

Technical Appendix

A Millennium of UK Business Cycles: insights from a structural VAR analysis*

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August 17, 2026

Abstract

The appendix presents estimation algorithms and additional results.

Keywords: Long-run data; Business Cycle shock; time-varying VAR.

JEL Codes: CC32; E32; E43; N10.

*We thank participants at presentations at the Workshop in Empirical and Theoretical Economics at King's College London, the 47th Meeting of the Brazilian Econometric Society, the 33rd Symposium of the Society for Nonlinear Dynamics & Econometrics, and the University of São Paulo (FEA/RP) for their useful comments. The views expressed in this paper are those of the authors and do not represent those of the Bank for International Settlements, the Central Bank of Brazil, or any of their Committees.

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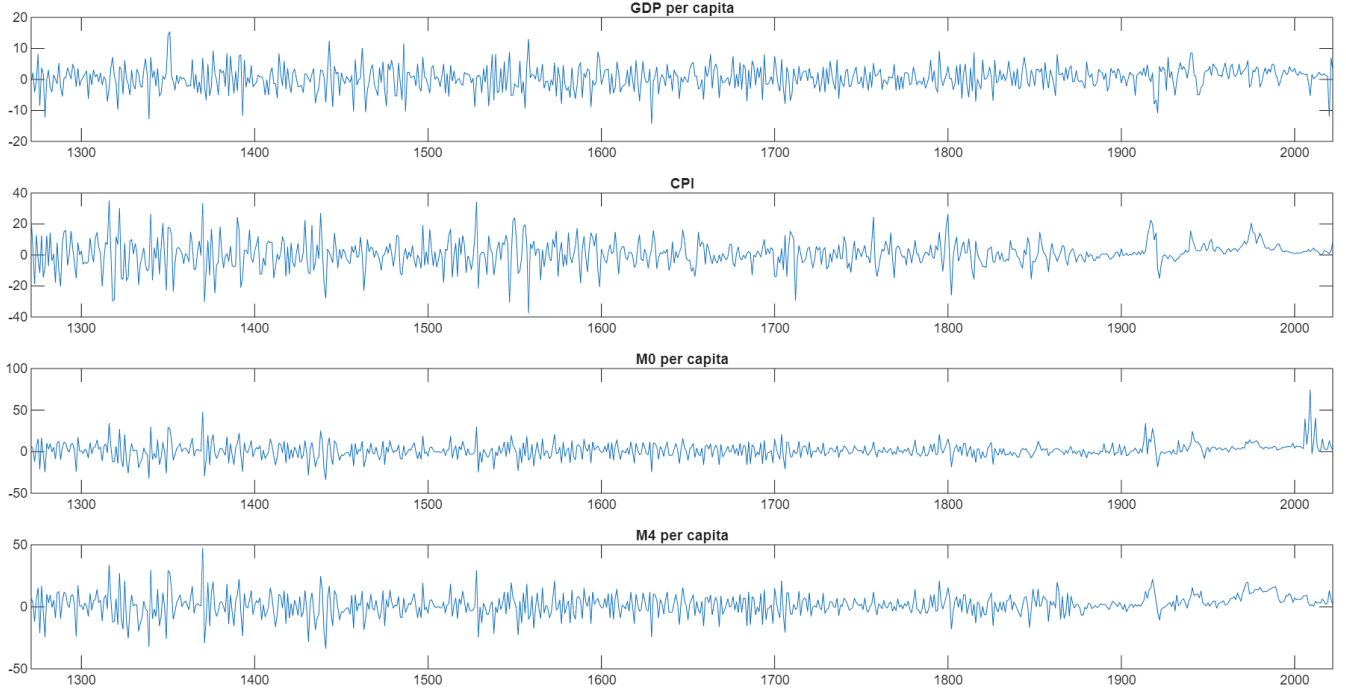
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Appendix

A Data

- GDP per capita: the measure of activity is the real GDP (at market prices) per capita. For 1271-1700, we use the data for England, and for 1701-1870, we use the data for Great Britain, as harmonised by [Broadberry et al. \(2026\)](#). From 1871 on, we use UK post-1922 definition. For the 1701-1870 period, where these three series are available their pairwise correlation is very strong: 99%.
- Inflation: the measure of inflation is the CPI variation. For 1271-1869, the series measures the cost of living in England only. For 1870 onwards, it measures inflation in the entire UK.
- Broad Money: the measure of broad money is M4x (M4 excluding intermediate OFCs) from 1997 on; from 1996 to 1963 the variation in M4 is used to backward extrapolate it; from 1962 to 1880, the variation of a synthetic M4 (M3 + building society shares + building society deposits - building societies holdings of cash) is used to construct the series. Then the variation of M3 is applied to construct the series until 1870; and, finally, from 1869 to 1271, the variation of an M3 estimate is used.
- Money: from 1833, the measure of money is the result of the merging of three different series of M0. From 1832 to 1271, the variation of an M3 estimate is used to backward extrapolate this series. Thus, for the first centuries of the sample our measures of money and broad money exhibit the same behaviour.

Figure A.1: Data



Notes: The blue lines depicts the log differences of the series, as used in the estimation.

B Prior distributions and starting values

The initial conditions for the VAR coefficients ϕ_0 are obtained via an OLS estimate of a fixed coefficient VAR using the first 50 observations of the sample period.

Let $\hat{v}^{ols}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data. For identification, the initial value of the stochastic volatility σ_0^2 is fixed to the diagonal elements of $\hat{v}^{ols}$.

The prior for the off-diagonal elements A_t is $A_0 \sim N(\hat{a}^{ols}, V(\hat{a}^{ols}))$ where $\hat{a}^{ols}$ are the off-diagonal elements of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V(\hat{a}^{ols})$ is assumed to be diagonal with the elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$. The prior on Q is assumed to be inverse Wishart $Q_0 \sim IW(\bar{Q}_0, T_0)$ where $\bar{Q}_0$ is assumed to be $var(\hat{\phi}^{OLS}) \times 10^{-4}$ and T_0 is the length of the sample used to for calibration. The results are not sensitive to this prior. We obtain similar results for larger values of the scaling parameter. The prior distribution for the blocks of S is inverse Wishart: $S_{i,0} \sim IW(\bar{S}_i, K_i)$ where i indexes the blocks of S . $\bar{S}_i$ is calibrated using $\hat{a}^{ols}$.

Specifically, $\bar{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by 10^{-3} . Following [Carriero et al. \(2022\)](#) we postulate an inverse-gamma distribution for the elements of G , $\sigma^2 \sim IG((n+3) \times 0.15 \times I_n, n+3)$.

Following [Koop \(2003\)](#), the prior for $\lambda_{k,i} = 1/\varpi_{k,i}$ is gamma

$$p(\lambda_k) \sim \Gamma(1, v_{\lambda,k})$$

$$p(v_{\lambda,k}) \sim \Gamma(v_0, 2)$$

where $v_0 = 20$ and $\Gamma(a, b)$ is the gamma density with mean a and df b .

C Simulating the posterior distribution

We use a Gibbs sampling algorithm to sample from the posterior distribution. The details of each conditional distribution are provided below.

C.1 Time-varying VAR coefficients

Conditional on the time-varying volatilities and contemporaneous coefficients, the model is a SUR system with time-varying parameters. The [Carter and Kohn \(2004\)](#) algorithm is used to sample from the conditional posterior of ϕ_t . The distribution of the time-varying VAR coefficients ϕ_t conditional on all other parameters and hyperparameters is linear and Gaussian: $\phi_t \setminus Z_t, \Xi \sim N(\phi_{T \setminus T}, P_{T \setminus T})$ and $\phi_t \setminus \phi_{t+1}, Z_t, \Xi \sim N(\phi_{t \setminus t+1, \phi_{t+1}}, P_{t \setminus t+1, \phi_{t+1}})$ where $t = T-1, \dots, 1$, Ξ denotes a vector that holds all the other VAR parameters and $\phi_{T \setminus T} = E(\phi_T \setminus Z_t, \Xi)$, $P_{T \setminus T} = Cov(\phi_T \setminus Z_t, \Xi)$, $\phi_{t \setminus t+1, \phi_{t+1}} = E(\phi_t \setminus Z_t, \Xi, \phi_{t+1})$ and $P_{t \setminus t+1, \phi_{t+1}} = Cov(\phi_t \setminus Z_t, \Xi, \phi_{t+1})$. As shown by [Carter and Kohn \(2004\)](#) the simulation proceeds as follows. First we use the Kalman filter to draw $\phi_{T \setminus T}$ and $P_{T \setminus T}$ and then proceed backwards in time using $\phi_{t|t+1} = \phi_{t|t} + P_{t|t} P_{t+1|t}^{-1} (\phi_{t+1} - \phi_t)$ and $\phi_{t|t+1} = \phi_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}$.

C.2 Elements of H_t

Following [Primiceri \(2005\)](#) and [Del Negro and Primiceri \(2015\)](#), the diagonal elements of the VAR covariance matrix are sampled based on the method presented in [Kim et al. \(1998\)](#), which consists of transforming a nonlinear and non-Gaussian state space form in a linear and approximately Gaussian. In order to that, a mixture of normals approximation of the $\log \chi^2$ distribution is used. [Kim et al. \(1998\)](#) select a mixture of 7 normal densities with component probabilities q_j , means $m_j - 1.2704$, and variances v_j^2 , $j = 1, \dots, 7$, chosen to match a number of moments of the $\log \chi^2(1)$ distribution. Mixtures states are then used to select which member of the mixture of normal approximation has to be used. Conditional on the data, the remaining parameter block and the states, H_t can be drawn relying on the state space representation and [Carter and Kohn \(1994\)](#).

C.3 Elements of A_t

Given a draw for ϕ_t , the VAR model can be written as $A_t'(\tilde{Z}_t) = u_t$, where $\tilde{Z}_t = Z_t - \sum_{l=1}^L \phi_{l,t} Z_{t-l} = v_t$ and $VAR(u_t) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard methods for state space models described in [Carter and Kohn \(2004\)](#) can be applied.

C.4 VAR hyperparameters

Conditional on Z_t , $\phi_{l,t}$, H_t , and A_t , the innovations to $\phi_{l,t}$, H_t , and A_t are observable, which allows us to draw the hyperparameters—the elements of Q , S , and the σ_i^2 —from their respective distributions.

C.5 Elements of λ_t

The conditional posterior distributions related to the t-distributed shock structure of the model are described in [Koop \(2003\)](#). Note that conditional on Φ_t and A_t , the orthogonalized residuals can be obtained as $\tilde{\varepsilon}_t = A_t u_t$. The conditional posterior distribution for $\lambda_{i,t}$ derived in [Geweke \(1993\)](#) applies to each column of $\tilde{\varepsilon}_t$. As shown in [Koop \(2003\)](#), this posterior density is a gamma

distribution with mean $(v_{\lambda,i} + 1) / \left(\frac{1}{\sigma_{i,t}^2} \tilde{\varepsilon}_{i,t}^2 + v_{\lambda,i} \right)$ and degrees of freedom $v_{\lambda,i} + 1$. Note that $\tilde{\varepsilon}_{i,t}$ is the i th column of the matrix $\tilde{\varepsilon}_t$.

C.6 Degrees of freedom $v_{\lambda,i}$

The conditional distribution for the degree of freedom parameter is non-standard and given by:

$$G(v_{\lambda,i} \setminus \lambda_i) \propto \left(\frac{v_{\lambda,i}}{2} \right)^{\frac{T v_{\lambda,i}}{2}} \Gamma \left(\frac{v_{\lambda,i}}{2} \right)^{-T} \exp \left(- \left(\frac{1}{v_0} + 0.5 \sum_{t=1}^T [\ln(\lambda_{t,i}^{-1}) + \lambda_{t,i}] \right) v_{\lambda,i} \right) \quad (\text{C.1})$$

As in [Geweke \(1993\)](#) we use the Random Walking Metropolis Hastings Algorithm to draw from this conditional distribution. More specifically, for each of the n equations of the VAR, we draw $v_{\lambda,i}^{new} = v_{\lambda,i}^{old} + c^{1/2} \epsilon$ with $\epsilon \sim N(0, 1)$. The draw is accepted with probability $\frac{G(v_{\lambda,i}^{new} \setminus \lambda_i)}{G(v_{\lambda,i}^{old} \setminus \lambda_i)}$ with c chosen to keep the acceptance rate close to the range 20%-40%.

D The Gibbs sampling algorithm

The Gibbs algorithm for the model cycles through the following eight conditional posterior distributions:

1. $G(A_t \setminus \Psi)$ where Ψ denotes the remaining parameters of the model.
2. $G(Q_A \setminus \Psi)$
3. $G(\Phi_t \setminus \Psi)$
4. $G(Q_\Phi \setminus \Psi)$
5. $G(g \setminus \Psi)$
6. $G(\lambda_{i,t} \setminus \Psi)$
7. $G(v_{\lambda,i} \setminus \lambda_{i,t})$
8. $G(\sigma_t^2 \setminus \Psi)$

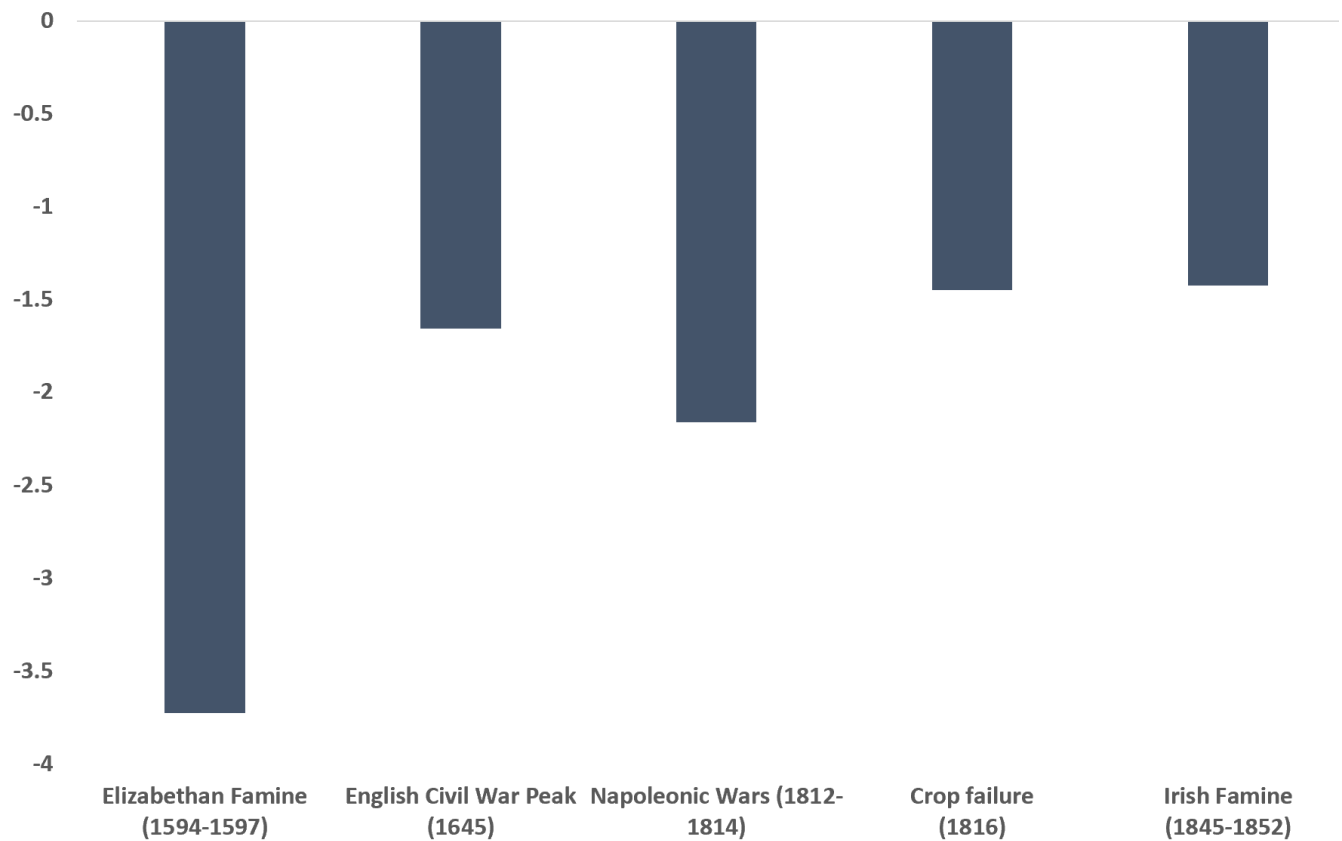
We employ 60,000 iterations, with a burn-in of 10,000 draws. Moreover, in order to reduce the correlation across draws and increase efficiency, only every 10th is kept.

E Additional results

E.1 Matching the Business Cycle Shocks with Narrative Accounts

This figure shows that during episodes such as the Elizabethan famine (1594–1597), the military peak of the English Civil War (1645), the Napoleonic Wars (1812–1814), the 1816 crop failure, and the Irish Famine (1845–1852), the model consistently identifies large negative business cycle shocks. These are characterized by declining output and rising prices, confirming the supply-side nature of macroeconomic disruptions in this era ([Broadberry et al., 2015](#)).

Figure E.1: Narrative evidence



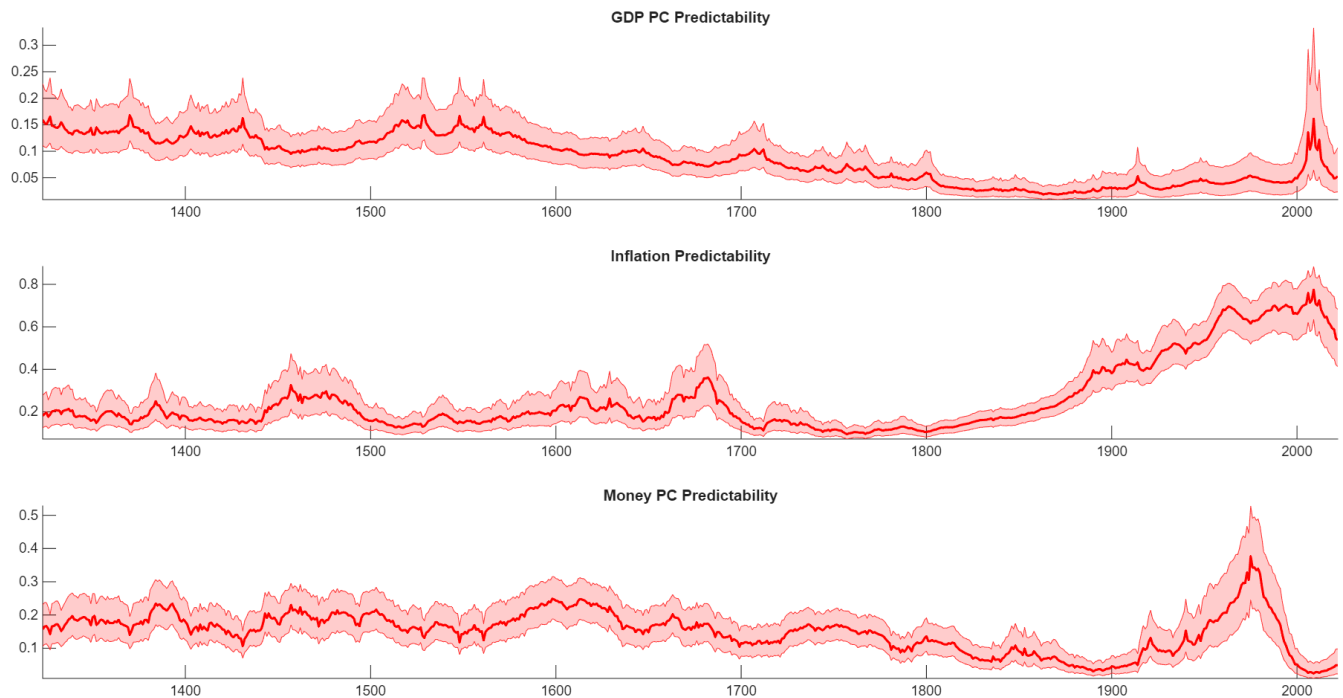
Notes: The bars show the main business cycle shocks cumulated over the years for several famous episodes of British history.

E.2 Predictability

We also examine the predictability of output, inflation, and money using forecast R^2 ([Cogley et al., 2010](#)) from the VAR model. As shown in Figure E.2, the rise in inflation predictability is a relatively recent phenomenon, consistent with the findings of [Cogley et al. \(2010\)](#) for

post-war U.S. data. Similar to [Nason and Smith \(2023\)](#), we find that inflation predictability remained low throughout earlier centuries. Predictability increases substantially in the twentieth century. Output, on the other hand, shows a contrasting trend: output predictability has actually declined since the 1300s with the exception of the Great Financial Crisis.

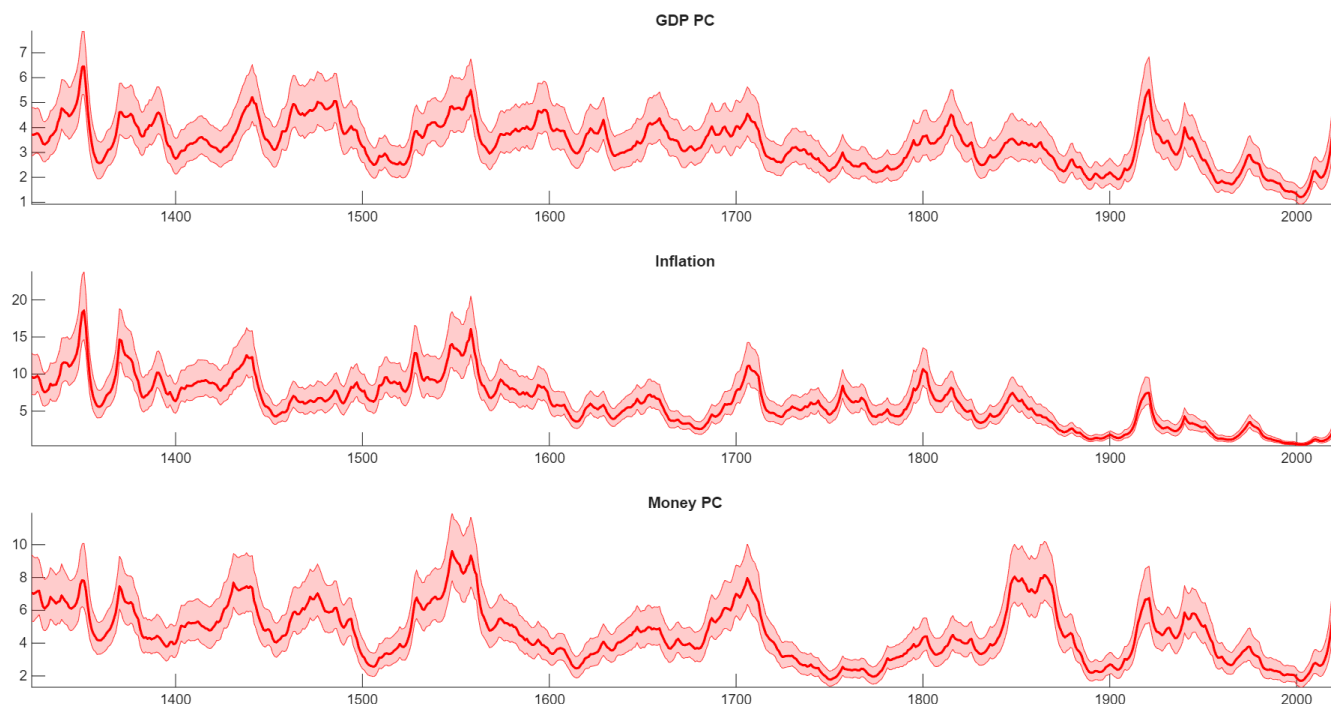
Figure E.2: Predictability



Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

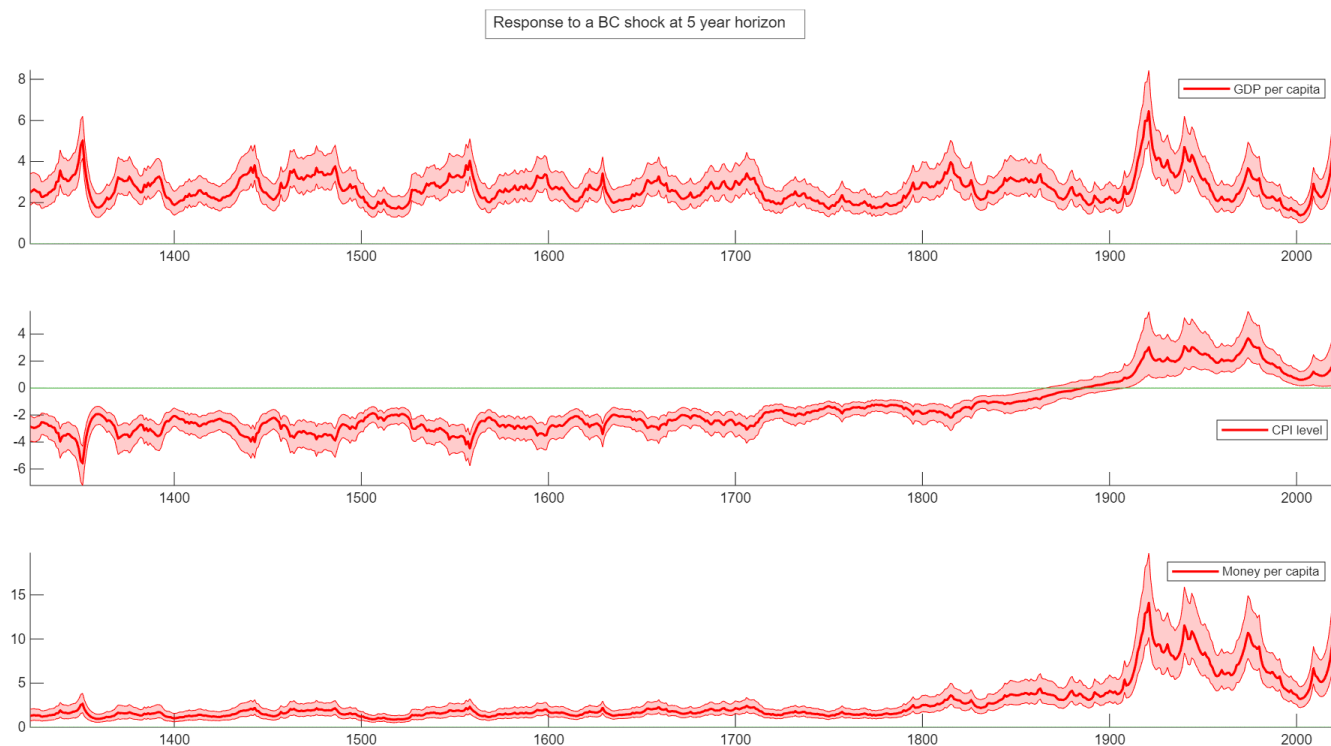
E.3 Three-variable VAR with M4

Figure E.3: Stochastic Volatility



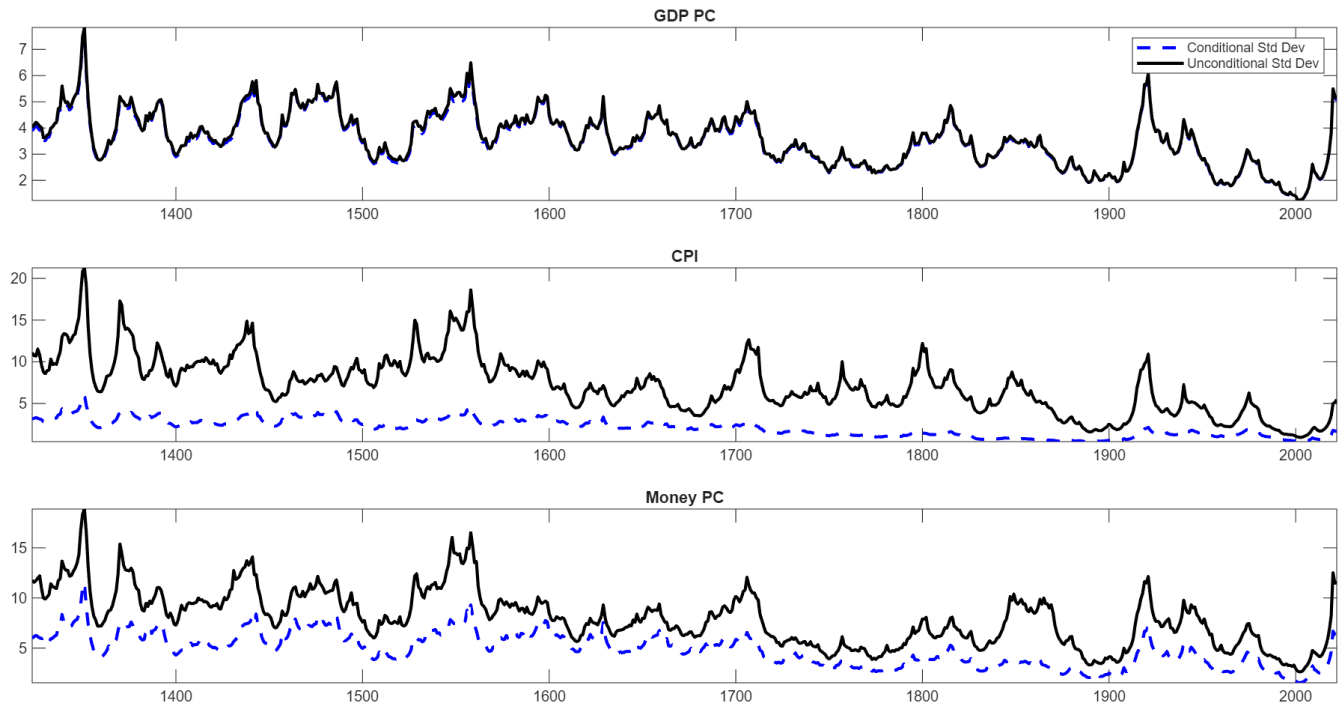
Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

Figure E.4: Impulse responses



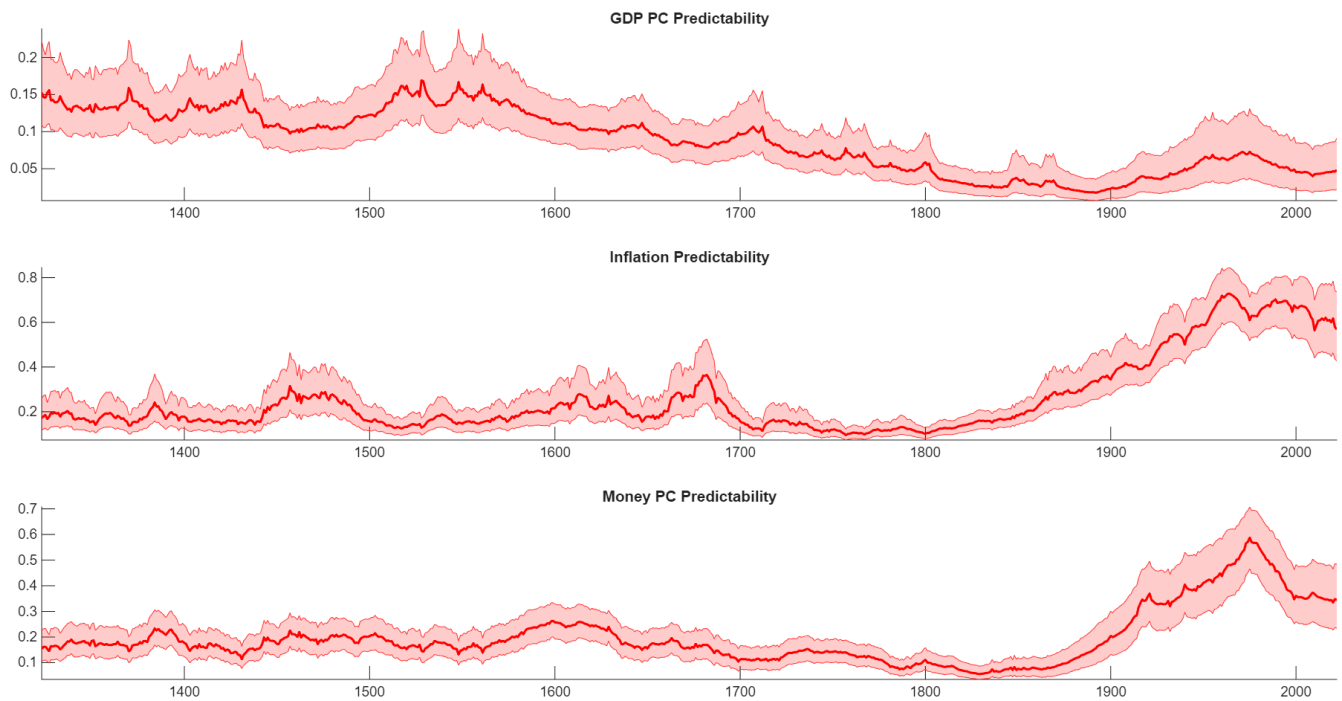
Notes: The pink shaded areas represent the 68 percent credible sets for the IRFs and the red lines depict the median IRFs using M4 as the measure of money.

Figure E.5: Conditional Volatility



Notes: The dashed blue lines are the estimates of the time-varying volatilities of each variable conditional on the business cycle shock. The solid black lines are the estimates of the unconditional volatilities of each of these variables.

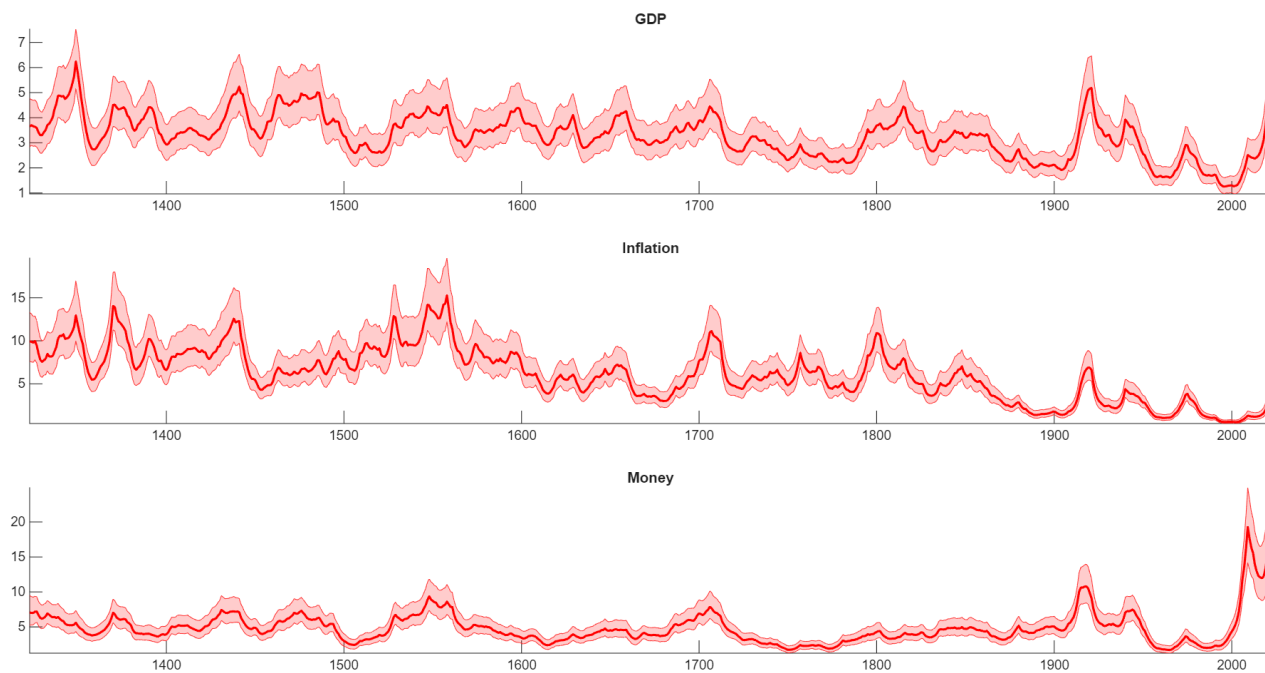
Figure E.6: Predictability



Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

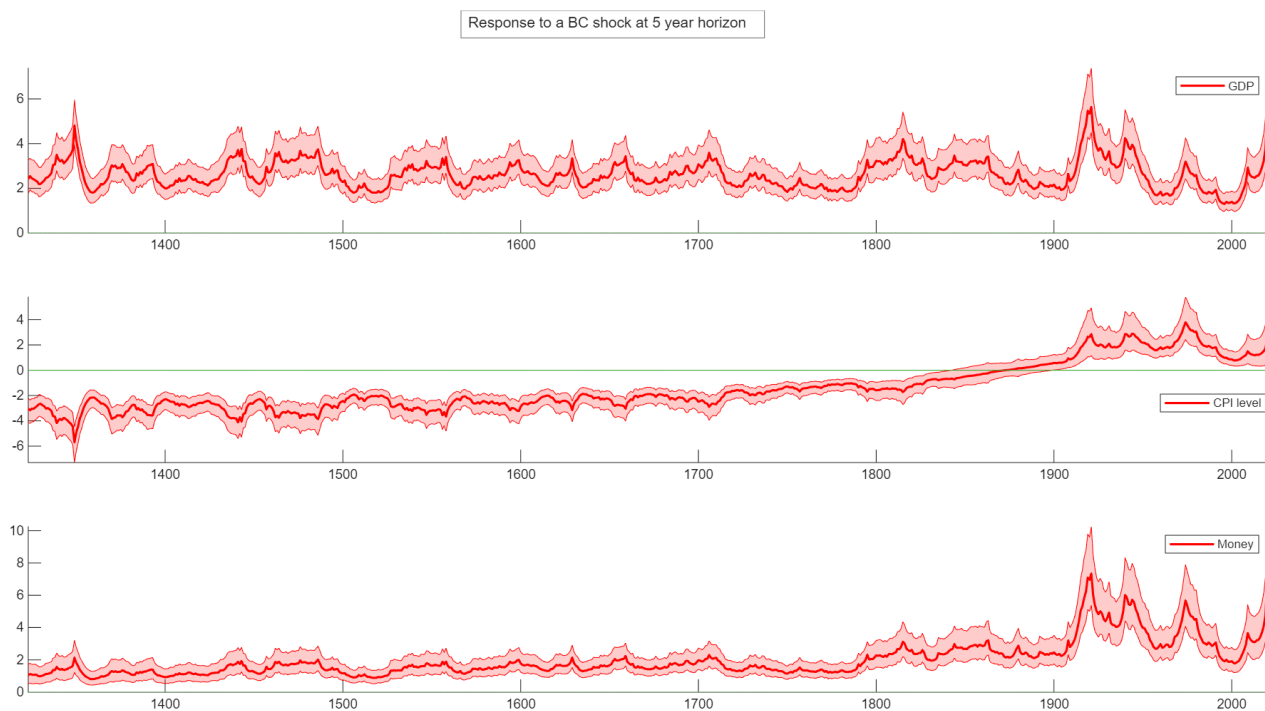
E.4 Three-variable VAR using aggregate (non-per-capita) variables

Figure E.7: Stochastic Volatility



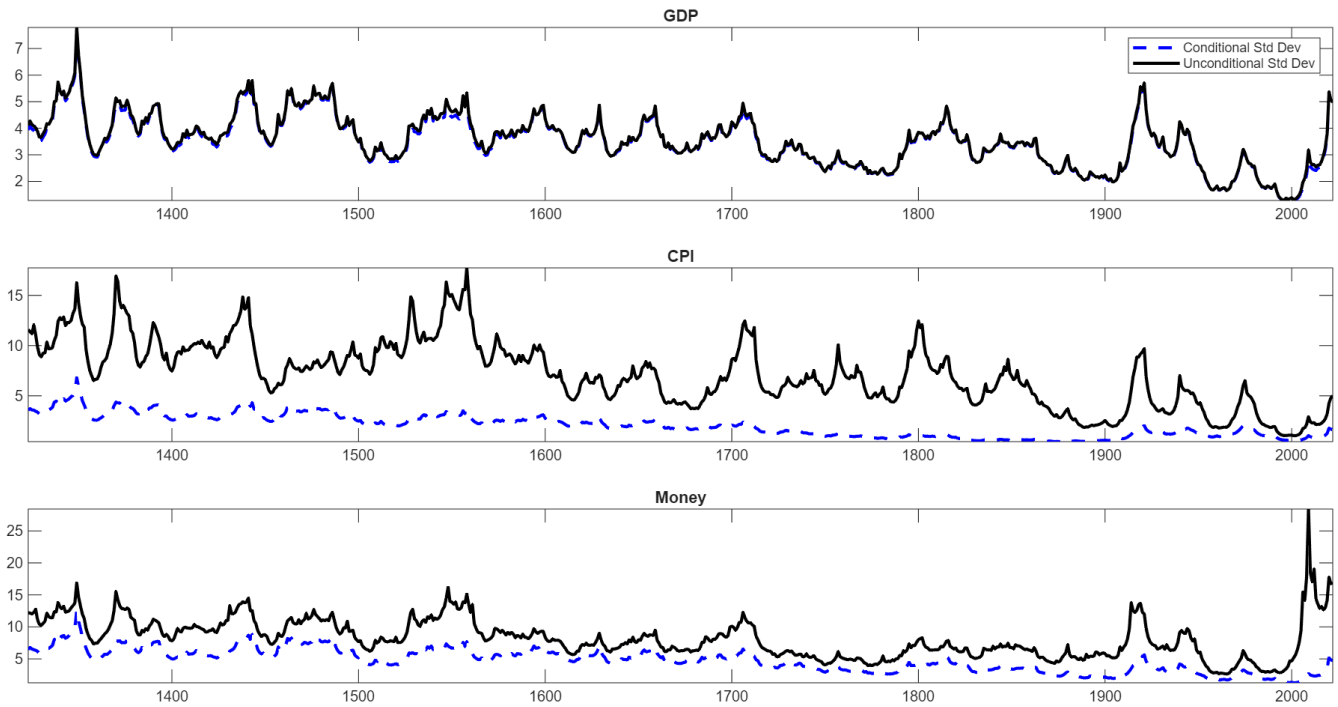
Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

Figure E.8: Impulse responses



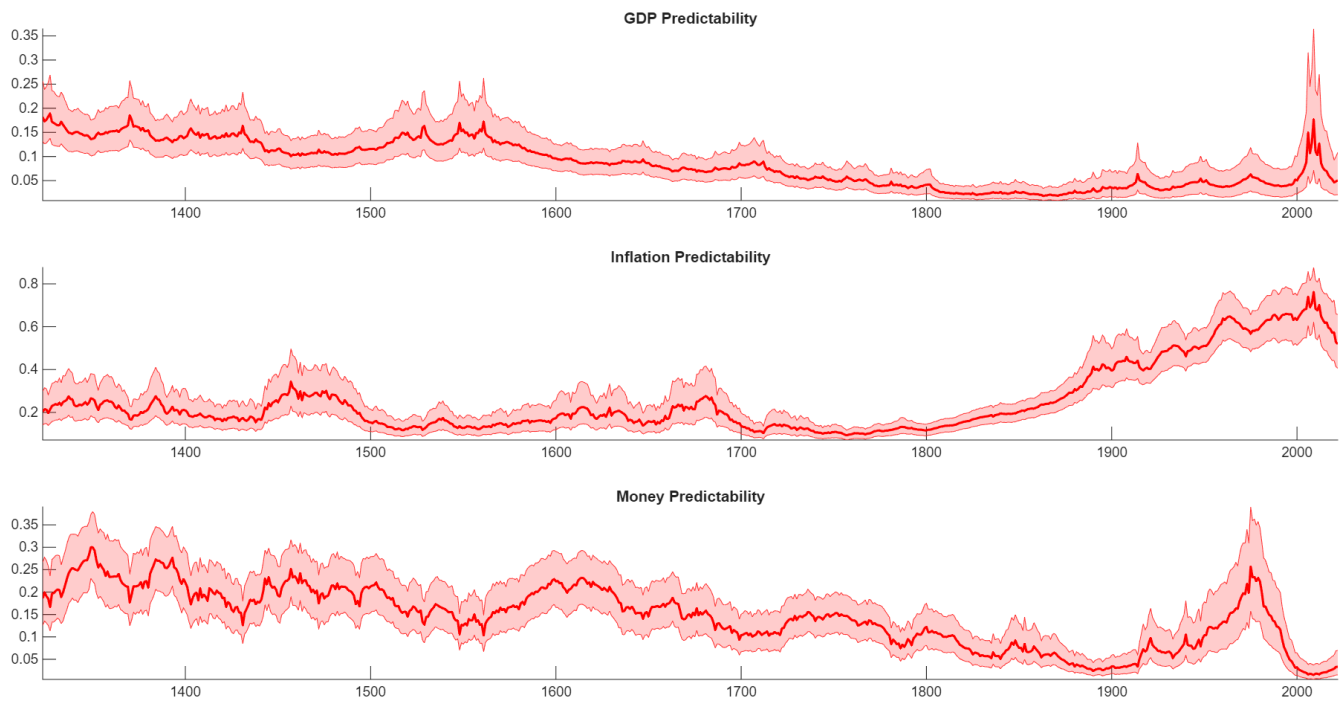
Notes: The pink shaded areas represent the 68 percent credible sets for the IRFs and the red lines depict the median IRFs using aggregate (non-per-capita) variables.

Figure E.9: Conditional Volatility



Notes: The dashed blue lines are the estimates of the time-varying volatilities of each variable conditional on the business cycle shock. The solid black lines are the estimates of the unconditional volatilities of each of these variables.

Figure E.10: Predictability



Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

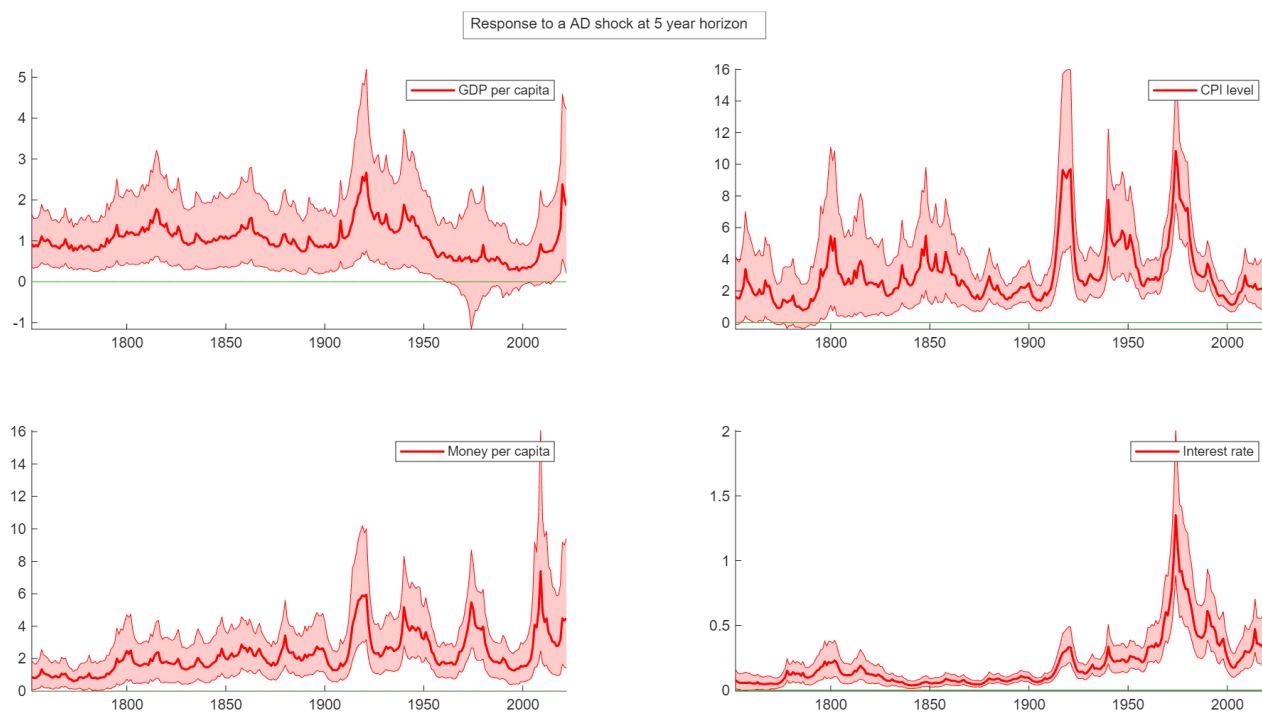
E.5 Extra sign restriction plots

The sign restrictions follow this pattern:

Table E.1: Sign Restrictions on Impact

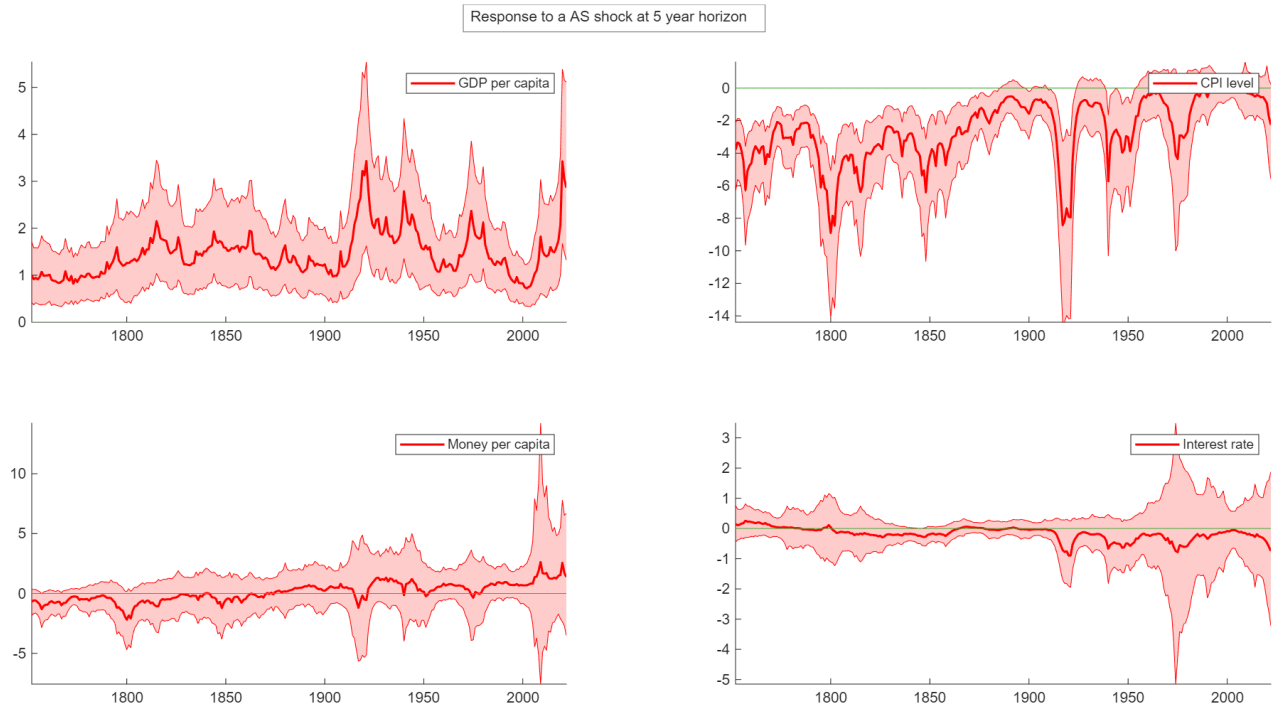
Shock / Variable	GDP per capita	Inflation	Money per capita	Interest rate
Demand shock	+1	+1	+1	+1
Supply shock	+1	-1	0	0
Money demand shock	-1	-1	+1	+1
Money supply shock	+1	+1	+1	-1

Figure E.11: IRF: aggregate demand



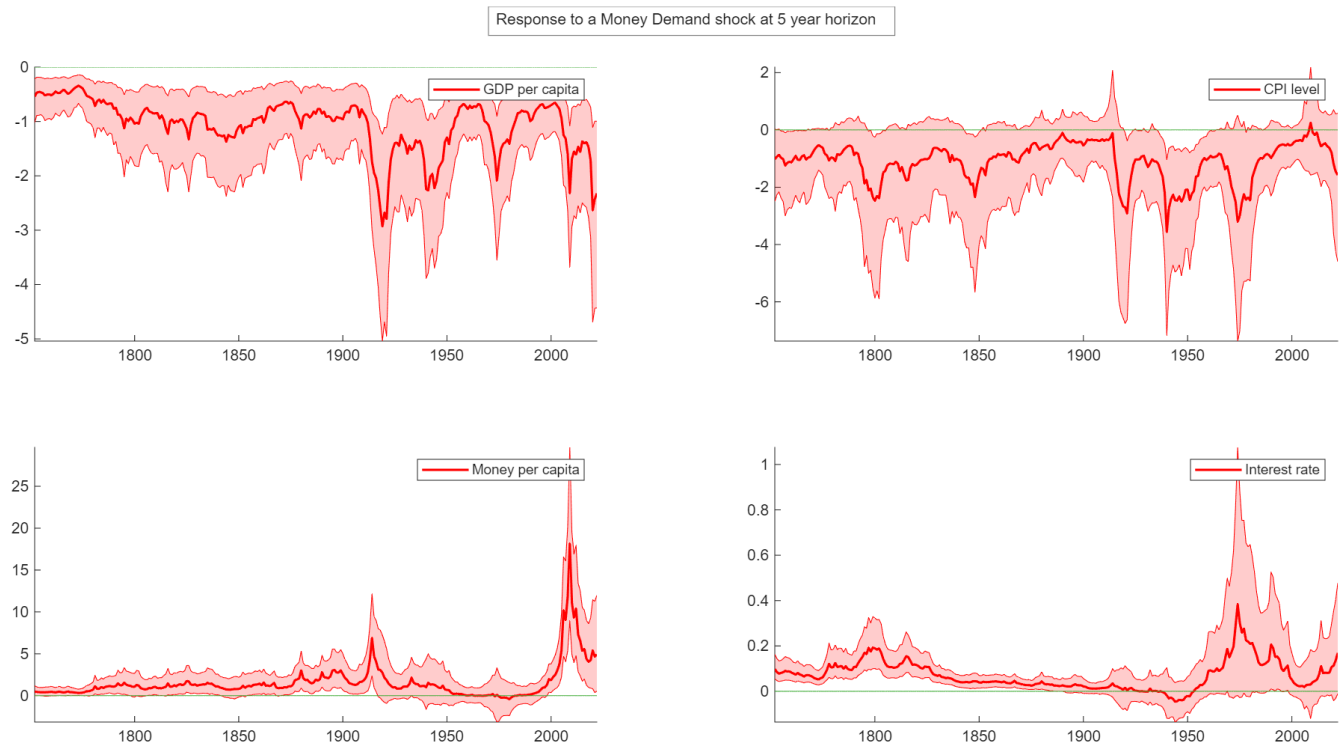
Notes: The pink shaded areas represent the 68 percent credible sets for the IRFs and the red lines depict the median IRFs using sign restrictions.

Figure E.12: IRF: aggregate supply



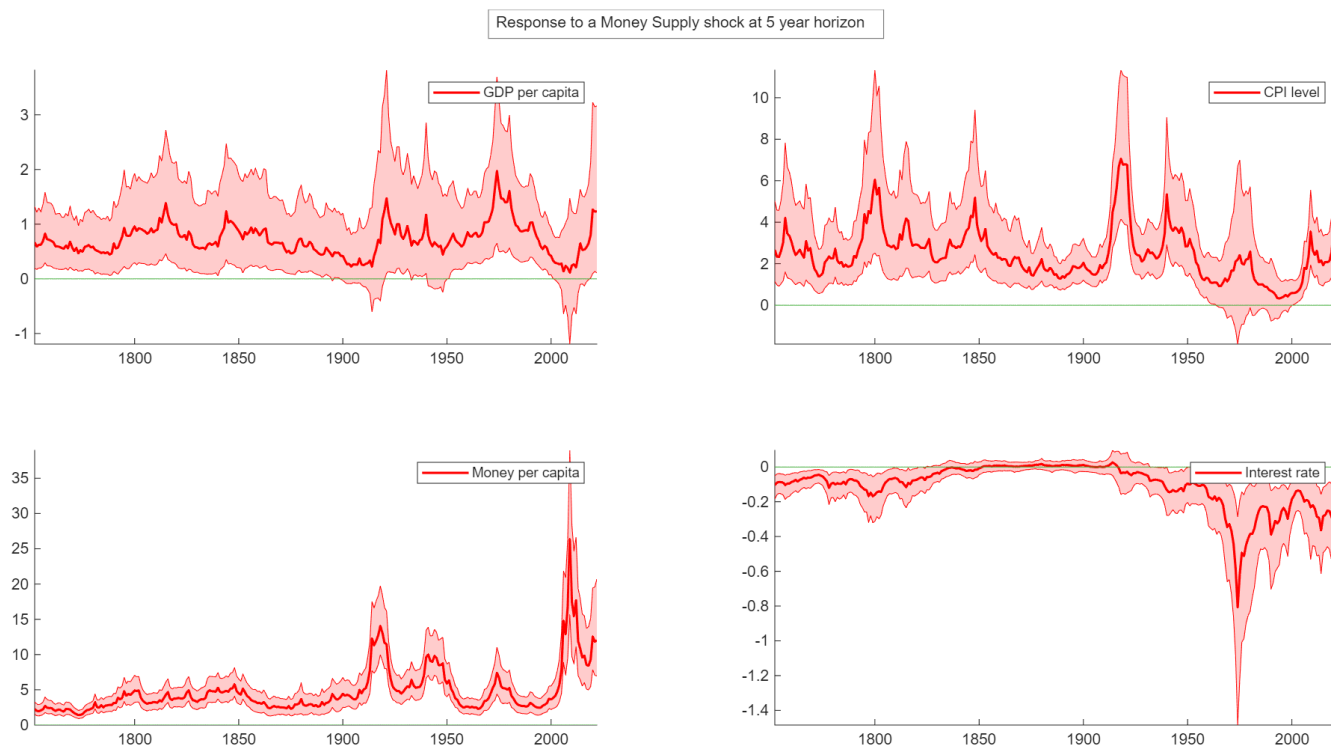
Notes: The pink shaded areas represent the 68 percent credible sets for the IRFs and the red lines depict the median IRFs using sign restrictions.

Figure E.13: IRF: money demand



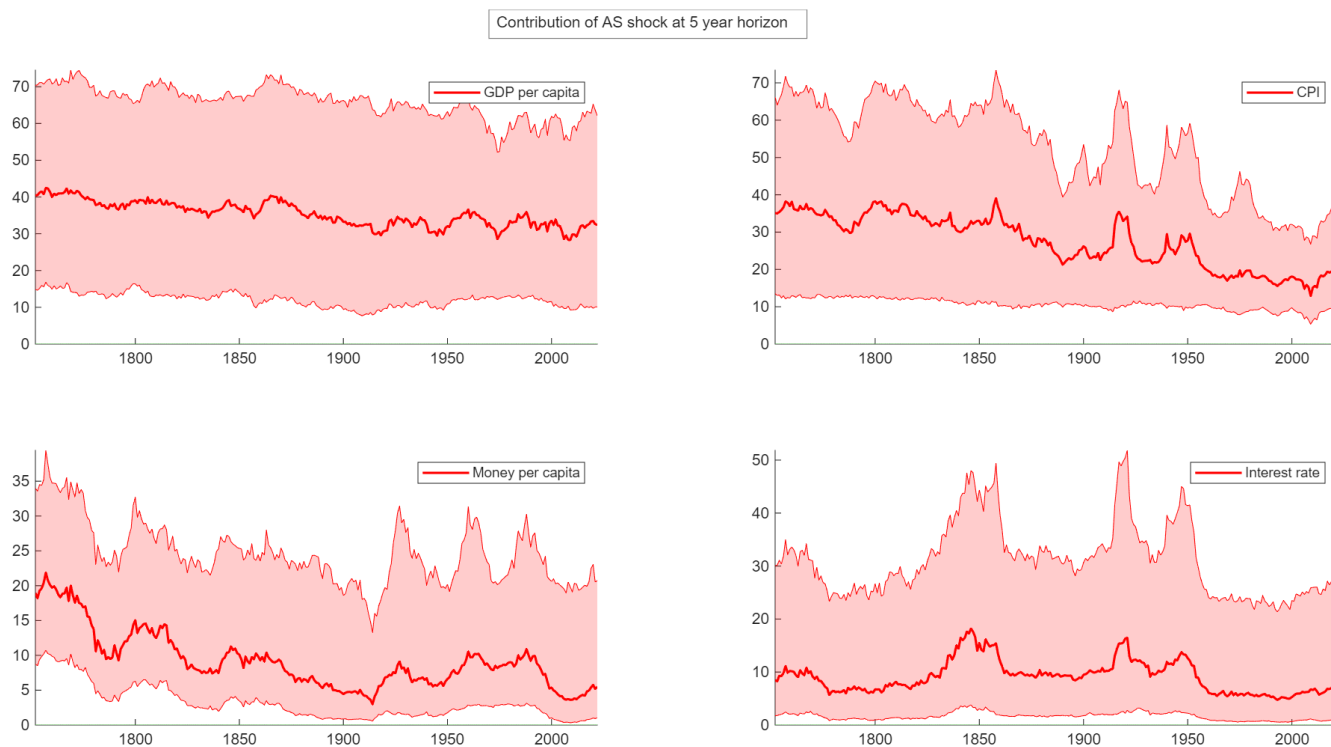
Notes: The pink shaded areas represent the 68 percent credible sets for the IRFs and the red lines depict the median IRFs using sign restrictions.

Figure E.14: IRF: money supply



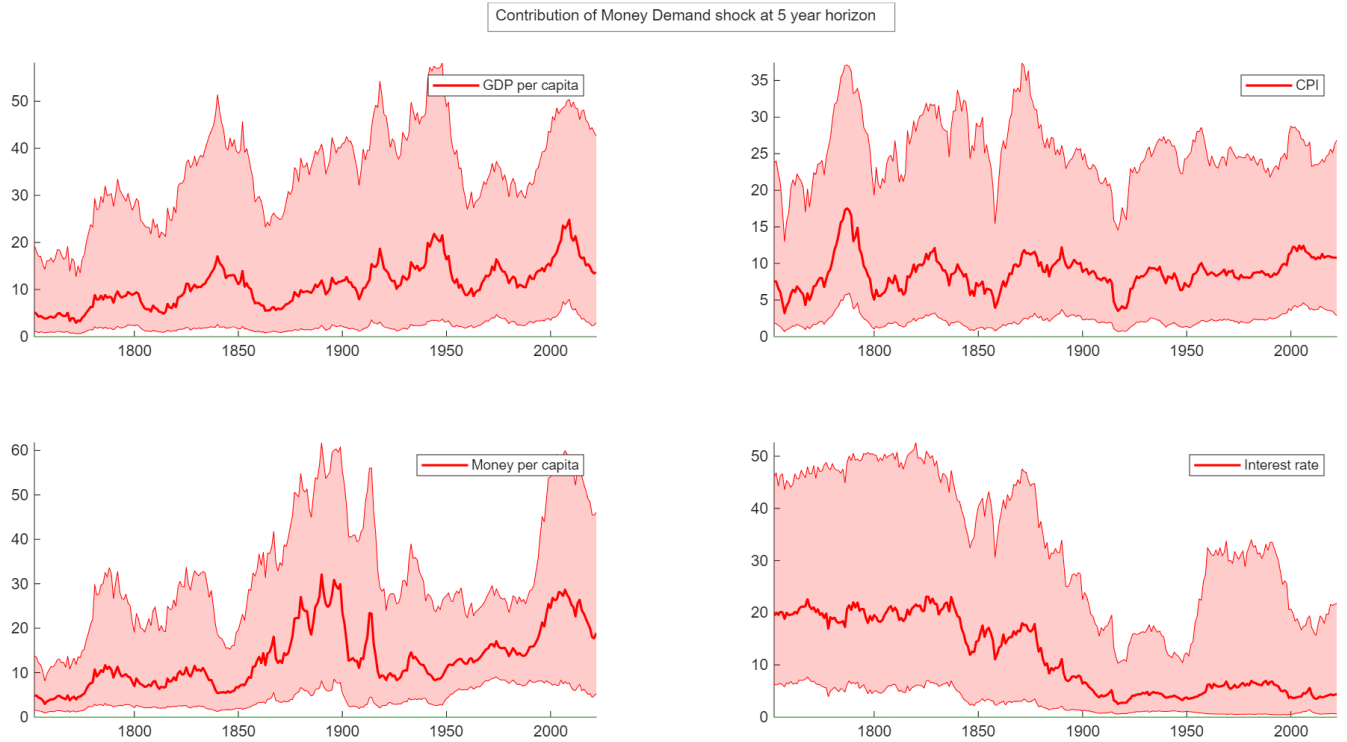
Notes: The pink shaded areas represent the 68 percent credible sets for the IRFs and the red lines depict the median IRFs using sign restrictions.

Figure E.15: FEVD: aggregate supply



Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

Figure E.16: FEVD: money demand

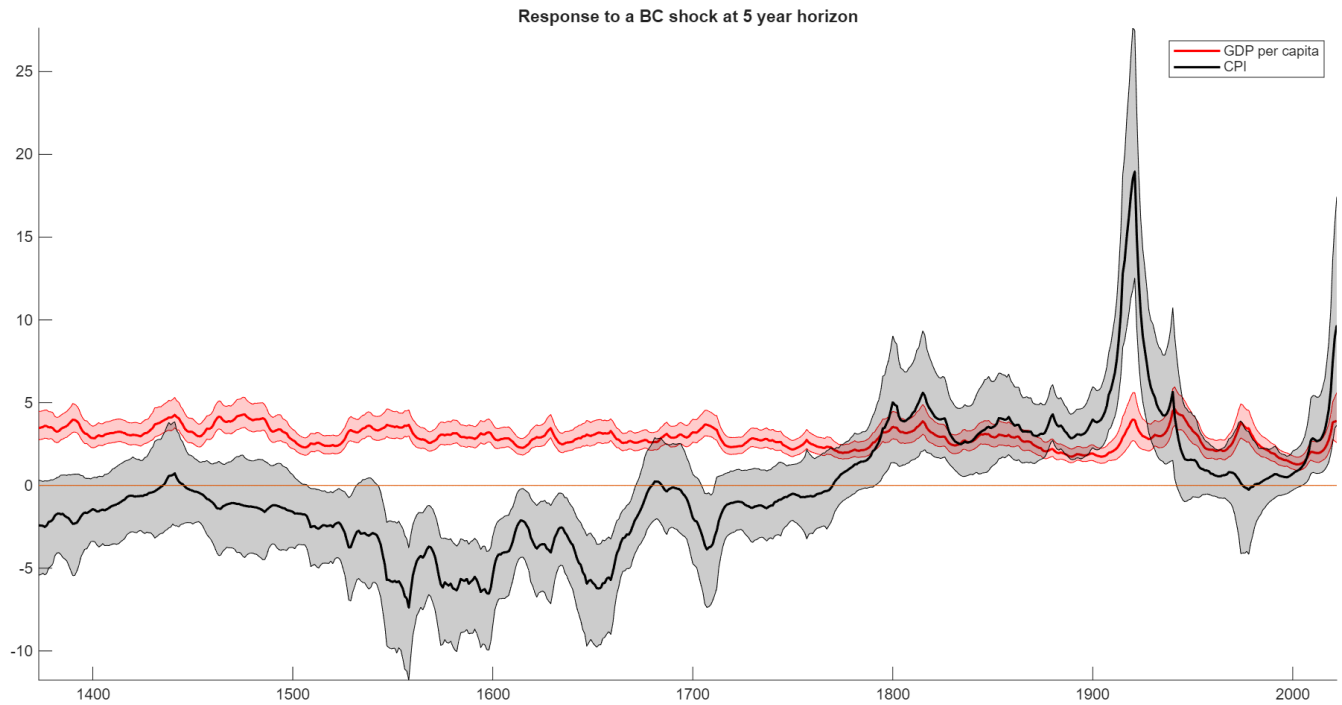


Notes: The pink shaded areas represent the 68 percent credible sets and the red lines depict the medians.

E.6 Dynamic Factor Model

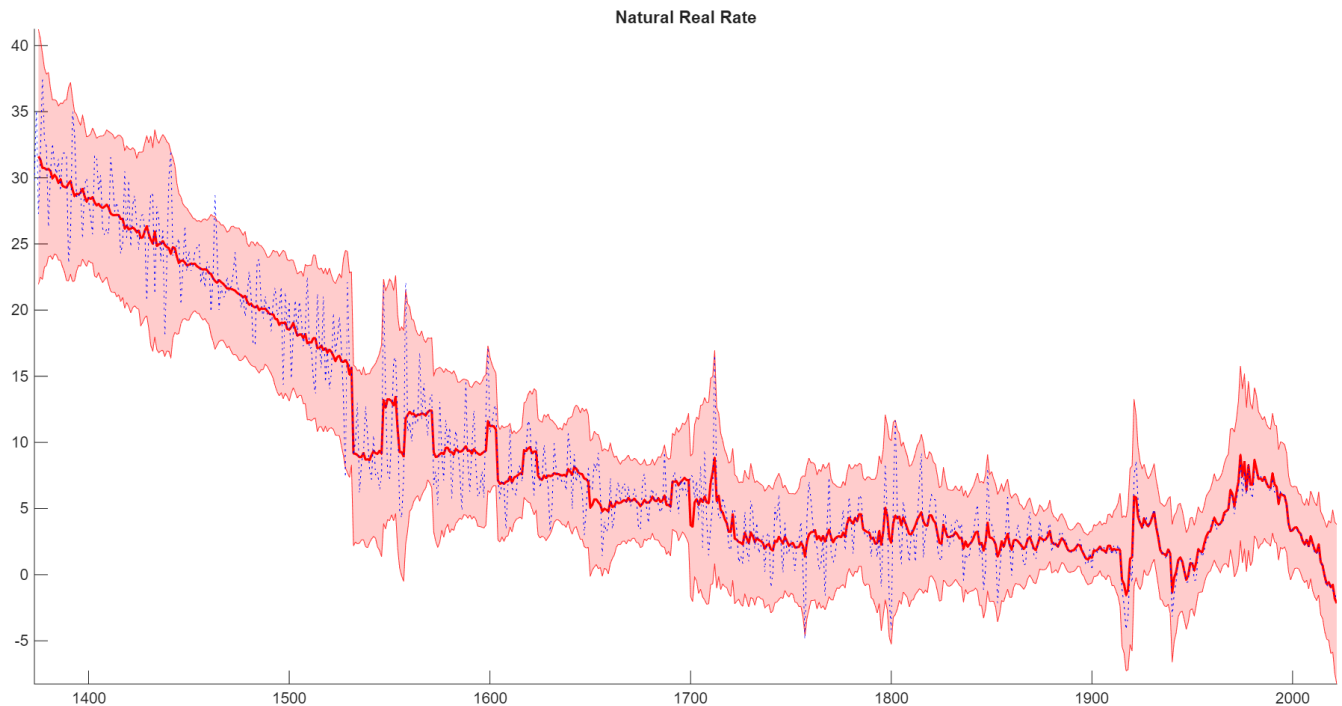
Alternatively, we address measurement error by estimating a Dynamic Factor Model (DFM). Figure E.17 shows that IRFs are similar to baseline results. A by-product of this approach is an estimate of the natural rate of interest. Following [Lubik and Matthes \(2015\)](#), we estimate r -star as the 5-year-ahead forecast of the observed real interest rate. Figure E.18 shows this estimate for the UK over seven centuries. In line with [Schmelzing \(2020\)](#), our estimate exhibits a declining trend throughout the early centuries of the sample, followed by a long period of relative stability. This stability was disrupted in the late twentieth century, with the natural rate peaking at 8.7% in 1974.

Figure E.17: Main IRFs from DFM model



Notes: The grey shaded area represents the 68 percent credible set and the black line depicts the median IRFs of CPI produced by a DFM. The pink shaded area and the red line display the equivalent quantities for the GDP per capita.

Figure E.18: Estimated Natural Rate



Notes: The pink shaded area represents the 68 percent credible set and the red line is the median natural rate of interest.

Estimation and specification We model the dynamics of 12 variables denoted by X_t using a Dynamic Factor model with time-varying parameters that allows for an unbalanced panel. These variables are: earnings, GDP deflator, CPI, exports, commodity prices, GDP, GDP per capita, M4, share prices, the dollar rate, the real interest rate, and public sector net lending.¹ Variables are included in the model in log first differences, with the exception of the interest rate (levels) and the public sector net lending (ratio of nominal GDP).

The observation equation of the model is defined as:

$$\begin{aligned} X_t &= BF_t + v_t \\ v_t &= \rho v_{t-1} + \sigma_t^{\frac{1}{2}} e_t \\ \ln(\sigma_t) &= \ln(\sigma_{t-1}) + g_t \end{aligned}$$

F_t denotes k common factors in the dataset X_t , and the idiosyncratic components e_t accounts for measurement error.

The transition equation is:

$$\begin{aligned} F_t &= \beta_t F_{t-1} + A_t^{-1} H_t^{\frac{1}{2}} \epsilon_t \\ \tilde{\beta}_t &= \tilde{\beta}_{t-1} + u_{\beta,t} \\ h_t &= h_{t-1} + u_{h,t} \\ a_t &= a_{t-1} + u_{a,t} \end{aligned}$$

The model is estimated using an MCMC algorithm. Priors for $\text{var}(u_{\beta,t})$, $\text{var}(u_{h,t})$ and $\text{var}(g_t)$ are based on previous studies such as [Cogley and Sargent \(2005\)](#) and [Primiceri \(2005\)](#) like in the VAR.

¹The real interest rate is obtained from [Schmelzing \(2020\)](#) and the other variables are obtained from [Thomas and Dimsdale \(2018\)](#) and [Broadberry et al. \(2026\)](#) as before.

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