Estimating Strategic Complementarity in a State-Dependent Pricing Model

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Overview

1. Motivation
2. Theoretical Model
3. Econometric Methodology and Identification
4. Dataset and Some Results
5. Preliminary Conclusions and Further Research
Price rigidity is an important assumption in Macroeconomics.

Two classes of sticky price models:

1. Time-Dependent Pricing Model (TDP)
   - Calvo (1983), New Keynesian Literature

2. State-Dependent Pricing Model (SDP)
   - Caplin and Spulber (1987), Dotsey, King and Wolman (1999)

A recent (old) discussion: Can sticky price models generate large real effects from monetary shocks?

- Result: State-dependent model presents smaller real effects from monetary shocks than time-dependent pricing model.
Can state-dependent pricing models behave like time-dependent pricing models?
- Golosov and Lucas (2007)
- Gertler and Leahy (2008), Midrigan (2009), Woodford (2009)

Menu cost model + strategic complementarity: Yes, they can!

Strategic complementarity: Decisions of two or more players are called strategic complements if they mutually reinforce one another.

Bils, Klenow and Malin (2009): Reset Price Inflation
They found that a SDP model with no strategic complementarities aligns more closely with the data.
We propose a methodology to estimate directly from microdata the structural parameter related to strategic complementarity in a SDP model.

We estimate some parameters defining the (S,s) pricing rule, as well as the variance of shocks affecting the firms in each sector. We relate these parameters to the price rigidity behavior in each sector.

We use microdata underlying the Brazilian CPI to estimate the model. Additionally, we document some stylized facts about price rigidity in Brazil and relate them to our results.
The Model

The model has three main elements:

- Households obtain utility from consumption goods. Firms supply differentiated goods in a monopolistically competitive environment. In the (segmented) labor markets households and firms behave competitively.

- Firms follow a state-dependent pricing rule.

- There are aggregate shocks and idiosyncratic productivity shocks.
Households: The representative household seeks to maximize

\[
E_0 \left\{ \sum_{t=0}^{\infty} e^{-\rho t} \left[ u(C_t; \xi_t) - \int_0^1 v(L_{i,t}; \xi_t) \, di \right] \right\}
\]

where \( C_t = \left[ \int_0^1 C_{i,t}^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)} \) and \( P_t = \left[ \int_0^1 P_{i,t}^{1-\theta} \, di \right]^{1/(1-\theta)} \)

The expenditure minimization problem implies that the demand for an individual product has the familiar form:

\[
C_{i,t} = C_t \left( \frac{P_{i,t}}{P_t} \right)^{-\theta}
\]

The optimal quantity of labor is implicitly given by

\[
\frac{v_L(L_{i,t}; \xi_t)}{u_C(C_t; \xi_t)} = \frac{W_{i,t}}{P_t}
\]
Firms: There is a continuum of monopolistically competitive firms supplying differentiated goods. Each firm has the production function:

\[ Y_{i,t} = A_{i,t} L_{i,t}^\alpha M_t^{1-\alpha} \]  

(4)

This leads to the following real marginal cost function:

\[
\psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}) = \frac{\lambda}{A_{i,t}} \left\{ \frac{v_L(Y_{i,t} ; \xi_t)}{u_C(Y_t ; \xi_t)} \right\}^\alpha E_t^{1-\alpha}
\]

(5)

Perfectly Flexible Prices: The firm maximizes

\[
P_{i,t} Y_{i,t} - W_{i,t} L_{i,t} - E_t M_t
\]

(6)

subject to (4), and perfect information about the cost structure (5) and the demand (2). This results in the following equation for the frictionless optimal price:

\[
\frac{P_{i,t}^*}{P_t} = \mu \psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t})
\]

(6)
A first-order log-linearization of equation (6) around the steady-state equilibrium with flexible prices leads to

\[
\log P_{i,t}^* = \kappa + \zeta \log y_t + (1 + \zeta) \log P_t + \frac{(1 + \alpha)}{1 + \alpha \omega \theta} \log E_t + a_{i,t}
\]

(7)

where \( a_{i,t} = \frac{1}{1 + \alpha \omega \theta} \frac{\partial \log \psi_{i,t}}{\partial \xi_t} \xi_t - \frac{1}{1 + \alpha \omega \theta} \log A_{i,t} \)

- Strategic complementarity: The lower the value of \( \zeta = \frac{\alpha(\omega + \sigma^{-1})}{1 + \alpha \omega \theta} \)

We will write equation (7) as (variables in logs):

\[
p_{i,t}^* = \kappa + \zeta y_t + (1 + \zeta) p_t + \frac{(1 + \alpha)}{1 + \alpha \omega \theta} e_t + a_{i,t}
\]

(8)

\[
= \kappa + x'_t \beta + a_{i,t}
\]

We also assume that \( a_{i,t} = \eta + a_{i,t-1} + \epsilon_{i,t} \), \( \epsilon_{i,t} \sim N(0, \sigma^2) \)
The firm follows a (s,S)-type rule:

Graph: Adjustment rule in a S,s model

\[ p^*(t) - p(\tau) \]
The Econometric Model

Define the latent variable \( y_{i,t}^* = p_{i,t}^* - p_{i,\tau} \) and \( y_{i,t} \) as:

\[
y_{i,t} = \begin{cases} 
1, & \text{if } p_{i,t} > p_{i,t-1} \\
0, & \text{if } p_{i,t} = p_{i,t-1} \\
-1, & \text{if } p_{i,t} < p_{i,t-1}
\end{cases}
\]

By the pricing rule, at the moment of price change: \( p_{i,\tau}^* - p_{i,\tau} = c \).

Then,

\[
y_{i,t}^* \equiv p_{i,t}^* - p_{i,\tau} = (x_i'\beta + a_{i,t}) - (x_{\tau}\beta + a_{i,\tau} - c)
\]

\[
= (x_t - x_{\tau})'\beta + c + (a_{i,t} - a_{i,\tau})
\]

\[
= z_{i,t}'\beta + c + (a_{i,t} - a_{i,\tau})
\]

But, \( a_{i,t} - a_{i,\tau} = \eta \delta_{i,t} + u_{i,t} \), where \( u_{i,t} \) follows a MA(\( \delta_{i,t} - 1 \)) process. \( \delta_{i,t} \) is the difference between \( t \) and \( \tau \). Then,

\[
y_{i,t}^* \equiv p_{i,t}^* - p_{i,\tau} = z_{i,t}'\beta + c + (a_{i,t} - a_{i,\tau})
\]

\[
= \eta \delta_{i,t} + z_{i,t}'\beta + c + u_{i,t}
\]

and \( u_{i,t} \sim N(0, \delta_{i,t} \sigma^2) \)
Then, defining \( w_{i,t} = (\delta_{i,t}, z_{i,t}) \), we can derive the probability of observing a price increase:

\[
\Pr\left[ y_{i,t} = 1 \mid w_{i,t} \right] = \Pr\left[ y_{i,t}^* \geq S \mid w_{i,t} \right] = \Pr\left[ \eta \delta_{i,t} + z_{i,t}' \beta + c + u_{i,t} \geq S \mid w_{i,t} \right] = \Pr\left[ \frac{u_{i,t}}{\sqrt{\delta_{i,t} \sigma}} \geq \frac{S - c - \eta \delta_{i,t} - z_{i,t}' \beta}{\sqrt{\delta_{i,t} \sigma}} \right] = 1 - \Phi \left( \frac{S - c}{\sigma} \frac{1}{\sqrt{\delta_{i,t}}} - \eta \sqrt{\delta_{i,t}} - \frac{z_{i,t}'}{\sqrt{\delta_{i,t} \sigma}} \frac{\beta}{\sigma} \right) = 1 - \Phi \left( \pi_1 \tilde{\delta}_{i,t} - \gamma \tilde{\delta}_{i,t} - \tilde{z}_{i,t}' \theta \right)
\]
We have the following Ordered Probit Model:

- **Probability of Price Increase:**

\[
\Pr \left[ y_{i,t} = 1 | w_{i,t} \right] = 1 - \Phi \left( \frac{S - c}{\sigma} - \eta \sqrt{\delta_{i,t}} - \frac{z'_{i,t}}{\sqrt{\delta_{i,t}}} \frac{\beta}{\sigma} \right) \\
= 1 - \Phi \left( \pi_1 \tilde{I}_{i,t} - \gamma \tilde{\delta}_{i,t} - \tilde{z}'_{i,t} \theta \right)
\]

- **Probability of Maintaining Price:**

\[
\Pr \left[ y_{i,t} = 0 | w_{i,t} \right] = \Phi \left( \pi_1 \tilde{I}_{i,t} - \gamma \tilde{\delta}_{i,t} - \tilde{z}'_{i,t} \theta \right) - \Phi \left( \pi_0 \tilde{I}_{i,t} - \gamma \tilde{\delta}_{i,t} - \tilde{z}'_{i,t} \theta \right)
\]

- **Probability of Price Decrease:**

\[
\Pr \left[ y_{i,t} = -1 | w_{i,t} \right] = \Phi \left( \frac{s - c}{\sigma} - \eta \sqrt{\delta_{i,t}} - \frac{\tilde{z}'_{i,t}}{\sqrt{\delta_{i,t}}} \frac{\beta}{\sigma} \right) \\
= \Phi \left( \pi_0 \tilde{I}_{i,t} - \gamma \tilde{\delta}_{i,t} - \tilde{z}'_{i,t} \theta \right)
\]
Estimation and Identification

- **Estimation:** By Quasi-Maximum Likelihood and robust variance-covariance matrix for heteroskedasticity and autocorrelation.

- **Identification:**

  Observe that:
  \[
  \theta_1 = \frac{\zeta}{\sigma} \quad \text{and} \quad \theta_2 = \frac{1 - \zeta}{\sigma}
  \]

  Then,
  \[
  \frac{\theta_1}{\theta_2} = \frac{\zeta}{1 - \zeta} \quad \Rightarrow \quad \zeta = \frac{\theta_1}{\theta_1 + \theta_2}
  \]

  Additionally, we have:
  \[
  \theta_1 = \frac{\zeta}{\sigma} \quad \Rightarrow \quad \sigma = \frac{1}{\theta_1 + \theta_2}
  \]

Primary information of price quotes collected and used by IBRE-FGV to compute the CPI-FGV. Collected in 12 metropolitan regions.

The CPI-FGV comprises 456 products and services grouped in 7 sectors. Approximately 2500 outlets.

Typical item: black beans of type 1, of the brand Combrasil, in a package of 1kg, which is sold in the outlet number 16,352 in Belém.

Our sample: A very representative sample of the overall CPI-FGV (around 85%), from 1996 to 2006.
## Dataset: Some Information

<table>
<thead>
<tr>
<th>Sector</th>
<th>Original Dataset</th>
<th>Treated Dataset</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># of observations</td>
<td># of trajectories</td>
</tr>
<tr>
<td>Food</td>
<td>3,973,527</td>
<td>36,400</td>
</tr>
<tr>
<td>Other Goods and Services</td>
<td>411,560</td>
<td>8,006</td>
</tr>
<tr>
<td>Education and Recreation</td>
<td>346,095</td>
<td>13,613</td>
</tr>
<tr>
<td>Housing</td>
<td>961,755</td>
<td>21,438</td>
</tr>
<tr>
<td>Medical and Personal Care</td>
<td>1,087,647</td>
<td>17,922</td>
</tr>
<tr>
<td>Transportation</td>
<td>149,185</td>
<td>5,026</td>
</tr>
<tr>
<td>Apparel</td>
<td>501,202</td>
<td>19,919</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>7,430,971</strong></td>
<td><strong>122,324</strong></td>
</tr>
</tbody>
</table>
### Descriptive Statistics of the Price Changes

Information about the distribution of the price changes, conditional on adjustment

| Sector                        | Duration (days) | Mean of $|\Delta p|$ % | % of Reduction | Average reduction % | Average increase % |
|-------------------------------|-----------------|----------|------------------|-------------------|--------------------|--------------------|
| All sectors                   | 59.27           | 15.99    | 0.44             | -16.67           | 16.67              | 15.45              |
| Food                          | 50.53           | 16.27    | 0.45             | -16.70           | 16.70              | 15.92              |
| Apparel                       | 53.89           | 25.32    | 0.48             | -25.46           | 25.46              | 25.19              |
| Housing                       | 61.59           | 13.27    | 0.42             | -14.04           | 14.04              | 12.72              |
| Other Goods and Services      | 63.42           | 10.75    | 0.43             | -11.05           | 11.05              | 10.53              |
| Transportation                | 64.82           | 7.58     | 0.39             | -7.13            | 7.13               | 7.86               |
| Medical and Personal Care     | 75.90           | 12.35    | 0.41             | -13.37           | 13.37              | 11.67              |
| Education and Recreation      | 134.61          | 15.82    | 0.36             | -17.66           | 17.66              | 14.78              |

<table>
<thead>
<tr>
<th>Sector</th>
<th>mean of $\Delta p$ %</th>
<th>Std deviation</th>
<th>Kurtosis</th>
<th>% of small $\Delta p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sectors</td>
<td>1.31</td>
<td>19.38</td>
<td>4.15</td>
<td>37.94</td>
</tr>
<tr>
<td>Food</td>
<td>1.15</td>
<td>19.41</td>
<td>3.81</td>
<td>37.27</td>
</tr>
<tr>
<td>Apparel</td>
<td>1.15</td>
<td>30.08</td>
<td>3.45</td>
<td>33.57</td>
</tr>
<tr>
<td>Housing</td>
<td>1.60</td>
<td>15.80</td>
<td>3.89</td>
<td>38.63</td>
</tr>
<tr>
<td>Other Goods and Services</td>
<td>1.31</td>
<td>12.48</td>
<td>3.16</td>
<td>33.69</td>
</tr>
<tr>
<td>Transportation</td>
<td>2.06</td>
<td>8.93</td>
<td>4.34</td>
<td>40.59</td>
</tr>
<tr>
<td>Medical and Personal Care</td>
<td>1.53</td>
<td>14.84</td>
<td>3.80</td>
<td>39.30</td>
</tr>
<tr>
<td>Education and Recreation</td>
<td>2.95</td>
<td>18.70</td>
<td>4.60</td>
<td>32.62</td>
</tr>
</tbody>
</table>

Note: 1) $p$ is defined as the natural logarithm of the item price
2) All statistics are calculated based on unweighted price changes. Kurtosis is calculated excluding the top and bottom 1% of observations
3) Small $\Delta p$ is defined as any price change whose absolute value is lower than 0.5 of the mean of $|\Delta p|$
Some Stylized Facts

- **Fact 1:** Prices change frequently, but the degree of price rigidity is quite different among the sectors.

- **Fact 2:** On average price changes (in absolute values) are large.

- **Fact 3:** Price decreases are frequent events.

- **Fact 4:** On average price decreases are larger than price increases.

- **Fact 5:** A large percentage of price changes is of small changes.
Models Fit

Probit Models:

Dependent variable: \( y_t \)

Explanatory variables: \( w_{i,t} = \left( \frac{1}{\sqrt{\delta_{i,t}}}, \sqrt{\delta_{i,t}}, \frac{y_i - y_{i,t}}{\sqrt{\delta_{i,t}}}, \frac{p_i - p_{i,t}}{\sqrt{\delta_{i,t}}}, \frac{e_i - e_{i,t}}{\sqrt{\delta_{i,t}}} \right) \)

| Sector                        | \( \text{Pr}[y_i = -1 \text{ or } y_i = 1|\text{mean}(w_i)] \) | Implied Duration (in days) | Duration (in days) |
|-------------------------------|---------------------------------------------------------------|----------------------------|--------------------|
| All sectors                   | 0.56                                                          | 53.24                      | 59.27              |
| Food                          | 0.63                                                          | 47.35                      | 50.53              |
| Apparel                       | 0.60                                                          | 50.03                      | 53.89              |
| Housing                       | 0.54                                                          | 55.41                      | 61.59              |
| Other Goods and Services      | 0.53                                                          | 56.82                      | 63.42              |
| Transportation                | 0.52                                                          | 57.34                      | 64.82              |
| Medical and Personal Care     | 0.48                                                          | 62.35                      | 75.90              |
| Education and Recreation      | 0.25                                                          | 118.81                     | 134.61             |
## Estimation Results: Strategic Complementarity

### Estimated Parameters of Probit Models and Strategic Complementarity

<table>
<thead>
<tr>
<th>Sector</th>
<th>$\gamma_t$</th>
<th>$p_t$</th>
<th>$e_t$</th>
<th>$\zeta$</th>
<th>Conf. interval for $\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sectors</td>
<td>0.92</td>
<td>9.15</td>
<td>-0.09</td>
<td>0.09</td>
<td>0.07 - 0.11</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.011)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>0.20</td>
<td>8.79</td>
<td>0.07</td>
<td>0.02</td>
<td>0.01 - 0.03</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.020)</td>
<td>(0.004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apparel</td>
<td>1.09</td>
<td>4.91</td>
<td>-0.17</td>
<td>0.18</td>
<td>0.15 - 0.22</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.071)</td>
<td>(0.012)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>1.36</td>
<td>11.02</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10 - 0.12</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.106)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Goods and Services</td>
<td>1.43</td>
<td>10.45</td>
<td>-0.36</td>
<td>0.12</td>
<td>0.09 - 0.15</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td>(0.358)</td>
<td>(0.027)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td>1.36</td>
<td>14.29</td>
<td>0.42</td>
<td>0.09</td>
<td>0.05 - 0.12</td>
</tr>
<tr>
<td></td>
<td>(0.141)</td>
<td>(0.311)</td>
<td>(0.025)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medical and Personal Care</td>
<td>1.24</td>
<td>10.06</td>
<td>-0.70</td>
<td>0.11</td>
<td>0.10 - 0.12</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.118)</td>
<td>(0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education and Recreation</td>
<td>6.09</td>
<td>11.81</td>
<td>0.32</td>
<td>0.34</td>
<td>0.32 - 0.36</td>
</tr>
<tr>
<td></td>
<td>(0.122)</td>
<td>(0.382)</td>
<td>(0.024)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
The standard deviation is in parenthesis
The confidence interval is 95% of confidence
The standard deviation of $\zeta$ was obtained by the Delta method
## Estimation Results: Pricing Rule

Estimated parameters related to the optimal pricing rule

<table>
<thead>
<tr>
<th>Sector</th>
<th>$\pi_0=(s-c)/\sigma$</th>
<th>$\pi_1=(S-c)/\sigma$</th>
<th>$\sigma$</th>
<th>$s-c$</th>
<th>$S-c$</th>
<th>$S-s$</th>
<th>$(S-s)/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All sectors</td>
<td>-0.93*</td>
<td>0.63*</td>
<td>0.10</td>
<td>-0.09</td>
<td>0.06</td>
<td>0.15</td>
<td>1.55</td>
</tr>
<tr>
<td>Food</td>
<td>-0.70*</td>
<td>0.48*</td>
<td>0.11</td>
<td>-0.08</td>
<td>0.05</td>
<td>0.13</td>
<td>1.18</td>
</tr>
<tr>
<td>Apparel</td>
<td>-0.72*</td>
<td>0.62*</td>
<td>0.17</td>
<td>-0.12</td>
<td>0.10</td>
<td>0.22</td>
<td>1.34</td>
</tr>
<tr>
<td>Housing</td>
<td>-1.05*</td>
<td>0.61*</td>
<td>0.08</td>
<td>-0.08</td>
<td>0.05</td>
<td>0.13</td>
<td>1.65</td>
</tr>
<tr>
<td>Other Goods and Services</td>
<td>-1.06*</td>
<td>0.70*</td>
<td>0.08</td>
<td>-0.09</td>
<td>0.06</td>
<td>0.15</td>
<td>1.76</td>
</tr>
<tr>
<td>Transportation</td>
<td>-1.21*</td>
<td>0.60*</td>
<td>0.06</td>
<td>-0.08</td>
<td>0.04</td>
<td>0.12</td>
<td>1.81</td>
</tr>
<tr>
<td>Medical and Personal Care</td>
<td>-1.30*</td>
<td>0.81*</td>
<td>0.09</td>
<td>-0.11</td>
<td>0.07</td>
<td>0.19</td>
<td>2.10</td>
</tr>
<tr>
<td>Education and Recreation</td>
<td>-2.12*</td>
<td>1.45*</td>
<td>0.06</td>
<td>-0.12</td>
<td>0.08</td>
<td>0.20</td>
<td>3.58</td>
</tr>
</tbody>
</table>

Note: Asterisk means significant at 1% of significance.
We propose a method to directly estimate strategic complementarity in pricing decisions. The methodology uses a microfounded model to derive a structural, non-standard ordered probit model.

The results indicate that the parameter $\zeta$ is about 0.1, implying a substantial degree of strategic complementarity, in line with assumptions assumed in Gertler and Leahy (2008), for example.

Differently from Bils, Klenow and Malin (2009), we did not find that a state-dependent pricing model with strategic complementarity is fundamentally at odds with the data.

In addition, the methodology allows us to estimate some parameters related to the pricing rules. In general, the results seem to explain the stylized facts.
Further Research

- Do these results of substantial strategic complementarity come from the fact that we are using a state-dependent pricing model?
  - Development of similar methodology for estimating a time-dependent model and the degree of strategic complementarity.

- We would like to separate each parameter of the pricing rule: S, s and c.