Equilibrium approach to financial stability: the Colombian case

Dairo Estrada
Agustín Saade O.
Daniel Osorio R.

May 27, 2008
Introduction

During the last three decades, Central Banks have been successful in achieving low levels of inflation and in keeping them under control. At the same time with inflationary success, however, central banks have been growingly concerned with financial stability. This is understandable in face of the huge costs of recent financial crises experienced almost worldwide.

Among the tools to assess the stability of the financial system, structural macroeconomic models, complex frameworks that allow interactions between economic agents, are becoming common.
Financial (In)stability Definitions

Numerous authors have defined financial stability, and, at one level, the various definitions seem to be familiar both from a theoretical as well as a practical viewpoint. However, few attempts have been made to define and formally characterize this term in an analytically rigorous manner.

Academics and policy makers have suggested a big set of definitions. We can categorize them into two broad categories:

- Information-based definition: Mishkin (19).
- Institutionally oriented definition: Haldane et al. (13).
Goodhart and Tsomocos’s financial instability definition:

A combination of probability of default - variously measured - of both banks as well as economic agents together with low bank profitability characterize financially unstable (fragile) regimes of the economy. This has the added advantage that it can be applied at both the individual and aggregate levels. **Thus, financial instability is characterized by both high probabilities of default and low profits.**
Demand for Models

The work by Bårdsen, Lindquist and Tsomocos [2006] contains an useful and detailed analysis of the comparative performance of several types of macroeconomic models. According to the authors, despite no model is able to answer all the questions, some of them share some *desirable* features that make them useful for financial stability analysis.

Desirable characteristics in a model for the analysis of financial stability:

<table>
<thead>
<tr>
<th>Contagion</th>
<th>Macroeconomic conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous Default</td>
<td>Micro-foundations</td>
</tr>
<tr>
<td>Incomplete financial markets</td>
<td>Empirically tractable</td>
</tr>
<tr>
<td>Heterogenous agents</td>
<td>Forecasting and policy analysis</td>
</tr>
<tr>
<td>Testing device</td>
<td></td>
</tr>
</tbody>
</table>
Pagan’s Diagram

The diagram shows a comparison between theory and data for different models. The models labeled are RBC, GE, and DSGE.
Dynamic Stochastic General Equilibrium Models

- RBC models. See Leao and Leao (18).

- Representative agent models with asymmetric information. See Bernanke et al. (3).

- Overlapping generation (OLG) models. See Azariadis and Smith (1), Reichlin and Siconolfi (23), Zichino (28).

- Finite horizon general equilibrium (FHGE) models. See Haldane et al. (14), Tsomocos (25).
Goodhart, Sunirand and Tsomocos[2006b]

- It is a general equilibrium model without production. It has endogenous default. It includes active and heterogeneous banks that are subject to balance sheet regulatory constraints.

- In the model it is possible to study the effect of capital constraints on the intertemporal decision making of banks.

- It is possible within the model to measure explicitly financial instability: a situation characterized by low profits in financial institutions and a high extent of default in markets. It is therefore possible to analyze the effect of policy measures undertaken before a financial crisis actually occurs.

- A limitation of the model is its difficulty to compute and calibrate.
Why to use a model like this one?

To analyze the applicability, it is useful to study a particular case of the Colombian economic history, namely, the financial crisis of the late nineties. From December 1997 to July 1999, the assets of the financial system fell 15.8% in real terms. Further, total loans as a percentage of GDP fell from 42.9% in 1997 to 35.5% in 1999. The financial crisis of this period was one of the most hurting economic events of the XXth century in Colombia. Figure 1 shows the recent evolution of the profitability of the financial institutions and of non-performing loans as a percentage of total loans (which is a proxy of default in credit markets).
The period of financial instability of 1998-9 was associated with huge losses to credit institutions and a low repayment of credits. These facts justify the use of the aforementioned *operational* definition for the analysis of the extent of financial stability in Colombia.
A model: Tsomocos et. al. 2006b

- Agents:
  - Heterogenous banks: $B = \{\gamma, \delta, \tau\}$
  - Private agents: $H = \{\alpha, \beta, \theta, \phi\}$
  - Central Bank ~ Regulator

- Markets:
  - Interbank market ($B, CB$)
  - Loan markets ($B, H$)
  - Deposits markets ($B, H$)

- Time horizon: $t \in \{0, ..., \infty\}$

- $S = \{i \ (good), ii \ (bad) \}$. $P (s = i) = p$. 
Equilibrium approach to financial stability: the Colombian case
Markets open simultaneously:
- Interbank Market
- Credit Markets
- Deposit Markets

Policy rules are defined

Nature chooses the state $s \in S$, according to a distribution $f(s)$

Markets re-open.

- Settlements of deposits and credit are carried out.
- Default may appear in all markets
- Penalties on violations of CAR and default

Given all transactions, bank benefits are calculated
It’s a set of parameters that affect the objective function and constraints of the banks. Various shocks of regulation may be designed by changing these parameters.

- Regulator
  - Sets capital adequacy requirements: $k_{t+1,s}^{b}$
  - Imposes penalties for failure to meet such requirements: $\lambda_{k,s}^{b}$
  - Imposes penalties on default: $\lambda_{s}^{b}$
  - Sets the risk weight on market book, loans and interbank loans: $\bar{\omega} \tilde{\omega} \omega$ (Basel 2)

- Central Bank
  - Conducts open market operations (OMOs)
  - Decides interbank rate $\rho$
Banks’ optimization problem
List of variables:

\[ \mu^b_t : \text{bank } b\text{'s debt in the interbank market in period } t \]

\[ d^b_t : \text{bank } b\text{'s interbank lendings} \]

\[ \mu^b_{d,t} : \text{bank } b\text{'s deposits} \]

\[ \nu^b_{t+1,s} : \text{repayment rate of bank } b \text{ in } t+1, \ s \]

\[ m^b_t : \text{amount of credit that bank } b \text{ offers in the period } t \]

\[ A^b_t : \text{Other assets of bank } b \]

\[ e^b_t : \text{bank } b\text{'s capital} \]

\[ r^b_t : \text{lending rate offered by } b \]

\[ r^b_{d,t} : \text{deposit rate offered by } b \]

\[ \rho_t : \text{interbank rate in period } t \]

\[ \nu^h_{t+1,s} : \text{repayment rate of } h^b \text{ in } t+1, \ s \]

\[ R_{t+1,s} : \text{repayment rate expected by banks from their interbank lending in } t+1 \]

\[ k^b_{t+1,s} : \text{Capital adequacy ratio (CAR)} \]
Banks’ optimization problem

\[ \text{Max}_{m_t^b, \mu_t^b, d_t^b, \mu_{d,t}^b, \nu_{t+1,s}^b, s \in S} E_t[\Pi_{t+1}^b] = \sum_{s \in S} p_s \{ \pi_{t+1}^b - c_s(\pi_{t+1,s}^b)^2 \} \]

\[-\sum_{s \in S} p_s \left( \lambda_{ks}^b \max[0, k_{t+1,s}^b - k_{t+1,s}^b] + \lambda_s^b (1 - \nu_{t+1,s}^b)(\mu_t^b) + \lambda_s^b (1 - \nu_{t+1,s}^b)(\mu_{d,t}^b) \right) \]

Subject to balance sheet constraint

\[ \bar{m}_t^b + d_t^b + A_t^b = \frac{\mu_t^b}{1 + \rho_t} + \frac{\mu_{d,t}^b}{1 + r_{d,t}^b} + e_t^b + \text{others}_t^b \]

and

\[ (1 + \rho_t)\nu_{t+1,s}^b \mu_d^b + (1 + r_{d,t}^b)\nu_{t+1,s}^b \mu_{d,s}^b + \text{others}_t^b + e_t^b \leq \nu_{t+1,s}^b (1 + r_{d,t}^b)\bar{m}_t^b + (1 + r_t^A)A_t^b + \tilde{R}_{t+1,s}d_t^b(1 + \rho_t) \quad s \in S \]
where:

\[
\pi_{t+1,s} = \nu_{t+1,s} (1 + r_{d,t}) m_t^b + (1 + r^A_t) A_t^b \\
+ \tilde{R}_{t+1,s} d_{t}^b (1 + \rho_t) - ((1 + \rho_t) \nu_{t+1,s} \mu_d^b \\
+ (1 + r_{d,t}) \nu_{t+1,s} \mu_d^b + \text{others}^b + e_t^b) \quad s \in S
\] (4)

\[
e_{t+1,s} = e_t^b + \pi_{t+1,s}, \quad s \in S
\] (5)

Capital adequacy ratio:

\[
k_{t+1,s}^b = \frac{e_{t+1,s}^b}{\omega \nu_{t+1,s} (1 + r^b_t) m_t^b + \tilde{\omega} (1 + r^A_t) A_t^b + \omega \tilde{R}_{t+1,s} d_{t}^b (1 + \rho_t)}, \quad s \in S
\] (6)
Private agent sector

- Demand for loans

\[
\ln(\mu_t^{h_b}) = a_{h_b,1} + a_{h_b,2} \text{trend} + a_{h_b,3} \ln[p(GDP_{t+1,i}) + (1 - p)(GDP_{t+1,ii})] + a_{h_b,4} r_t^b \tag{7}
\]

- Loan repayment rates (1-default)

\[
\ln(\nu_{t+1,s}^{h_b}) = g_{h_b,s,1} + g_{h_b,s,2} \ln(GDP_{t+1,s}) + g_{h_b,s,3} \ln(\overline{m_\gamma} + \overline{m_\delta} + \overline{m_\tau}) \tag{8}
\]

- \(\phi\)'s supply of deposits

\[
\ln(d_{b,t}^\phi) = z_{b,1} + z_{b,2} \ln[p(GDP_{t+1,i}) + (1 - p)(GDP_{t+1,ii})] + z_{b,3} (r_{d,t}^b [p\nu_{t+1,i}^b + (1 - p)\nu_{t+1,ii}^b]) \\
+ z_{b,4} \sum_{b' \neq b} (r_{d,t}^{b'} [p\nu_{t+1,i}^{b'} + (1 - p)\nu_{t+1,ii}^{b'}]) \tag{9}
\]

- GDP:

\[
\ln(GDP_{t+1,s}) = u_{s,1} + u_{s,2} \text{trend} + u_{s,3} \ln(\overline{m_\gamma} + \overline{m_\delta} + \overline{m_\tau}) \tag{10}
\]
Market Clearing Conditions

- Bank $b$’s credit market clears

$$1 + r^b_t = \frac{\mu^b_t}{m^b_t}$$  \hspace{1cm} (11)

- Bank $b$’s deposit market clears

$$1 + r^b_{d,t} = \frac{\mu^b_{d,t}}{d^b_{o,t}}$$  \hspace{1cm} (12)

- Interbank market clears

$$1 + \rho_t = \frac{\overline{B}_t + \sum_{b \in B} \mu^b_t}{M_t + \sum_{b \in B} d^b_t}$$  \hspace{1cm} (13)
See GST[2006b] for more details. The monetary equilibrium with commercial banks and default (MECBD) in time $t$ is a set of endogenous variables such that:

- All banks maximize their expected future payoff: $E_t[\Pi^b_t[\pi^b_{t+1,s}]]$.
- All markets clear.
- Banks form correctly their expectations about the repayment rate they receive from their interbank lending:

$$\tilde{R}_s = \sum_{b \in B} \nu_{t+1,s}^b \mu_t^b \sum_{b \in B} \mu_t^b, \quad s \in S$$

(14)

- The reduced form equations for GDP, deposits supply, credit demands, and household repayment rates are satisfied.
Implementation

The equation system to solve is, for bank $b$, with:

$$x^b = (m^b_t, \mu^b_t, d^b_t, \mu^b_{d,t}, \nu^b_{t,s}) \quad s \in S$$

$$\text{Min}_{x^b} \quad f^b(x^b) = -E_t[\Pi^b_{t+1}]$$

such that:

$$g^b_1(x^b) = m^b_t + d^b_t + A^b_t - \frac{\mu^b_t}{1 + \rho_t} + \frac{\mu^b_{d,t}}{1 + r^b_{d,t}} + e^b_t + \text{others}^b_t = 0$$

$$g^b_2(x^b) = (1 + \rho_t)\nu^b_{t+1,i} \mu^b_t + (1 + r^b_{d,t})\nu^b_{t+1,i} \mu^b_t + \text{others}^b_t + e^b_t$$

$$- \nu^b_{t+1,i} (1 + r^b_t)\bar{m}^b_t + (1 + r^A_t)A^b_t + \bar{R}_{t+1,i}d^b_t (1 + \rho_t) \leq 0$$

$$g^b_3(x^b) = (1 + \rho_t)\nu^b_{t+1,ii} \mu^b_t + (1 + r^b_{d,t})\nu^b_{t+1,ii} \mu^b_t + \text{others}^b_t + e^b_t$$

$$- \nu^b_{t+1,ii} (1 + r^b_t)\bar{m}^b_t + (1 + r^A_t)A^b_t + \bar{R}_{t+1,ii}d^b_t (1 + \rho_t) \leq 0$$
Implementation

Optimization conditions for banks:

• 6 equations per bank = 18 equations

\[ \nabla f^b(x^b) + \lambda_1^b \nabla g_1^b(x^b) + \lambda_2^b \nabla g_2^b(x^b) + \lambda_3^b \nabla g_3^b(x^b) = 0 \]  \hspace{2cm} (20)

• 2 equation per bank = 6 equations

\[ \lambda_2^b \times g_2^b(x^b) = 0, \quad \lambda_3^b \times g_3^b(x^b) = 0, \]  \hspace{2cm} (21)

\[ \lambda_2^b \geq 0, \quad \lambda_3^b \geq 0 \]  \hspace{2cm} (22)

• Additionally, 1 equality constraint per bank = 3 equations

\[ g_1^b(x^b) = 0 \]  \hspace{2cm} (23)
Implementation

- Every bank is correct about the repayment rate that it receives from its interbank lending = 2 equations

\[ \tilde{R}_s = \frac{\sum_{b \in B} \nu_{t+1, s} \mu_t^b}{\sum_{b \in B} \mu_t^b}, \quad s \in S \]  

(24)

- Reduced form equations: credit demands (3 equations), supply of deposits (3 equations), household repayment rates (3 households \( \times \) 2 states = 6 equations), and future GDP (2 equations).

- Market clearing conditions: three credit markets, three deposit markets, and the interbank market. Each one has an equation describing the market clearing conditions.
Implementation

The system can be written:

\[ F(x^{\gamma}, x^{\delta}, x^{\tau}, c) = \vec{O} \]  

(25)

The algorithm of solution searches for a zero in this function, solving the following problem:

\[
\text{Min } \sum_i F_i(x^{\gamma}, x^{\delta}, x^{\tau}, c)^2
\]  

(26)

We used MATLAB’s Optimization Toolbox.

- \( t = 0 \): Calibration
For more details, see Saade, Osorio and Estrada (24). The Colombian banking system is divided as follows:

- **BECH** ($\gamma$): mortgage banks
- **NATIONAL** ($\delta$): national banks (non BECH)
- **FOREIGN** ($\tau$): foreign banks (non BECH)
Calibration

The following reduced-forms were calibrated using several econometric techniques:

- Household’s demand for credit: Following Chrystal and Mizen[2001], long-run elasticities were estimated using the error-correction representation of a cointegrated system between credit, money, consumption, inflation, interest rates and unemployment.

- Agent $\phi$’s supply of deposits: The parameters were obtained from the estimation of a random-effects model on a panel data set containing bank-specific information about deposits, interest rates and real GDP.

- Household’s repayment rate: The parameters were obtained from the estimation of a random-effects model on a panel data set containing bank-specific information about non-performing loans, credit and real GDP.

- Real GDP: Like the demand for loans, the parameters were extracted from a cointegration vector for credit and real GDP, characterized by the presence of a drift.
Performance of the model

The simulations were conducted under two initial conditions. The first subsection shows the set of the results of a simulation whose initial period mimics the structure of the Colombian financial system in the first quarter of 1997. The second set of results corresponds to a simulation that uses the last quarter of 1999 as initial period.

The only reason for using these periods as starting conditions is to explore the performance of the model in replicating the behavior of the financial system during a crisis and during a recovery. The Colombian economy featured during these two initial periods the onset of a recession (in the case of 1997-I) and of an economic recovery (1999-IV).
The recession period: $t_0=1997-I$

Figure 2: I. Loans. $t_0=1997q1$. 
Figure 3: II. Deposits. $t_0=1997q1$
Figure 4: III. Household repayment rate. $t_0=1997q1$
Figure 5: $t_0=1997q1$

IV. Interbank repayment rate.  

V. GDP.

![Graph showing interbank repayment rate from 1997q1 to 1999q1.]

![Graph showing GDP from 1997q1 to 1999q1.](#)
The recovery period: 1999-IV to 2002-IV

Figure 6: I. Loans. $t_0=1999q4$

[Graphs showing loan data for different periods and models.]

BECH: loans SOE2007
BECH: loans obs
BECH: loans FSR2008_03

DB: loans SOE2007
DB: loans obs
DB: loans FSR2008_03

FB: loans SOE2007
FB: loans obs
FB: loans FSR2008_03

SOE2007 - COP94 trillions
Figure 7: II. Deposits. $t_0=1999q4$
Figure 8: III. Household repayment rate. $t_0=1999q4$
Figure 9: \( t_0 = 1999q4 \)

IV. Interbank repayment rate. V. GDP.
Concluding comments

The empirical fit of the model is remarkable (especially in the short-run), as well as its capacity to mimic some of the most important stylized facts of the Colombian financial system.

However, the model cannot solve all the puzzles that arise in the analysis of financial stability (e.g. the model was not designed for GDP forecasting). The comparative advantage of this model is its special focus on the behavior (and the interaction with the rest of the economy) of the financial institutions.
Research Agenda

Two complementary tasks are open for this research agenda in the future.

- To use the model in the assessment of the effect that policy variables (such as CAR, policy rate or penalties) have on financial stability. This is a very important *acid test* for the use of the model as a tool for financial stability analysis, and it is likely to be its most important function in the future.

- To improve on the empirical fit of the model. This might be done using two complementary approaches:
  - Improvement (and eventually, rethinking) of the econometric calibration strategies. Despite the suppression of one of the sources of overestimation in the model (endogenous GDP), the problem remained in some cases. This leads to think that the evaluation of calibration techniques goes next.
  - The second approach is the incorporation of some particular (idiosyncratic) features of the Colombian financial system into the model.

- Additionally, the Financial Stability Department is currently working in the implementation of a dynamic model in the spirit of Leao and Leao (18).
References


sounder banks in emerging markets? The Latin American experience. *Federal Reserve Bank of New York Staff Papers.*


[23] Reichlin P. and P. Siconolfi (2002). Optimal debt contract and moral hazard along the business cycle, manuscript.


