Conservatism and Liquidity Traps

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Re-examining inflation targeting

- In the light of the liquidity trap conditions prevailing in many advanced economies an increasing number of policymakers and economists have called for a reassessment of the inflation targeting framework.

- Several studies argue that we might have underestimated the severity of ZLB events and that the ZLB might bind more frequently in the future (IMF WEO, April 2014).
Two snapshots from the U.S.

- FOMC members median estimate of the appropriate longer-run level of the real federal funds rate is 1.5% according to the March 2015 SEP (compared to 2% in March 2013)

- Market participants attach 15-20% probability to a return to the ZLB during the two years following the first increase in the federal funds target rate/range (Survey of Primary Dealers, March 2015)
This paper

- What are the implications of heightened 'ZLB risks' for the design of inflation targeting?
- How much weight should a central bank put on inflation stabilization relative to output stabilization when the ZLB is an occasionally binding constraint?
- Focus on optimal time-consistent policy
- Abstract from original inflation bias problem (Rogoff, 1985)
Outline

- The model
- Analytical results
- Numerical illustration
- Extensions
  - A model with demand and supply shocks
  - A continuous-state model
A standard sticky-price model

- Identical, infinitely-living **households** maximize expected lifetime utility. Utility is separable in private consumption and leisure.

- **Firms** hire labor to produce consumption goods. They act under monopolistic competition and maximize profits s.t. staggered price setting.

- A benevolent, discretionary **central banker** decides about the one-period nominal interest rate.

- Uncertainty arises from a stochastic natural real rate of interest.
Aggregate private sector behavior

Log-linearized behavioral constraints

\[ \pi_t = \kappa y_t + \beta E_t \pi_{t+1} \]

\[ y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1} - r^*) + d_t, \]

where the parameters satisfy

\[ \kappa > 0 \]

\[ \beta \in (0, 1) \]

\[ \sigma > 0 \]

\[ r^* = \frac{1}{\beta} - 1. \]
Social welfare

Society’s preferences are represented by a linear-quadratic approximation to household welfare

\[ V_t = u(\pi_t, y_t) + \beta E_t V_{t+1}, \]

where

\[ u(\pi_t, y_t) = -\frac{1}{2} \left[ \pi_t^2 + \bar{\lambda} y_t^2 \right] \]

and \( \bar{\lambda} = \frac{\kappa}{\theta} \).
The central banker

- The central banker does not have access to a commitment technology.

- The central bank’s objective function is given by

$$V_{t}^{CB} = u^{CB}(\pi_{t}, y_{t}) + \beta E_{t}V_{t+1}^{CB},$$

where

$$u^{CB}(\pi_{t}, y_{t}) = -\frac{1}{2} \left[ \pi_{t}^{2} + \lambda y_{t}^{2} \right]$$

and $\lambda \geq 0$. 
The monetary policy problem

Each period $t$, the central bank chooses the vector $\{\pi_t, y_t, i_t\}$ in order to maximize its instant objective function

$$\max_{\{\pi_t, y_t, i_t\}} -\frac{1}{2} [\pi_t^2 + \lambda y_t^2]$$

subject to

$i_t \geq 0$

NKPC, Euler equation

$d_t$ given

$$\{\pi_{t+j}, y_{t+j}, i_{t+j} \geq 0\} \text{ given for } j \geq 1.$$
A Markov-Perfect equilibrium is a set of time-invariant value and policy functions \( \{V^{CB}(\cdot), y(\cdot), \pi(\cdot), i(\cdot)\} \) that solves the central bank’s problem, together with society’s value function \( V(\cdot) \), which is consistent with \( y(\cdot) \) and \( \pi(\cdot) \).
A two-state shock version of the model

Assume, $d_t$ follows a two-state Markov process, taking the value of either $d_H$ or $d_L$, where

$$d_H > -\sigma r^*$$

$$d_L < -\sigma r^*.$$ 

The transition probabilities are given by

$$\text{Prob}(d_{t+1} = d_L|d_t = d_H) = p_H$$

$$\text{Prob}(d_{t+1} = d_L|d_t = d_L) = p_L.$$
Markov-Perfect equilibria

- Two Markov-Perfect equilibria
  - *standard* MPE
  - *deflationary* MPE

- Paper provides analytical characterisations of the conditions for equilibrium existence

- Conditions are identical for the two MPE

- *In the remainder, we focus on the standard MPE*
The standard Markov-Perfect equilibrium

The standard MPE is given by a vector \( \{y_H, \pi_H, i_H, y_L, \pi_L, i_L\} \) that solves

\[
y_H = [(1 - p_H)y_H + p_H y_L] + \sigma[(1 - p_H)\pi_H + p_H \pi_L - i_H + r^*] + d_H
\]

\[
\pi_H = \kappa y_H + \beta[(1 - p_H)\pi_H + p_H \pi_L]
\]

\[
0 = \lambda y_H + \kappa \pi_H
\]

\[
y_L = [(1 - p_L)y_H + p_L y_L] + \sigma[(1 - p_L)\pi_H + p_L \pi_L - i_L + r^*] + d_L
\]

\[
\pi_L = \kappa y_L + \beta[(1 - p_L)\pi_H + p_L \pi_L]
\]

\[
i_L = 0
\]

\[
i_H > 0
\]

\[
0 > \lambda y_L + \kappa \pi_L.
\]
Existence of the standard MPE

The standard Markov-Perfect equilibrium exists if and only if

\[ p_L \leq p_L^*(\Theta(-p_L)) \]

and

\[ p_H \leq p_H^*(\Theta(-p_H)). \]
The signs of variables in the standard MPE

For any $\lambda \geq 0$, in the low state

- $\pi_L < 0$
- $y_L < 0$

and in the high state

- $\pi_H \leq 0 \rightarrow \textit{deflationary bias}$ of discretionary policy
- $y_H > 0$

Note: If $\lambda = 0$, then $\pi_H = 0$. 
The role of conservatism

For any $\lambda \geq 0$

- $\frac{\partial \pi_H}{\partial \lambda} < 0$

- $\frac{\partial y_H}{\partial \lambda} < 0$ iff $\beta p_H - (1 - \beta) \left( \frac{1-p_L}{\kappa \sigma} (1 - \beta p_L + \beta p_H) - p_L \right) < 0$

- $\frac{\partial \pi_L}{\partial \lambda} < 0$

- $\frac{\partial y_L}{\partial \lambda} < 0$

Monetary conservatism mitigates the deflationary bias and improves stabilization outcomes at the ZLB.
The optimal degree of conservatism

We quantify welfare costs by the perpetual consumption transfer (as a share of its steady state) that would make a household in the economy indifferent to living in the economy without any fluctuations

\[ W := (1 - \beta) \frac{\theta}{\kappa} (\sigma^{-1} + \eta) \mathbb{E}[V]. \]

Proposition: Welfare is maximized at \( \lambda = 0. \)
Numerical illustration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Economic interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.5</td>
<td>Intertemporal elasticity of substitution in consumption</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.47</td>
<td>Inverse labor supply elasticity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>10</td>
<td>Price elasticity of demand</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8106</td>
<td>Share of firms per period keeping prices unchanged</td>
</tr>
<tr>
<td>$d_H$</td>
<td>0</td>
<td>Demand shock in the high state</td>
</tr>
<tr>
<td>$d_L$</td>
<td>-0.0113</td>
<td>Demand shock in the low state</td>
</tr>
<tr>
<td>$p_H$</td>
<td>0.005</td>
<td>Frequency of contractionary demand shock</td>
</tr>
<tr>
<td>$p_L$</td>
<td>0.875</td>
<td>Persistence of contractionary demand shock</td>
</tr>
</tbody>
</table>
Policy functions and the degree of conservatism

High State

Output Gap (%)

Inflation (Annualized %)

Nominal Interest Rate (Annualized %)

Low State
Welfare and the degree of conservatism
Welfare gains from conservatism
Extension I: A model with demand and supply shocks

We now extend the analysis to an economy that is subject to both demand and supply shocks. In this case, the NKPC becomes

\[ \pi_t = \kappa y_t + \beta E_t \pi_{t+1} + u_t, \]

where \( u_t \) is a cost-push shock.

We assume that the cost-push shock takes two values, \( u_H = c \geq 0 \) and \( u_L = -c \), with probability 0.5 regardless of the state today.
The optimal degree of conservatism in the model with supply shocks
Policy functions and the degree of conservatism
Extension II: A continuous-state model

We now assume that $d_t$ follows a stationary AR(1)-process

$$d_t = \rho_d d_{t-1} + \epsilon^d_t$$

where $\epsilon^d_t$ is an i.i.d. $N(0, \sigma^2_\epsilon)$ innovation. We set $\rho_d = 0.9$ and $\sigma_\epsilon = 0.176$ so that the ZLB frequency amounts to about 30%.

There are no cost-push shocks.

The policy functions are approximated using a projection method.
Approximated policy functions

Note: $\lambda = 0$ (solid lines) and $\lambda = \bar{\lambda}$ (dashed lines). $r_t = r^* + \frac{1}{\sigma} d_t.$
Conditional expectations

Output Gap

Inflation

Nominal Interest Rate

ZLB not binding

ZLB binding
Welfare in the continuous-state model
Conclusion

- An economy that experiences occasional ZLB episodes can improve welfare by appointing an inflation conservative central banker.

- Under flexible IT, the presence of the ZLB faces the central bank with a stabilization trade-off when the ZLB is not binding.

- This trade-off results in a deflationary bias that exacerbates the decline in inflation and output at the ZLB.

- A conservative central banker counteracts this vicious cycle by mitigating (or eliminating) the deflationary bias.
Appendix
Context: Policy options to deal with ZLB

1. Change policy at the ZLB:
   - fiscal policy
   - quantitative easing, asset purchase programs

2. Design regime that temporarily changes policy out of the ZLB conditional on a preceding ZLB event:
   - price level targeting
   - nominal income level targeting
   - several other proposals that introduce history dependence
3. Change policy outside of the ZLB:

- positive inflation target
- Rogoff’s (1985) conservative central banker

Most proposals in the literature fit into the first two categories.

Our paper is one of the few that considers a policy option belonging to the third category.
Policy functions (Two-state model)

\[ \pi_H = -\frac{\beta \lambda p_H}{E(\lambda)} r_L \]

\[ \pi_L = -\frac{\kappa^2 + \lambda(1 - \beta + \beta p_H)}{E(\lambda)} r_L \]

\[ y_H = \frac{\beta \kappa p_H}{E(\lambda)} r_L \]

\[ y_L = -\frac{(1 - \beta p_L) \kappa^2 + \lambda(1 - \beta)(1 - \beta p_L + \beta p_H)}{\kappa E(\lambda)} r_L \]

where \( E(\lambda) = \beta \lambda p_H - (\kappa^2 + \lambda(1 - \beta)) \left[ \frac{1-p_L}{\kappa \sigma} (1 - \beta p_L + \beta p_H) - p_L \right] < 0 \) if standard MPE exists.
Numerical procedure

- Equilibrium conditions $H(z, s)$, where $z$ are the free endogenous variables.
- Want to approximate unknown policy functions $z = f(s)$.
- Use a weighted sum of known basis functions $\psi_1, \ldots, \psi_n$:
  \[
  f(s) \approx C\Psi(s)
  \]
- Choose coefficients in $C$ such that $H(C\Psi(s), s)$ is close to zero for $s \in S$.
  - linear splines (finite elements)
  - collocation method