Monetary Policy with Ambiguity Averse Agents

Riccardo M. Masolo and Francesca Monti

Bank of England and Centre for Macroeconomics


Disclaimer: Any views expressed are solely those of the author and so cannot be taken to represent those of the Bank of England or to state Bank of England policy
Motivation

- In the last 50 years central bank transparency and communication have increased enormously.
- Since the beginning of the Great Recession, communication has become an even more important tool.
- Lately policymakers have been explicitly mentioning their desire to “clarify their reaction function.”

*The real questions have to do with how much a central bank should reveal about its own decision processes, something that the standard literature treats as being so well understood by everyone that they require no discussion.*

Michael Woodford, 2007
Our paper

We study a simple model (e.g. Galí 2008), augmenting it so that

- there is scope for clarifying the reaction function
- there is a role for changes in the confidence the agents have about their model/information

⇓

① The agents face ambiguity about the reaction function of the Central Bank

② The agents dislike such ambiguity and base their decisions as if the worst case scenario materialised
Main results

1. A reduction in ambiguity can be an effective way to reduce rates, reduce inflation and increase welfare.

2. Inflation responsiveness interacts with ambiguity and regains relevance, unlike in the standard case (Schmitt-Grohé and Uribe, 2007)

3. Knightian uncertainty matters for the optimal choice of an inflation target

4. Credibility can affect the economic transmission (e.g. the slope of the Phillips Curve)
Interest Rate

The Central Bank follows a very simple reaction function (which, however, is optimal in our simple setting):

\[ R_t = R_t^n \prod_t^\phi \]

Where \( R_t^n = \mathbb{E}_t \frac{A_{t+1}}{\beta A_t} \) is the natural rate of interest.

To build some intuition we jump all the way to its linear counterpart.

In a standard rational-expectations model:

\[ \mathbb{E}_t[r_t] = \mathbb{E}_t[r_t^n + \phi \pi_t] = r_t^n + \phi \pi_t \]

Under ambiguity:

\[ \mathbb{E}_t^\mu[r_t] = \mathbb{E}_t[r_t] + \mu_t = r_t^n + \phi \pi_t + \mu_t \quad \mu_t \in [-\mu_t, \mu_t] \]
Worst-case characterization

In our baseline scenario (with a production subsidy and the response function on the previous slide), it is easy to verify that:

- For all economically sensible parameter values, the value function is strictly concave around $\mu = 0$, where it achieves its maximum.

- As $\beta \to 1^-$ the worst-case scenario corresponds to $\mu = -\bar{\mu}$ for any sufficiently small $\bar{\mu}$, given any economically sensible value for the other parameters.
Inflation and interest rate determination

- Suppose agents "underestimate" the policy rate

- Their intertemporal Euler equation will make them want to bring consumption forward

- This creates inflationary pressures

- The Central Bank will increase the policy rate more than one for one
Figure: Steady State policy rate (solid) and interest rate expected by ambiguity-averse agents (dashed) as a function of $\mu$. 
Effects of a reduction in ambiguity on inflation and the policy rate

Result

Let $\beta \to 1^-$ and let all the other parameters take on any economically sensible value, then for any small enough $\bar{\mu} > 0$:

$$\Pi_w(\bar{\mu}') < \Pi_w(\bar{\mu}) \quad R_w(\bar{\mu}') < R_w(\bar{\mu}) \quad \forall \ 0 \leq \bar{\mu}' < \bar{\mu}$$

where $\Pi_w(\bar{\mu})$ and $R_w(\bar{\mu})$ are defined as the levels of inflation and the policy rate prevailing in the worst-case state state over the interval $[-\bar{\mu}, \bar{\mu}]$. 
Inflation Target

• In these models, absent ambiguity, an inflation target can be set optimally to reduce distortions (from monopolistic competition in our case)

• The choice of the target should also account for the degree of uncertainty in the economy

• Targets based on models that disregard uncertainty can be sub-optimal when Knightian uncertainty plays a role in agents’ decisions.
An Inflation Target in our model

- Our model is simple enough that a Taylor rule and a subsidy can implement the first-best allocations at all times.

- In larger models it would not be so.

- To get a sense of the potential effects of working in such an environment we study what happens in our simple model in the absence of the production subsidy.
Welfare function - no subsidy

Figure: On the left pane, the first-best welfare function (red) and that in the absence of the subsidy (blue). On the right pane a detail of the welfare function without subsidy around zero.
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Optimal Target disregarding ambiguity

• The optimal inflation target is the one that makes $\nabla_\ast'(0) = 0$ in the absence of a subsidy.

• The Taylor rule would become:

$$R_t = R_t^n \left( \frac{\Pi_t}{\Pi_\ast} \right)^\phi$$

$$\Pi_\ast \equiv e^{-\frac{\mu_{SB}}{\phi}} \mu_{SB} : \nabla_{NT}'(\mu_{SB}) = 0$$
Figure: Welfare function without subsidy in blue and without subsidy but with the optimal inflation target in green.
Worst-Case Inflation

\[ \Pi_{WS} \]

with the subsidy
without the subsidy
no subsidy and target

\[ \bar{\mu} \]

\[ x \times 10^{-4} \]
Optimal Inflation Target: Some Considerations

• Under our proposed target, inflation would be higher than in the first-best economy (for any level of ambiguity):

$$\Pi(\mu, \cdot) = e^{-\frac{\mu}{\phi-1}} \Pi^*_{\phi-1}, \quad \Pi^* > 1$$

• Even for $\mu = 0$, $\Delta > 1$, i.e., there is some welfare loss

• Target+No Ambiguity: Second Best

• This level of the target is optimal in the absence of ambiguity, but uncertainty persists a different level of the target should be implemented.
Figure: Difference in the worst-case welfare between the case in which an inflation target is implemented and that in which is not for different values $\bar{\mu}$. In formulas:

$$\min_{\mu \in [-\bar{\mu}, \bar{\mu}]} V_*(\mu, \cdot) - \min_{\mu \in [-\bar{\mu}, \bar{\mu}]} V_{NT}(\mu, \cdot)$$
General Conclusions

In a small New-Keynesian model we show that ambiguity on the policymaker’s response function:

- has first-order effects on welfare. In general credibility seems to be disinflationary

- interacts in a non-trivial way with commonly studied policy-design concepts such as the Central Bank’s responsiveness to inflation. High $\phi$ can substitute for lower ambiguity

- should be taken into consideration for the evaluation of standard policy-design experiments such as an inflation target
Mainly, two alternative preferences specifications used for representing ambiguity aversion in macro:

   - Multiple priors utility is not smooth when belief sets differ in means.
   - Effects of ambiguity show up in a first order approximation (Ilut and Schneider, 2014)

   - Fear of misspecification: statistical perturbation around an approximating model.
   - Smooth utility function
I. Optimal MP design in small NK models. An incomplete list includes:
   - King and Wolman 1996,
   - Schmitt-Grohé and Uribe, 2007
   - Ascari and Ropele, 2007
   - Yun, 2005

II. Ambiguity:
   - Ilut and Schneider, 2014 (first-order effects of ambiguity)
   - Gilboa and Schmeidler, 1998

III. Ambiguity and Monetary Policy:
   - Hansen and Sargent, 2005
   - Adams and Woodford, 2012
   - Benigno and Paciello, 2014
Timing

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III. If the private sector was to fully trust the Central Bank (the standard modeling assumption) then clearly there would be no uncertainty about the policy rate.

IV. In our model, however, agents are ambiguous and will act on the worst-case interest rate.
Households’ problem

The households solve this problem, which reflect their aversion to ambiguity:

\[ U_t(\mathbf{C}^t; s^t) = u(\mathbf{C}_t) + \beta \min_{\mu \in [-\bar{\mu}_t, \bar{\mu}_t]} \mathbb{E}^{\mu} U_{t+1}(\mathbf{C}; s_t, s_{t+1}) \]

\[ P_t C_t + B_{t+1} = R_{t-1} B_t + W_t N_t + T_t \]

where their felicity is described by:

\[ u(\mathbf{C}_t) = \log[C_t] - \chi \frac{N_t^{1+\sigma}}{1 + \sigma} \]
Households’ First-Order Conditions

\[
\frac{1}{C_t} = \mathbb{E}_t^\mu \left[ \frac{\beta R_t}{C_{t+1} \Pi_{t+1}} \right] \\
\chi N_t^\sigma C_t = \frac{W_t}{P_t}
\]

Hence the intertemporal Euler equation becomes:

\[
\frac{1}{C_t} = \mathbb{E}_t \left[ \frac{\beta \tilde{R}_t}{C_{t+1} \Pi_{t+1}} \right]
\]
Firm’s problem

Firms maximize expected profits subject to Calvo frictions:

$$\max_{P_t^*} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \theta^s Q_{t+s} \left( \left( \frac{P_t^*}{P_{t+s}} \right)^{1-\epsilon} Y_{t+s} - \Psi \left( \left( \frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right) \right) \right]$$

Which result in the following standard first-order conditions:

$$\frac{P_t^*(i)}{P_t} = \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t+s} \left( \frac{P_{t+j}}{P_t} \right)^{\epsilon} \frac{\epsilon}{\epsilon-1} MC_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t+s} \left( \frac{P_{t+j}}{P_t} \right)^{\epsilon-1}}$$

$$\frac{P_t^*(i)}{P_t} = \left( \frac{1 - \theta \Pi_t^{\epsilon-1}}{1 - \theta} \right)^{\frac{1}{1-\epsilon}}$$
The government taxes to finance the subsidy. We lump the profits together with the tax, which results in the following:

\[
T_t = P_t \left( -\tau \frac{W_t}{P_t} N_t + Y_t \left( 1 - (1 - \tau) \frac{W_t \Delta_t}{P_t A_t} \right) \right) \\
= P_t Y_t \left( 1 - \frac{W_t \Delta_t}{P_t A_t} \right)
\]
Steady State

\[ \Pi(\mu, \cdot) = e^{-\frac{\mu}{\phi - 1}} \]

\[ R(\mu, \cdot) = \frac{1}{\beta} e^{-\frac{\phi \mu}{\phi - 1}} \quad \tilde{R}(\mu, \cdot) = \frac{1}{\beta} e^{-\frac{\mu}{\phi - 1}} \]

\[ N(\mu, \cdot) = \left( \frac{(1 - \theta \Pi(\mu, \cdot)^{\epsilon - 1}) (1 - \beta \theta \Pi(\mu, \cdot)^{\epsilon - 1})}{\chi (1 - \beta \theta \Pi(\mu, \cdot)^{\epsilon - 1}) (1 - \theta \Pi(\mu, \cdot)^{\epsilon})} \right) \frac{1}{1 + \psi} \]

\[ \Delta(\mu, \cdot) = \frac{(1 - \theta) \left( \frac{1 - \theta \Pi(\mu, \cdot)^{\epsilon - 1}}{1 - \theta} \right)^{\frac{\epsilon}{\epsilon - 1}}}{1 - \theta \Pi(\mu, \cdot)^{\epsilon}} \]

\[ C(\mu, \cdot) = \frac{A}{\Delta(\mu, \cdot)} N(\mu, \cdot) \]

\[ \nabla(\mu, \cdot) = \frac{1}{1 - \beta} \left( \log(C(\mu, \cdot)) - \chi \frac{N(\mu, \cdot)^{1+\psi}}{1 + \psi} \right) \]
Worst-case characterization

Concavity:

Result

For $\beta \in [0,1)$, $\epsilon \in (1,\infty)$, $\theta \in [0,1)$, $\phi \in (1,\infty)$, $\psi \in [0,\infty)$, $V(\mu, \cdot)$ is continuously differentiable around $\mu = 0$ and:

$$\frac{\partial V(0, \cdot)}{\partial \mu} = 0 \quad \text{and} \quad \frac{\partial^2 V(0, \cdot)}{\partial \mu^2} < 0$$

As a consequence, for small enough $\bar{\mu}$, there are no minima in $\mu \in (-\bar{\mu}, \bar{\mu})$.

... and ”skew”:

Result

As $\beta \to 1^{-}$ and all the other parameters are in the intervals defined above, $\mu = -\bar{\mu}$ minimizes $V(\mu, \cdot)$ over $[-\bar{\mu}, \bar{\mu}]$, for any sufficiently small $\bar{\mu} > 0$. 
Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>Discount Factor</td>
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<tr>
<td>$\psi$</td>
<td>Inverse Frisch</td>
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</tr>
<tr>
<td>$\chi$</td>
<td>Labor Disutility Scaling</td>
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<td>$\phi$</td>
<td>Inflation Responsiveness</td>
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<tr>
<td>$\theta$</td>
<td>Calvo parameter</td>
<td>.83</td>
</tr>
</tbody>
</table>
Figure: Steady-state marginal costs for different values of $\mu$. 
A Simple Equivalence: Characterization

Result

For all economically sensible parameter values, $\beta \to 1^-$, and $\bar{\mu}$ a small positive number; given any pair $(\mu, \phi) \in [-\bar{\mu}, 0) \times (1, \infty)$, for any $\mu' \in [-\bar{\mu}, 0)$ there exists $\phi' \in (1, \infty)$ such that:

$$\forall (\mu, \phi') = \forall (\mu', \phi)$$

(1)

And $\phi' \geq \phi$ iff $\mu' \geq \mu$.

A corresponding equivalence holds for $\mu \in (0, \bar{\mu}]$. 