Samba: Stochastic Analytical Model with a Bayesian Approach

DSGE Model Project for Brazil’s economy

Working in Progress - Preliminary results

DSGE team: Solange Gouvea, André Minella, Rafael Santos, Nelson Souza-Sobrinho, and Tomiê Sugahara

Banco Central do Brasil – Research Department
X Seminar on Inflation Targeting
August 4th, 2008
Outline

- Introduction
- Model
- Estimation results
- Challenges and next steps
Purposes of the project

- Provide the Banco Central do Brasil with a Dynamic Stochastic General Equilibrium (DSGE) model to be used as a tool for:
  - policy analysis
    * framework for policy discussions; qualitative and quantitative assessment of shock effects, monetary policy decisions and different scenarios, etc.
  - medium-term forecast

- “All models are wrong! Some are useful” George Box

- Models and judgement are complements, not substitutes
Model features

• Microfounded model developed for the inflation targeting period (started in mid-1999)

• Small open economy model

• Aggregate demand \((C + I + G + X - M)\):
  
  – Households \(\rightarrow\) private consumption and investment
  
  – Firms \(\rightarrow\) import demand
  
  – Government \(\rightarrow\) government consumption
  
  – Rest of the world \(\rightarrow\) export demand
Model features

- Supply side (Y)
  - Competitive firms -> assemble differentiated goods supplied by monopolistic competitive firms and sell them in
    * Local markets (domestic consumption and investment goods)
    * Abroad (export goods)
  - Monopolistic competitive firms -> production of differentiated goods
    * Inputs: labor, capital services, and imports
    * Price rigidity (à la Calvo) with forward- and backward-looking behavior (Galí and Gertler, 1999)
Model features

- Government:
  - Monetary policy: Taylor rule
  - Fiscal policy rule

- Rest-of-the-world variables: interest rate, inflation, world imports, and foreign investors’ "risk aversion".
Main loglinear equations

- **Aggregate Demand: Consumption**
  
  - Optimizing households
    \[
    c_t^o = \left( \frac{1}{1 + h} \right) E_t (c_{t+1}^o) + \left( \frac{h}{1 + h} \right) c_{t-1}^o - \frac{1}{\sigma} \left( \frac{1 - h}{1 + h} \right) E_t (r_t - \pi_{t+1}) + \ldots + \frac{1}{\sigma} \left( \frac{1 - h}{1 + h} \right) (1 - \rho_c) z_t^c
    \]
  
  - Rule-of-thumb households
    \[
    c_t^{rot} = w_t^r + n_t^{rot}
    \]
  
  - Aggregate consumption
    \[
    c_t = (1 - \varpi_c) c_t^o + \varpi_c c_t^{rot}
    \]

  $r_t$ - interest rate; $\pi_t$ - inflation; $z_t^c$ - shock to consumption; $w_t^r$ - real wages; $n_t^{rot}$ - employment
• **Aggregate Demand:** Investment:

\[ i_t = \frac{1}{\delta_s (1 + \beta)} q_t^I + \frac{\beta}{1 + \beta} E_t i_{t+1} + \frac{1}{1 + \beta} i_{t-1} + \left( \frac{1 - \rho I^\beta}{1 + \beta} \right) z_t^I \]

Shadow price of capital

\[ q_t^I = E_t \left\{ \beta (1 - \delta) q_{t+1}^I + (1 - \beta(1 - \delta)) \hat{r}_t^k - (r_t - \pi_{t+1}) \right\} \]

• **Aggregate Demand:** Net Exports

  - Exports

\[ x_t = m_t^* + \kappa q_t \]

  - Imports

\[ m_t = y_t - \rho (q_t - m_{ct}) \]

\( \hat{r}_t^k \) - rental rate of capital; \( z_t^I \) - shock to investment; \( m_t^* \) - world imports; \( q_t \) - real exchange rate; \( y_t \) - (gross) output; \( m_{ct} \) - real marginal cost
- **Aggregate Supply**

  - Production function

\[
y_t = f(k_t, u_t, n_t, m_t, a_t)
\]

  - Labor market

    * Labor supply

\[
n_t = (1 - \varpi_n) n_t^o + \varpi_n n_t^{rot}
\]

    * Labor demand

\[
n_t = y_t - [(1 - \varrho) + \varrho s_d] a_t - [\alpha + \varrho (1 + s_d) (1 - \alpha) ] w_t^r +
+ \alpha [1 - \varrho (1 - s_d)] r_t^k + \varrho (1 - s_d) q_t
\]

  \(k_t\) - physical capital; \(u_t\) - rate of capital utilization; \(a_t\) - productivity shock
- Capital services

* Demand

\[ k_t + u_t = y_t - [(1 - \varrho (1 - s_d)] a_t - [(1 - \alpha) + \alpha \varrho (1 - s_d)] \hat{r}_t^k + ... + (1 - \alpha) [(1 - \varrho (1 - s_d)] w_t^r + \varrho (1 - s_d) q_t \]

* Supply

\[ u_t = \frac{1}{\delta_\alpha} \hat{r}_t^k \]

* Law of motion for capital

\[ k_{t+1} = (1 - \delta) k_t + \left( \frac{I}{K} \right) i_t \]
- Phillips curve

\[ \pi_t = \lambda m c_t + \lambda_b \pi_{t-1} + \lambda_f E_t \pi_{t+1} \]

where:

\[ m c_t = s_d \left( \alpha \hat{r}^k_t + (1 - \alpha) w^r_t - a_t \right) + (1 - s_d) q_t \]

\[ (\lambda, \lambda_b, \lambda_f) = f (\theta, \varpi_b, \beta) \]
• Financial variables

- Real exchange rate (UIP)

\[ q_t = E_t q_{t+1} - \left[ (r_t - E_t \pi_{t+1}) - (r_t^* + \phi_t - E_t \pi_t^*_{t+1}) \right] \]

- Country-risk premium

\[ \phi_t = -\psi b^y_{t+1} + \nu z^\phi_t + z^\phi_t \]

\( r_t^* \) - world interest rate; \( \pi_t^* \) - world inflation;
\( z^\phi_t \) - international investors’ risk averstion; \( z^\phi_t \) - shock to country-risk premium
• Government

  – Monetary policy (Taylor rule)

  \[ r_t = \gamma_r r_{t-1} + (1 - \gamma_r) \left[ \gamma_\pi E_t (\pi_{t+1} - \bar{\pi}_{t+1}) + \bar{\pi}_t + \gamma_y y_t^{VA} \right] + z^r_t \]

  – Fiscal policy rule

  \[ g^y_t = \gamma_g g^y_{t-1} + (1 - \gamma_g) \left( \gamma_s \tilde{s}^y_{t-1} - \gamma_b b^y_{t} \right) + z^g_t \]

  \( \bar{\pi}_t \) - inflation target; \( z^r_t \) - shock to monetary policy;

  \( g^y_t \) - government consumption-to-GDP ratio; \( \tilde{s}^y_t \) - primary fiscal surplus target;

  \( z^g_t \) - shock to fiscal policy; \( \tilde{s}^y_{t-1} \) - primary fiscal surplus deviation from the target
• **Shocks and rest-of-the-world variables:**

\[ z_t = \rho z_{t-1} + \varepsilon_t \]

• **Value added (GDP) - Equilibrium:**

\[ y_t^{VA} = s_{cc} c_t + s_{i} i_t + s_{gg} g_t + s_{xx} x_t - s_{mm} m_t \]
Estimation technique

- Bayesian estimation:

Estimated parameter distribution = prior distribution + likelihood information from the data

It is a bridge between calibration and maximum likelihood

Results: Model + Data + Priors
Estimation

- Sample period: 1999Q2 to 2008Q1 (36 obs)

- Data: 25 series:

- Data treatment: HP filter

- Number of model parameters: 58
  - 41 estimated: 17 structural parameters and 24 shock parameters
  - 17 calibrated: 3 structural parameters and 14 steady-state relationships
Posteriors distributions for selected parameters

- **h**
- **θ**
- **γπ**
Impulse responses to a consumption shock
Impulse responses to a monetary policy shock
Impulse responses to a world GDP shock
Challenges

• Common to DSGE models and their estimation:

  – Generation of slower and more persistent dynamics (enough propagation mechanisms, lags in the transmission mechanisms, etc.)

  – Identification of the main model channels in place

  – Large number of parameters to be estimated – calibration versus estimation
Challenges

- Brazilian economic features:
  - Small sample size
  - Specific features: administered prices
  - Large changes in some ratios over the sample (ex.: net external debt-to-GDP ratio)
Next steps

● Refining model setup:
  – Add nominal and real rigidities: wage rigidity, price rigidity in the import and export sectors, firm-specific capital
  – Disaggregate CPI inflation into administered and non-administered prices

● New estimation and model implementation
Thank you for your attention!