Aggregation and the PPP Puzzle in a Sticky Price Model

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*The views expressed in this presentation are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.
Real exchange rates and PPP

- Real exchange rate (RER): ratio of prices of basket of goods denominated in common currency
  \[ Q = \frac{E^P}{P} \]

- Purchasing power parity (PPP) hypothesis: “\( Q \) is constant”
  - \( Q = 1 \): “absolute PPP”
  - \( Q = \text{constant} \neq 1 \): “relative PPP”
The puzzle: data + model failure

- “PPP Puzzle” - Rogoff (1996)
  - High persistence and volatility of real exchange rates (deviations from PPP)
  - Hard to reconcile with economic models
  - In particular, when deviations arise from nominal shocks in the presence of price rigidity

- Quantitative framing of the puzzle:
  - Most estimates of half-life of deviations from PPP fall between 3 - 5 years
  - Standard sticky-price models with monetary shocks: half-life around 1 year

- “Too much” price rigidity needed to match persistence of the data
Our paper

- Build on ample evidence of heterogeneity in frequency of price adjustment (Bils and Klenow 2004 and others)
  - Average degree of price rigidity not the relevant statistic for aggregate dynamics (Carvalho 2006)

- Potential source of heterogeneity in dynamics of sectoral real exchange rates

- Multi-sector, two-country, sticky-price model:
  - Heterogeneity in frequency of price changes
  - Price discrimination and local currency pricing

⇒ Heterogeneous sectoral real exchange rates
Summary of the model - K sectors

Home

Consumers

Final Goods

Intermediate Goods

Intermediate Goods

Final Goods

Consumers

Foreign

Final Goods*

Intermediate Goods*

Intermediate Goods*

Final Goods*

Consumers*

\[ Y_{H,k,j} \]

\[ Y_{F,k,j} \]

\[ Y_{H,k,j} \]

\[ Y_{F,k,j} \]

\[ Y_{H,k,j} \]

\[ Y_{F,k,j} \]

\[ Y_{H,k,j} \]

\[ Y_{F,k,j} \]
Summary of the model - 1 sector

<table>
<thead>
<tr>
<th>Home</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Goods</td>
<td>Final Goods*</td>
</tr>
<tr>
<td>Consumers</td>
<td>Consumers*</td>
</tr>
<tr>
<td>Intermediate Goods</td>
<td>Intermediate Goods*</td>
</tr>
</tbody>
</table>

Symbols:
- C: Consumption
- L: Labor
- Y: Output
- Y_H,j: Output of home sector to intermediate goods
- Y_F,j: Output of foreign sector to intermediate goods
- Y*_H,j: Output of home sector to final goods
- Y*_F,j: Output of foreign sector to final goods

Note: The diagram illustrates the flow of goods and labor between the home and foreign sectors.
Summary of core quantitative results

- Quantitative model
  - Discipline our argument: distribution of price stickiness chosen to match micro data
  - Average frequency of price changes implies average spells of 4.4 months
  - Half-life of deviations from PPP in *heterogeneous economy*: 3.8 years
  - Half-life of deviations from PPP in *homogeneous economy*: 1.2 years
Core results: intuition I

- Sectors with more stickiness are disproportionately important for aggregate dynamics.

- Extreme case: 1 flexible- and 1 sticky-price sector, no pricing complementarities.
  - Aggregate RER dynamics driven only by sticky-price sector.

- Mathematically: measures of persistence (and volatility) are convex in the frequency of price changes.
Core results: intuition II

- Effects even **stronger** in the presence of **pricing complementarities**!

- Not-so-extreme version of “responders vs non-responders” of Haltiwanger and Waldman (1991)

- Bottom line in terms of persistence:
  - heterogeneous economy $\approx$ homogeneous economy with more price stickiness than what’s implied by average frequency of price changes
The model

- Two countries, Home and Foreign, with identical, infinitely lived consumers

- Commodities are labor, a consumption good and a continuum of intermediate goods

- Consumer supplies labor, invests in complete set of state-contingent assets, and consumes non-traded final good

- Competitive final good producers bundle intermediate goods; flexible prices

- Monopolistically competitive intermediate goods producers:
  - Price discriminate, setting prices in local currency
  - Sticky prices; adjustment frequency varies across sectors
The model - final good I

\[
\text{max } P_t Y_t - \left( \sum_{k=1}^{K} f_k \int_0^1 P_{H,k,j,t} Y_{H,k,j,t} \, dj + \sum_{k=1}^{K} f_k \int_0^1 P_{F,k,j,t} Y_{F,k,j,t} \, dj \right)
\]

s.t.

\[
Y_t = \left( \sum_{k=1}^{K} f_k^\eta Y_{k,t}^\eta \right)^{\frac{\eta}{\eta-1}}
\]

\[
Y_{k,t} = \left( \frac{1}{\omega^\rho Y_{H,k,t}^\rho} + (1 - \omega) \frac{1}{\omega^\rho Y_{F,k,t}^\rho} \right)^{\frac{\rho}{\rho-1}}
\]

\[
Y_{H,k,t} = \left( f_k^{\frac{\theta-1}{\theta}} \int_0^1 Y_{H,k,j,t}^\frac{\theta-1}{\theta} \, dj \right)^{\frac{\theta}{\theta-1}}
\]

\[
Y_{F,k,t} = \left( f_k^{\frac{\theta-1}{\theta}} \int_0^1 Y_{F,k,j,t}^\frac{\theta-1}{\theta} \, dj \right)^{\frac{\theta}{\theta-1}}
\]
The model - final good II

- Demands

\[ Y_{H,k,j,t} = \omega \left( \frac{P_{H,k,j,t}}{P_{H,k,t}} \right)^{-\theta} \left( \frac{P_{H,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t \]

\[ Y_{F,k,j,t} = (1 - \omega) \left( \frac{P_{F,k,j,t}}{P_{F,k,t}} \right)^{-\theta} \left( \frac{P_{F,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t \]

- Price Indices:

\[ P_t = \left( \sum_{k=1}^{K} f_k P_{k,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \]

\[ P_{k,t} = \left( \omega P_{H,k,t}^{1-\rho} + (1 - \omega) P_{F,k,t}^{1-\rho} \right)^{\frac{1}{1-\rho}} \]

\[ P_{H,k,t} = \left( \int_0^1 P_{H,k,j,t}^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} \]

\[ P_{F,k,t} = \left( \int_0^1 P_{F,k,j,t}^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}} \]
The model - intermediate firms I

- Intermediate firms:
  - divided into sectors that differ in the frequency of price changes
  - Calvo (1983) pricing: $\alpha_k =$ frequency of price changes in sector $k$
  - indexed by country, sector ($k \in K$), and by $j \in [0, 1]$
  - sectoral weights $f_k$
  - produce differentiated varieties using labor
Choose prices $X_{H,k,j,t}$, $X^*_{H,k,j,t}$, set in local currencies:

$$\max \ E_t \sum_{s=t}^{\infty} \Theta_{t,s} (1 - \alpha_k)^{s-t} \left[ X_{H,k,j,t} Y_{H,k,j,s} + \mathcal{E}_s X^*_{H,k,j,t} Y^*_{H,k,j,s} - W_s N_{k,j,s} \right]$$

s.t. $Y_{H,k,j,t} + Y^*_{H,k,j,t} = N^\chi_{k,j,t}$

and demands

Analogous for Foreign
The model - aggregate and sectoral RERs

- Aggregate:

\[ Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t} \]

- Sectoral:

\[ Q_{k,t} \equiv \frac{\mathcal{E}_t P_{k,t}^*}{P_{k,t}} \]
Closing the model

- Specifications for monetary policy that ensure existence and uniqueness of RE equilibrium

- Equilibrium: optimality and market clearing conditions

- Solution: loglinearization around zero inflation steady-state
Counterfactual homogeneous (one-sector) economy:

- Everything the same, except:

- One sector with frequency of price changes $\bar{\alpha} = \sum_{k=1}^{K} f_k \alpha_k$
Parametrization

• 36-sector model:
  
  - $\alpha_k = 1/k$, $k = 1, \ldots, 36$
  
  - Micro evidence from Nakamura-Steinsson (2008):
    
    • CPI data - no sales/substitutions
    
    • $f_k = \text{sum of expenditure weights for underlying categories}$
    
    • $36^{th}$ sector: $f_{36} = 4.2\%$
    
    • $\bar{\alpha} = 0.226 \Rightarrow \text{Prices changes, on average, every 4.4 months}$
Quantitative results

<table>
<thead>
<tr>
<th>Persistence measures:</th>
<th>$\mathcal{P}(q)$</th>
<th>$\mathcal{P}(q^{1\text{sec}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{CIR}</td>
<td>79.9</td>
<td>20.2</td>
</tr>
<tr>
<td>\textit{SAC}</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>\textit{LAR}</td>
<td>0.94</td>
<td>0.86</td>
</tr>
<tr>
<td>\textit{HL}</td>
<td>45</td>
<td>14</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>.98</td>
<td>.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatility measure:</th>
<th>$\nu(q)^{1/2}$</th>
<th>$\nu(q^{1\text{sec}})^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Using our structural model - extras

- We provide structural interpretation to the (reduced-form) empirical literature on heterogeneity, aggregation and PPP
  - “PPP Strikes Back/Aggregation Bias” debate between Imbs et al. (2005), and Chen and Engel (2005) / Crucini and Shintani (2008)
- Study persistence in the cross-section of sectoral exchange rates; touch base with Kehoe and Midrigan (2008) (this is work in progress)
Decomposition of effect of heterogeneity

\[
\mathcal{P}(q_t) - \mathcal{P}(q_t^{1\text{sec}}) \equiv \left[ \mathcal{P}(q_t) - \sum_{k=1}^{K} f_k \mathcal{P}(q_k) \right] + \left[ \sum_{k=1}^{K} f_k \mathcal{P}(q_k) - \mathcal{P}(q_t^{1\text{sec}}) \right]
\]

- **total heterogeneity effect**
- **aggregation effect**
- **counterfactual effect**
### Applying decomposition

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Effects</td>
<td>$\mathcal{P}(q)$</td>
<td>$\frac{1}{K} \sum \mathcal{P}(q_k)$</td>
<td>$\mathcal{P}(q^{1_{\text{sec}}})$</td>
</tr>
</tbody>
</table>

**Eurostat data**

<table>
<thead>
<tr>
<th>Persist. meas.</th>
<th>CIR</th>
<th>SAC</th>
<th>LAR</th>
<th>HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>64.39</td>
<td>0.98</td>
<td>0.97</td>
<td>46</td>
</tr>
<tr>
<td>Value</td>
<td>59.48</td>
<td>0.97</td>
<td>0.94</td>
<td>43.16</td>
</tr>
<tr>
<td>Value</td>
<td>33.19</td>
<td>0.97</td>
<td>0.95</td>
<td>26</td>
</tr>
</tbody>
</table>

**Simulated data**

<table>
<thead>
<tr>
<th>Economy/Estim. method</th>
<th>Heterog.</th>
<th>Heterog.</th>
<th>One-sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>$\mathcal{P}(q)$</td>
<td>$\frac{1}{K} \sum \mathcal{P}(q_k)$</td>
<td>$\mathcal{P}(q^{1_{\text{sec}}})$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>CIR</th>
<th>SAC</th>
<th>LAR</th>
<th>HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>73.4</td>
<td>0.98</td>
<td>0.96</td>
<td>43.1</td>
</tr>
<tr>
<td>Value</td>
<td>60.8</td>
<td>0.97</td>
<td>0.89</td>
<td>31.6</td>
</tr>
<tr>
<td>Value</td>
<td>36.9</td>
<td>0.97</td>
<td>0.87</td>
<td>21.8</td>
</tr>
</tbody>
</table>
Multi-sector, two-country, sticky-price model can produce RER that is as persistent as in the data.

One-sector version of same economy with average frequency of price changes fails to do so.

We use our model to explain the apparently conflicting findings in reduced-form empirical literature.

- Different papers measured different objects.
- As Chen and Engel (2005) and Crucini and Shintani (2008), we find aggregation effect to be small (in the parametrized model and in the data).
- As Imbs et al. (2005), we find total heterogeneity effect to be large (in the parametrized model and in the data). This is due to the counterfactual effect.
What matters for understanding the gap between data and standard one-sector models is total heterogeneity effect.

Heterogeneity in price stickiness goes a long way towards solving the PPP puzzle.

Such type of heterogeneity known to matter for policy:

- In open economies: ?