^M$$

$$Q_{I_{ss}}^M = \left[1 + \iota_I\left(R_{ss}^* S_{ss}^{B^*} - 1\right)\right] Q_{ss}^M$$

$$Q_{X_{ss}}^M = \left[1 + \iota_X\left(R_{ss}^* S_{ss}^{B^*} - 1\right)\right] Q_{ss}^M$$

$$MC_{ss}^G = Q_{ss}^D$$

11. Sectoral Relative Prices

$$Q_{ss}^I = \frac{\epsilon_I^P}{\epsilon_I^P - 1} MC_{ss}^I$$

$$Q_{ss}^X = \frac{\epsilon_X^P}{\epsilon_X^P - 1} MC_{ss}^X$$

$$Q^G_{ss} = \frac{\epsilon^P_G}{\epsilon^P_G - 1} MC^G_{ss}$$

12. Household consumption of domestic and imported goods

$$o_C = \frac{(1 - \frac{s_M}{s_C}) + \left[(\frac{s_I + s_X}{s_C})(1 - \varpi_{C^D}) \left(\frac{Q^M_{C^D_{ss}}}{MC^{C^D}_{ss}} \right)^{-\epsilon_{C^D}} \right]}{1 - \left[(1 - \varpi_{C^D}) \left(\frac{Q^M_{C^D_{ss}}}{MC^{C^D}_{ss}} \right)^{-\epsilon_{C^D}} \right]}$$

$$\varpi_A = \frac{peso\_A}{o_C}$$

$$\tilde{C}^D_{ss} = o_C \tilde{C}_{ss}$$

$$\tilde{C}^M_{ss} = (1 - o_C)\tilde{C}_{ss}$$

13. Demand for goods

$$\tilde{Y}^{C^D}_{ss} = \tilde{C}^D_{ss}, \tilde{Y}^I_{ss} = \tilde{I}_{ss}, \tilde{Y}^G_{ss} = \widetilde{G}_{ss}, \tilde{Y}^X_{ss} = \tilde{X}_{ss}$$

14. Sectoral Imports

$$\tilde{M}^{C^D}_{ss} = (1 - \varpi_{C^D}) \left(\frac{Q^M_{C^D_{ss}}}{MC^{C^D}_{ss}} \right)^{-\epsilon_{C^D}} \tilde{Y}^{C^D}_{ss}$$

$$\tilde{M}^I_{ss} = (1 - \varpi_I) \left(\frac{Q^M_{I_{ss}}}{MC^I_{ss}} \right)^{-\epsilon_I} \tilde{Y}^I_{ss}$$

$$\tilde{M}^X_{ss} = (1 - \varpi_X) \left(\frac{Q^M_{X_{ss}}}{MC^X_{ss}} \right)^{-\epsilon_X} \tilde{Y}^X_{ss}$$

$$\widehat{\tilde{M}}_{ss} = \tilde{M}^{C^D}_{ss} + \tilde{M}^I_{ss} + \tilde{M}^X_{ss} + \tilde{C}^M_{ss}$$

15. Demand for Domestic Intermediate Goods

$$\tilde{Y}^D_{C^D_{ss}} = \varpi_{C^D} \left(\frac{Q^D_{ss}}{MC^{C^D}_{ss}} \right)^{-\epsilon_{C^D}} \tilde{Y}^{C^D}_{ss}$$

$$\tilde{Y}^D_{I_{ss}} = \varpi_I \left(\frac{Q^D_{ss}}{MC^I_{ss}} \right)^{-\epsilon_I} \tilde{Y}^I_{ss}$$

$$\tilde{Y}^D_{G_{ss}} = \tilde{Y}^G_{ss}$$

$$\tilde{Y}^D_{X_{ss}} = \varpi_X \left(\frac{Q^D_{ss}}{MC^X_{ss}}\right)^{-\epsilon_X} \tilde{Y}^X_{ss}$$

$$\tilde{Y}^D_{ss} = \tilde{Y}^D_{C^D_{ss}} + \tilde{Y}^D_{I_{ss}} + \tilde{G}_{ss} + \tilde{Y}^D_{X_{ss}}$$

16. Additional Relative Prices

$$Q^{F\star}_{ss} = Q^F_{ss}, \quad Q^A_{ss} = Q^F_{ss}, \quad Q^{A\star}_{ss} = Q^A_{ss}, \quad Q^{I\star}_{ss} = Q^I_{ss}, \quad Q^{G\star}_{ss} = Q^G_{ss}, \quad Q^{X\star}_{ss} = Q^X_{ss}, \quad Q^{M\star}_{ss} = Q^M_{ss}$$

17. GDP Deflator

$$Q^Y_{ss} = \tilde{C}_{ss} + Q^I_{ss}\tilde{I}_{ss} + Q^G_{ss}\tilde{G}_{ss} + Q_{ss}Q^X_{ss}\tilde{X}_{ss} - Q_{ss}Q^{M*}_{ss}\tilde{M}_{ss}$$

18. Capital Stock and Domestic Intermediate Good Supply

$$N_{ss} = L_{ss}(1 - U^L_{ss})$$

$$\tilde{K}_{ss} = \frac{\tilde{I}_{ss}Z^Z_{ss}}{Z^Z_{ss} - (1-\delta)}$$

$$Q^K_{ss} = Q^I_{ss}$$

$$R^K_{ss} = Q^K_{ss}\left[\frac{R_{ss}}{\Pi^C_{ss}} - (1-\delta)\right]$$

$$\alpha_{aux} = \left[\left(\frac{Q^D_{ss}\tilde{Y}^D_{ss}}{R^K_{ss}\tilde{K}_{ss}/Z^Z_{ss}} - 1\right)\left(\frac{R^K_{ss}}{\left(Q^D_{ss}\tilde{Y}^D_{ss} - R^K_{ss}\tilde{K}_{ss}/Z^Z_{ss}\right)\left(N_{ss}\left(1-\overline{N}\right)\right)}\right)^{1-\epsilon_D} + 1\right]^{-1};$$

$$\alpha = 1/[(1/\alpha_{aux} - 1)^{1/\epsilon_D} + 1];$$

$$\tilde{W}_{ss} = (1-\alpha)\left[\left(\frac{1}{1-\alpha}\right)\left(\frac{Q^D_{ss}}{Z^D_{ss}}\right)^{1-\epsilon_D} - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{R^K_{ss}}{\alpha}\right)^{1-\epsilon_D}\right]^{\frac{1}{1-\epsilon_D}}$$

$$\tilde{W}^\star_{ss} = \tilde{W}_{ss} \text{ and } \widehat{N}_{ss} = N_{ss}$$

19. Variables expressed as GDP ratios

$$T_{ss} = S^y_{ss} + \frac{Q^G_{ss}\widetilde{G_{ss}}}{Q^Y_{ss}\widetilde{Y_{ss}}}$$

$$G^Y_{ss} = \frac{\widetilde{G_{ss}}}{\widetilde{Y_{ss}}}$$

$$NX^y_{ss} = \frac{Q_{ss}Q^{X^*}_{ss}\tilde{X}_{ss}}{Q^Y_{ss}\tilde{Y}_{ss}} - \frac{Q_{ss}Q^{M^*}_{ss}\tilde{M}_{ss}}{Q^Y_{ss}\tilde{Y}_{ss}}$$

$$L^{*y}_{ss} = \sum_{H\in\{C^D,I,X\}} \iota_H \left(R^*_{ss}S^{B^*}_{ss} - 1\right) \frac{Q^M_{ss}\tilde{M}^H_{ss}}{Q^Y_{ss}\tilde{Y}_{ss}}$$

$$B^{*y}_{ss} = \frac{NX^y_{ss} - L^{*y}_{ss}}{\left(\frac{1}{R^*_t S^{B^*}_t}\right) - \left(\frac{1}{Z^Z_{ss}\Pi^*_{ss}}\right)}$$

20. Demand for Exports

$$\varpi^* = \frac{\tilde{X}_{ss}}{\tilde{Y}^*_{ss}\left(Q^X_{ss}\right)^{-\epsilon^*}}$$

21. Households

$$\widetilde{T}^{Lump}_{ss} = (1 - s_{TL})T_{ss}Q^Y_{ss}\widetilde{Y_{ss}}$$

$$\tau^N_{ss} = \frac{T_{ss}Q^Y_{ss}\widetilde{Y_{ss}} - \widetilde{T}^{Lump}_{ss}}{\tilde{W}_{ss}N_{ss}}$$

$$\tilde{C}^{RT}_{ss} = (1 - \tau^N_{ss})\tilde{W}_{ss}N_{ss}$$

$$\tilde{C}^O_{ss} = \frac{\tilde{C}_{ss} - \varpi_{RT}\tilde{C}^{RT}_{ss}}{(1 - \varpi_{RT})}$$

$$\tilde{\Lambda}_{ss} = \left(\tilde{C}^O_{ss} - \kappa\frac{\tilde{C}^O_{ss}}{Z^Z_{ss}}\right)^{-\sigma}$$

22. Labor Market: GSW

$$\widetilde{C}^S_{ss} = \left(\frac{Z^C_{ss}}{\tilde{\Lambda}_{ss}}\right)\left(Z^Z_{ss}\right)^{-\frac{(1-v)\sigma}{v}}$$

$$\varphi^S_{ss} = \frac{\widetilde{C}^S_t\tilde{\Lambda}_{ss}}{Z^C_{ss}}$$

$$\psi = \frac{(1-\tau^N_{ss})\tilde{W}_{ss}}{Z^L_{ss}\widetilde{C}^S_{ss}L^\eta_{ss}}$$

$$\epsilon^W = \frac{1}{1-\left(\frac{Z^C_{ss}Z^W_{ss}Z^L_{ss}\psi\varphi^S_{ss}}{\tilde{\Lambda}_{ss}(1-\tau^N_{ss})\tilde{W}_{ss}N^{-\eta}_{ss}}\right)}$$

23. Nominal Rigidities

$$SW^1_{ss} = \frac{\tilde{\Lambda}_{ss}(1-\tau^N_{ss})\tilde{W}^{\epsilon^W}_{ss}N_{ss}(\epsilon^W-1)}{1-\left[\theta_W\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SW^2_{ss} = \frac{Z^C_{ss}Z^W_{ss}Z^L_{ss}\psi\varphi^S_{ss}\tilde{W}^{\epsilon^W(1+\eta)}_{ss}N^{(1+\eta)}_{ss}\epsilon^W}{1-\left[\theta_W\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SM^1_{ss} = \frac{\left(Q^M_{ss}\right)^{\varepsilon^M}\widehat{\widetilde{M}}_{ss}}{1-\left[\theta_M\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SM^2_{ss} = \frac{\left(Q^M_{ss}\right)^{\varepsilon^M}Q_{ss}Q^{M*}_{ss}\widehat{\widetilde{M}}_{ss}}{1-\left[\theta_M\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SI^1_{ss} = \frac{\left(Q^I_{ss}\right)^{\varepsilon^P_I}\widetilde{Y}^I_{ss}}{1-\left[\theta_I\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SI^2_{ss} = \frac{\left(Q^I_{ss}\right)^{\varepsilon^P_I}\widetilde{Y}^I_{ss}MC^I_{ss}Z^P_{ss}}{1-\left[\theta_I\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SG^1_{ss} = \frac{\left(Q^G_{ss}\right)^{\varepsilon^P_G}\widetilde{Y}^G_{ss}}{1-\left[\theta_G\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SG^2_{ss} = \frac{\left(Q^G_{ss}\right)^{\varepsilon^P_G}\widetilde{Y}^G_{ss}MC^G_{ss}Z^P_{ss}}{1-\left[\theta_G\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SF^1_{ss} = \frac{(Q^{C^D}_{ss})^{\epsilon^P_{C^D}}\widetilde{Y}^{C^D}_{ss}}{1-\left[\theta_F\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SF^2_{ss} = \frac{(Q^{C^D}_{ss})^{\epsilon^P_{C^D}}\widetilde{Y}^{C^D}_{ss}MC^{C^D}_{ss}Z^P_{ss}}{1-\left[\theta_F\beta\left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SA^1_{ss} = \frac{(Q^{C^D}_{ss})^{\epsilon^P_{C^D}}\widetilde{Y}^{C^D}_{ss}}{1-\left[\theta_A \beta \left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SA^2_{ss} = \frac{(Q^{C^D}_{ss})^{\epsilon^P_{C^D}}\widetilde{Y}^{C^D}_{ss} MC^{C^D}_{ss} Z^P_{ss}}{1-\left[\theta_A \beta \left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SX^1_{ss} = \frac{\left(Q^X_{ss}\right)^{\varepsilon^P_X} Q_{ss}\widetilde{Y}^X_{ss}}{1-\left[\theta_X \beta \left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

$$SX^2_{ss} = \frac{\left(Q^X_{ss}\right)^{\varepsilon^P_X}\widetilde{Y}^X_{ss} MC^X_{ss} Z^{P*}_{ss}}{1-\left[\theta_X \beta \left(Z^Z_{ss}\right)^{1-\sigma}\right]}$$

C Data Treatment

Observed variables measured in per capita terms use the interpolated trend from the HP filter applied to annual population estimates provided by IBGE. These estimates are published for years where Census data is not available, and the use of the trend from the HP filter eliminates significant jumps in data when new information is considered to compute new estimates.

Table 6: Data used in estimation

Variable	Model representation	Description	Source	Treatment	Observation
Δy_t	$100 \times \left[\log\left(\frac{Y_t}{Y_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right)\right]$	Growth rate, GDP per capita	IBGE	I-L-D	National Accounts: GDP seas. adj.
Δc_t	$100 \times \left[\log\left(\frac{C_t}{C_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right)\right] + m.e.$	Growth rate, Consumption per capita	IBGE	I-L-D	National Accounts: Consumption seas. adj.
Δi_t	$100 \times \left[\log\left(\frac{I_t}{I_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right)\right] + m.e.$	Growth rate, Investment per capita	IBGE	I-L-D	National Accounts: Investment seas. adj.
Δg_t	$100 \times \left[\log\left(\frac{G_t}{G_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right)\right] + m.e.$	Growth rate, Gov. spending per capita	IBGE	I-L-D	National Accounts: Gov. spending seas. adj.
Δx_t	$100 \times \left[\log\left(\frac{X_t}{X_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right)\right] + m.e.$	Growth rate, Exports per capita	IBGE	I-L-D	National Accounts: Exports seas. adj.
Δm_t	$100 \times \left[\log\left(\frac{M_t}{M_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right)\right] + m.e.$	Growth rate, Imports per capita	IBGE	I-L-D	National Accounts: Imports seas. adj.
y_t	$100 \times \log\left(\frac{Y_t}{Y_{ss}}\right) + m.e$	Output gap, GDP per capita	IBGE & BCB	I-L-G	National Accounts: GDP seas. adj.; See 3.2
Δwn_t	$100 \times \left[\log\left(\frac{W_t}{W_{t-1}}\right) + \log\left(\frac{Z_t^Z}{Z_{ss}^Z}\right) + \log\left(\frac{\Pi_t^C}{\overline{\Pi}_{ss}^C}\right)\right] + m.e.$	Growth rate, Nominal wages	IBGE & MF-VAR	S-L-D	PNAD-C Survey
l_t	$100 \times (L_t - L_{ss})$	Participation rate	IBGE & MF-VAR	S-L-G	PNAD-C Survey
u_t	$100 \times (U_t^L - U_{ss}^L)$	Unemployment rate	IBGE & MF-VAR	S-L-G	PNAD-C Survey
p_t^C	$100 \times (\Pi_t^C - \overline{\Pi}_{ss}^C)$	CPI Inflation rate	IBGE	S-L-G	IPCA
p_t^A	$100 \times (\Pi_t^A - \overline{\Pi}_{ss}^C)$	Inflation rate, Administered prices	IBGE & BCB	S-L-G	
p_t^F	$\frac{p_t^C - weight_A p_t^A - \epsilon_t^{me,CPI}}{1-weight_A}$	Inflation rate, Free prices	IBGE & BCB	S-L-G	
$\overline{p}_t^C$	$100 \times (\overline{\Pi}_t^C - \overline{\Pi}_{ss}^C)$	Inflation target (IT)	CMN	L-G	

(Continued on next page)

Table 6 (continued from previous page)

Variable	Model representation	Description	Source	Treatment	Observation
$e_{t-2}^{\overline{\Pi},2}$	$\epsilon_{t-2}^{\overline{\Pi},2}$	Change in IT: two-quarters ahead	CMN	L-G	
$e_{t-6}^{\overline{\Pi},6}$	$\epsilon_{t-6}^{\overline{\Pi},6}$	Change in IT: six-quarters ahead	CMN	L-G	
$e_{t-10}^{\overline{\Pi},10}$	$\epsilon_{t-10}^{\overline{\Pi},10}$	Change in IT: ten-quarters ahead	CMN	L-G	
$E_t p_t^C$	$p_{t+1}^C + p_{t+2}^C + p_{t+3}^C + p_{t+4}^C$	Inflation expectations	BCB	L-G	Focus Survey
r_t	$100 \times (R_t - R_{ss})$	Interest rate	BCB	L-G	SGS: Selic rate
s_t^y	$100 \times (S_t^y - S_{ss}^y)$	Primary result – quarterly	BCB	S-G	SGS
$\overline{s}_t^y$	$100 \times (\overline{S}_t^y - \overline{S}_{ss}^y)$	Primary result target	BCB	G	SGS
q_t	$100 \times (\log Q_t - \log Q_{ss})$	Real exchange rate	BCB	L-G	SGS
Δq_t^{M*}	$100 \times (\log Q_t^{M*} - \log Q_{t-1}^{M*})$	Import price inflation	Funcex	I-S-L-D	
p_t^X	$100 \times (\Pi_t^X - \Pi_{ss}^X)$	Export prices	Funcex	L-G	
p_t^{co*}	$p_t^{co*} - p_{ss}^{co*}$	Commodity prices	BCB	G	See 3.1
r_t^*	$100 \times (R_t^* - R_{ss}^*)$	Foreign interest rates	St Louis Fed	L-HP-G	FRED
p_t^*	$100 \times (\Pi_t^* - \Pi_{ss}^*)$	Foreign inflation	St Louis Fed	G	FRED
y_t^*	$100 \times \log(Y_t^*/Y_{ss}^*)$	Foreign output gap	BCB	L-OHP-G	
$s_t^{B^*}$	$100 \times (S_t^{B^*} - S_{ss}^{B^*})$	Country risk premium	Bloomberg	G	CDS
v_t^*	$V_t^* - V_{ss}^*$	Foreign risk aversion	BCB	G	See 3.1

Note: In column "Model representation", *"m.e."* is the representation for measurement error described in equation (76). Legend for column "Treatment": *"I"* is new index combining two raw indexes (as in per capita variables combining the raw information with population data, or real variables combining raw data with a deflator); *"L"* is the natural logarithm of an index or rate; *"G"* is the gap with respect to steady state value; *"D"* is the first difference of an index; *"OHP"* is the one-sided HP Filter; *"HP"* is the two-sided HP Filter; *"S"* is seasonal adjustment based on ARIMA X-12 procedure.

D Priors and Histogram of Posterior Distribution

Figure 13: Prior and Posterior Distribution – 1

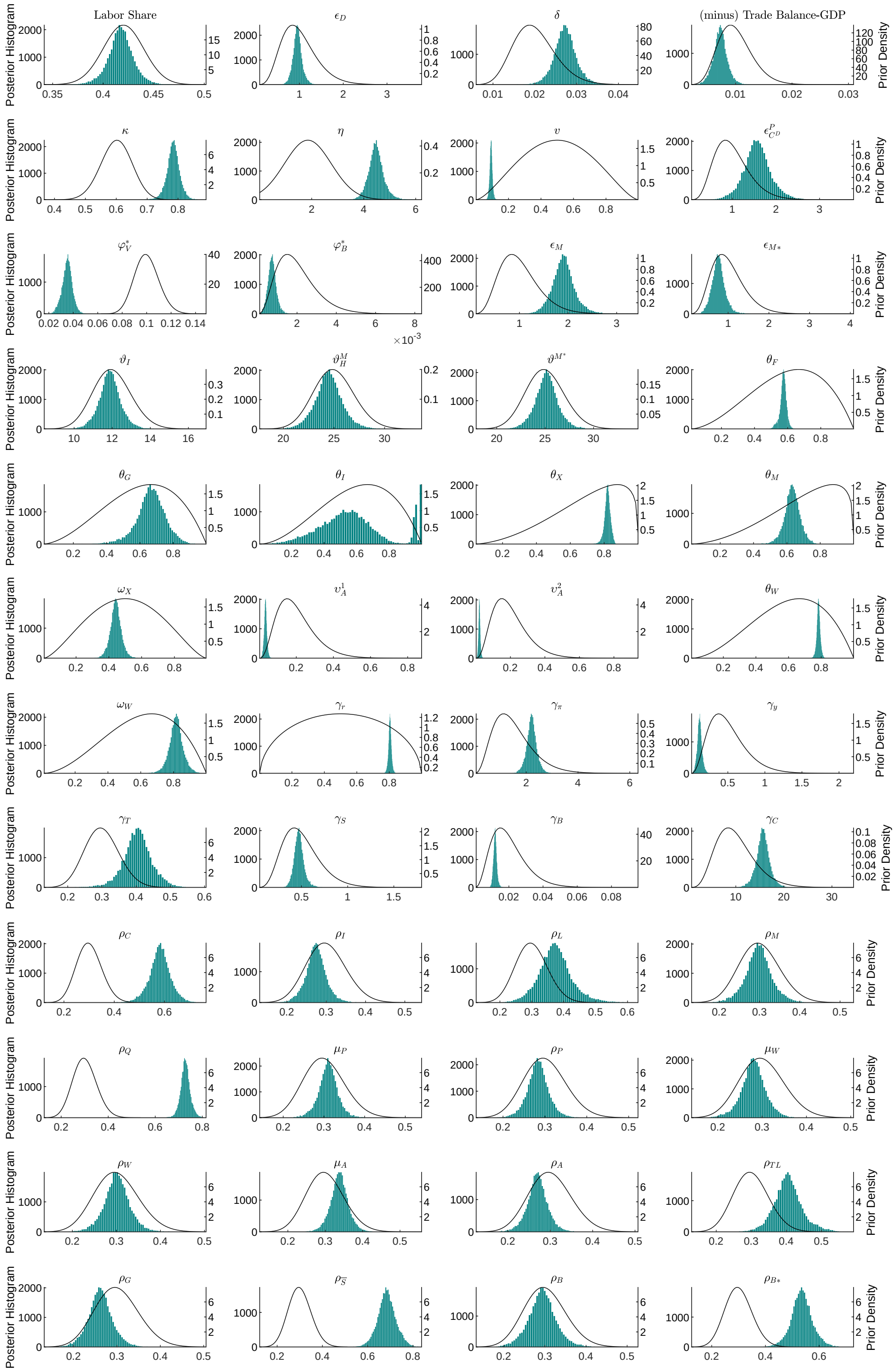
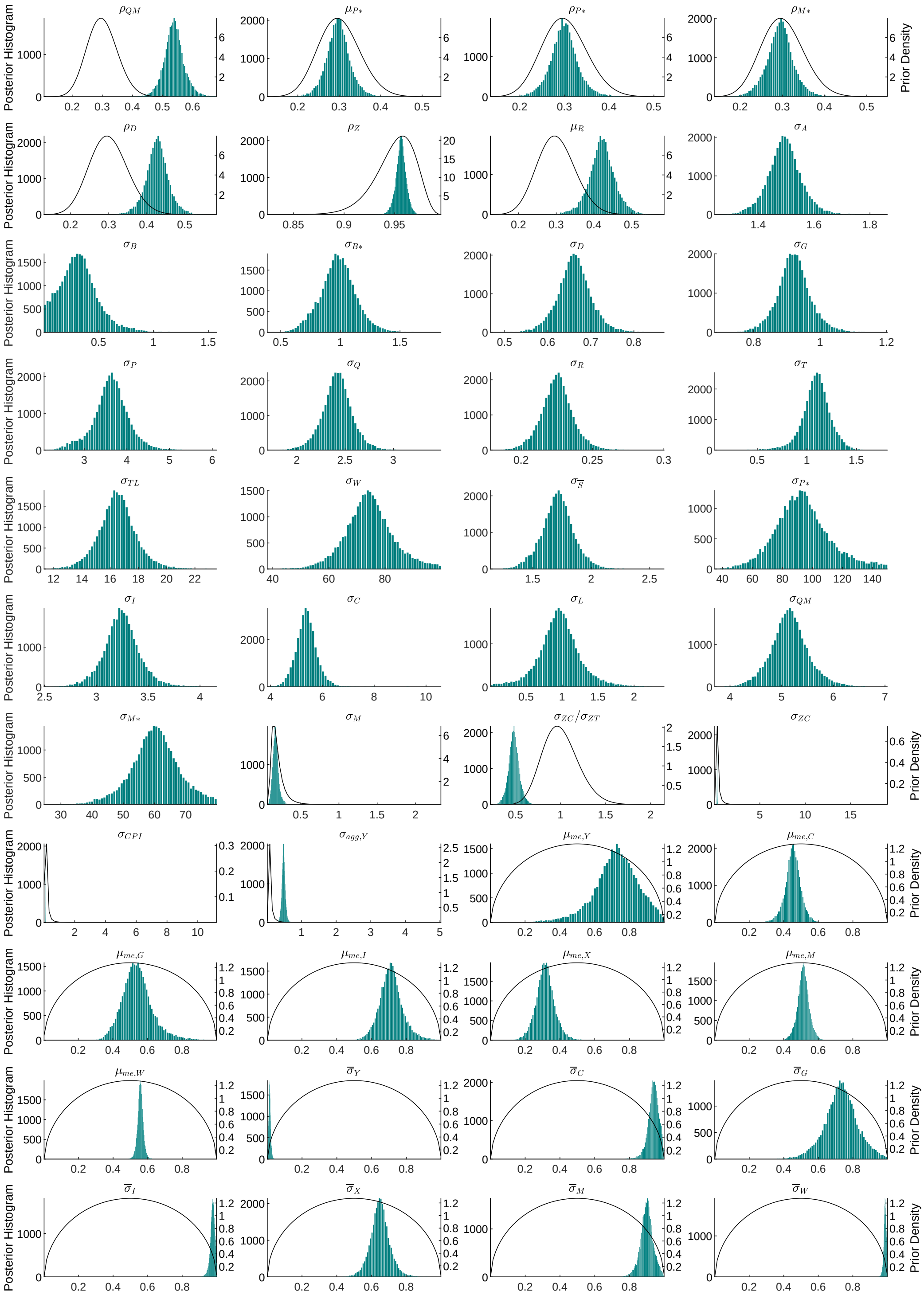


Figure 14: Prior and Posterior Distribution – 2



E Computational details

The SMC estimation procedure characterizes the posterior distribution as a set of "particles", where each particle is a vector representing a complete set of estimated parameter values. As detailed in Cai et al. (2021)[14], these particles are initially sampled from the prior distribution and then gradually updated to increasingly better approximate the posterior distribution. The procedure includes several steps, but computational time is dominated by the likelihood evaluation.

A critical property of SMC estimation is the independence of particle updating for each step of the algorithm. This property facilitates parallel implementations of the algorithm across particles. The likelihood evaluation, which is the focus of the discussion here, can be broken into two main parts: solving the model for each particle to obtain its decision rules and running the Kalman filter algorithm to obtain the likelihood for each particle. The algorithm for likelihood evaluation works dividing the cloud of particles in n chunks of s particles each[31]. Then, each chunk is co-distributed across all CPU cores, each core receiving roughly $d = s/C$ particles, where C is the number of cores in the CPU. All CPU cores available compute decision rules in parallel (inside each core the computation is sequential), and after this step each core runs the Kalman filter algorithm with the help of a GPU.

The GPU computation of the Kalman filter is carried out by all cores simultaneously calling the GPU. The data sample available for estimation has size T and each call sends C separated blocks of d particles to the GPU. Each block is converted to a GPU array[32]. Blocks are processed in GPU for each time slice of the sample following the traditional log-likelihood evaluation through Kalman filtering. In other words, for a given period $t < T$ of the data sample, GPU cores evaluate the density of particle i for the whole block of d particles before moving to the next period, $t + 1$.

Our computational resources comprises a workstation with a $C = 10$ cores CPU combined with a 32 Gb dedicated memory GPU and 256 Gb RAM. The following pseudo-code summarizes the procedure executed by each core:

Algorithm 1 Log-likelihood evaluation

```
procedure LogLikelihood(P)                              ▷ P is a d particles block
    for i ← 1 to d do
        D(i) ← ComputeDecisionRules(P(i))
    end for
    for t ← 1 to T do
        Loglikelihood(t) ← ComputeLogLikelihoodGPU(D, t)
    end for
end procedure
```

In Algorithm 1, *LogLikelihood* has T log-likelihood evaluations for each particle. Summing

[31] The last chunk may have size less than s.

[32] All code is written using MATLAB Parallel Computing Toolbox™ available for MATLAB®.

over t gives the log-likelihood for each particle. The final result is obtained combining the outputs of each core.

An important point to be observed is that the function *ComputeDecisionRules* was not implemented on GPU. It uses standard compiled code from Dynare[2]. Performance tests carried out for serial, parallel CPU and GPU computing of the Kalman filter are summarized in Table 7. The size for the set of "particles" in Table 7 approximates the full capacity of the GPU memory for the model presented in this paper. The final estimation of the model included 30,000 particles.

Table 7: Performance Tests – Kalman Filter

Time(s)	Particles	Device	Gain relative to serial code
3,516	8,000	CPU Serial	1.0x
686	8,000	CPU 10 cores	5.1x
236	8,000	GPU	14.9x