Expectations, Learning and Business Cycle Fluctuations

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The views expressed are those of the authors and are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System
Motivation

• Expectations as a source of business cycle fluctuation

• Such theories rely on
  – Multiple equilibria
  – Imperfect information
Pigou (1926):

"[...] a rise in prices, however brought about, by creating some actual and some counterfeit prosperity for business man, is liable to promote an error of optimism, and a fall in prices an error of pessimism, and this mutual stimulation of errors and price movements may continue in a vicious spiral until it is checked by some interference from outside."
What we do

- Stochastic growth model

- Propose model of Pigou Cycles. Agents have:
  - Completely specified belief system
  - Incomplete economic model
    - Minimal departure from neoclassical theory

- Consider implied time series properties
Findings

- Relative to rational expectations analysis
  - 20 percent smaller exogenous productivity shocks to match US post-war output variability
  - Higher relative volatility of investment and hours; captures persistence
Model

- Households
- Firms
- Competitive markets
- Near-rational expectations
Beliefs

Under rational expectations:

1. Agents optimize given beliefs

2. The probabilities they assign to future state variables coincide with the predictions of the model

This paper retains (1) and replaces (2) with

2'. Future state variables outside agent's control are forecasted using an econometric model.
Model Summary 1

- Market clearing, aggregating and log-linearizing around balanced growth path provides consumption decision rule

\[
\hat{c}_t = \frac{1 - \beta}{\epsilon_c} \left[ \beta^{-1} \left( \hat{k}_t - \hat{\gamma}_t \right) + \bar{R} \hat{R}^K_t + \epsilon_w \hat{w}_t \right] \\
+ \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ \frac{(1 - \beta)}{\epsilon_c} - \beta \right] \beta \bar{R} \hat{R}^K_{T+1} + \\
\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{(1 - \beta)}{\epsilon_c} \beta \epsilon_w \hat{w}_{T+1}
\]

where \( \epsilon_w, \epsilon_c \) composites of model primitives, \( \bar{R} > 0 \)
Model Summary II

- Log-linear approximation around balanced growth path implies

$$\hat{k}_{t+1} = \left[ \alpha \frac{\bar{y}}{\bar{k}} + \frac{(1 - \delta)}{\bar{\gamma}} \right] \hat{k}_t + (1 - \alpha) \frac{\bar{y}}{\bar{k}} \hat{H}_t - \frac{\bar{c}}{\bar{k}} \hat{c}_t - \frac{(1 - \delta)}{\bar{\gamma}} \hat{\gamma}_t.$$  
  \rightarrow \text{Evolution of capital stock}

$$\psi \hat{H}_t = -\hat{c}_t + \hat{w}_t$$  
  \rightarrow \text{Labor-leisure allocation}

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}} \hat{c}_t + \frac{\bar{i}}{\bar{y}} \hat{i}_t$$  
  \rightarrow \text{Resource constraint}

$$\hat{r}_t^K = (1 - \alpha) \left( \hat{y}_t - \hat{k}_t \right)$$

$$\hat{w}_t = \alpha \left( \hat{y}_t - \hat{H}_t \right)$$  
  \rightarrow \text{Wage and rental rate}
Beliefs I

- Estimate statistical model

\[ z_t = \omega_{0,t} + \omega_{1,t}\hat{k}_t + e_{z,t} \]

where \( z_t = \{\hat{w}_t, \hat{r}_t^K, \hat{k}_{t+1}\} \).

- Assumption: \( \hat{\gamma}_t \) not utilized in forecasts

- Recall: imperfect common knowledge models
  
  * Lorenzoni (2008)
Beliefs II

- Update their beliefs according to

\[ \hat{\omega}_t = \hat{\omega}_{t-1} + g R_t^{-1} q_{t-1} \left( z_t - \hat{\omega}'_{t-1} q_{t-1} \right)' \]

\[ R_t = R_{t-1} + g \left( q_{t-1} q'_{t-1} - R_{t-1} \right) \]

where \( \omega' = (\omega_0, \omega_1) \), \( z_t = (\hat{R}_t^K, \hat{\omega}_t, \hat{k}_{t+1}) \) and \( q_{t-1} = (1, \hat{k}_t) \)

- Actual dynamics depend on beliefs

\[ z_t = T_1 (\hat{\omega}_{t-1}) q_{t-1} + T_2 (\hat{\omega}_{t-1}) \hat{\gamma}_t \]

  - Rational expectations equilibrium: \( \omega^* = T_1 (\omega^*) \)

  - \( \hat{\omega} \) converge to an *invariant* distribution around \( \hat{\omega}^* \)
Model Calibration I

- Sample 1955Q1-2004QIV

- Choose technology shock and gain to match properties of output data:

  \[ \tilde{\gamma} = 1.0045 \]
  \[ \sigma_A = 0.98 \]
  \[ g = 0.0029 \]

  - Literature suggests values \( g \in (0.005, 0.05) \)
  
  - For \( g = 0.0029 \), weight on observation \( T = 200: 0.5 \)
Model Calibration II

- Remaining parameters:

\[ \alpha = 0.34 \]
\[ \beta = 0.99 \]
\[ \delta = 0.025 \]
\[ \sigma = 1 \]
\[ \psi = 0.0025 \]
\[ \bar{H} = 0.2 \]
Table 1: HP filtered moments

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>REE</th>
<th>Learning</th>
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<tr>
<td>Technology: $\sigma_A$</td>
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<td>1.22</td>
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<td>Output: $\sigma_Y$</td>
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<td>1.54</td>
<td>1.52</td>
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<td>Consumption: $\sigma_C/\sigma_Y$</td>
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<td>Investment: $\sigma_I/\sigma_Y$</td>
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<tr>
<td>Hours: $\sigma_H/\sigma_Y$</td>
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<td>0.71</td>
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<tr>
<td>Labor Prod: $\sigma_{Pr}/\sigma_Y$</td>
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<td>$\rho_{Y,C}$</td>
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<td>0.52</td>
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Table 3: Autocorrelation in growth rates

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<tr>
<td>Labor Prod.</td>
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<td>Hours</td>
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Impulse Response Functions

- Impulse to one percent technology shock state dependent
- Simulate model 2100 periods
- Given initial condition at 2001 consider one percent shock
  - Difference gives impulse response function
- Repeat 5000 times
Optimal Decisions

\[
\hat{c}_t = \frac{1 - \beta}{\epsilon_c} \left[ \beta^{-1} (\hat{k}_t - \hat{\gamma}_t) + \bar{R}\hat{R}^K_t + \epsilon_w \hat{w}_t \right] \\
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\hat{E}_t \sum_{T=t}^\infty \beta^{T-t} \frac{(1 - \beta)}{\epsilon_c} \beta \epsilon_w \hat{w}_{T+1}
\]
Figure 1: Dotted lines denote the 75% bands, solid line denotes the median impulse response under learning. The dashed line denotes the impulse response under rational expectations.
**Intuition**

- **Project**
  - Flatter profile of wages
  - Steeper profile for returns to capital

- **Consumption lower: high marginal value of income**
  - Strong labor supply effect

- **Amplifies standard substitution effects: volatility in** $Y, I, H$
Further Insights: Beliefs

• Consider econometrician estimating belief parameters under two scenarios

• True data generating process is generated by:
  – the rational expectations equilibrium
  – agents learning using optimal decisions
Figure 2: Kernel estimates of belief distributions. Dashed line denotes the no feedback case. The solid line denotes the case of feedback.
Robustness

- Earlier literature based economic decisions on Euler equation used to characterize rational expectations equilibrium

- In the context of our model, consumption determined by

\[ c_t = \hat{E}_t c_{t+1} - \hat{E}_t \left( \beta \hat{R}_t R_{t+1}^K + \hat{\gamma}_{t+1} \right) \]

  - Conclude learning does not provide amplification and propagation mechanism
Robustness II

- Consider sensitivity to

  - Choice of Frisch elasticity: \( \psi = \epsilon_w \bar{H} \left( 1 - \bar{H} \right)^{-1} = 0.2 \)

  - Choice of gain: \( g = 0.009 \)
## Table 5: Robustness

<table>
<thead>
<tr>
<th>Statistic</th>
<th>σ_Y</th>
<th>σ_C/σ_Y</th>
<th>σ_I/σ_Y</th>
<th>σ_H/σ_Y</th>
<th>ρ_Y,C</th>
<th>Δ_C</th>
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Conclusions

- Learning important as a amplification and propagation mechanism
  - single friction

- Learning acts as Endogenous news/demand shocks