Interest Rates and Default in Unsecured Loan Markets

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Abstract

This paper investigates how interest rates affect the probability of default (PD) in a general equilibrium incomplete market economy. We show that the PD depends positively on the loan interest rate and negatively on the economy’s basic interest rate. Empirically, this finding is confirmed by estimation of the Cox proportional hazard model with time-varying covariates using a sample of 445,889 individual contracts from a large Brazilian bank. Among the controls are macroeconomic variables and specific characteristics of the contract and borrowers. A lower basic interest rate, implied by easing monetary policy, leads banks to lend for riskier borrowers, increasing the PD.

Keywords: Default probability; Incomplete markets; Survival analysis.

JEL Code: D52; C81.

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1 Introduction

Recent developments of the international financial crisis, started in US late 2007, have shown that several factors might affect the borrowers’ capacity of debt repayment. The probability of default can be explained not only by individuals’ characteristics but also by macroeconomic conditions. For instance, a raise in unemployment would make it difficult for individuals to repay debts because of shortage in the personal income and so would push them into default. Similarly, there might be effects coming from interest rates, economic growth, inflation and other macroeconomic variables. Thus, when building credit score models to estimate and forecast probability of default, it is crucial to take those variables into consideration in addition to the traditional contract and individual’s characteristics.

This paper analyzes theoretically and empirically the probability of default in the Brazilian financial market taking into account both the contract and borrowers’ specific characteristics and the country’s macroeconomic conditions. The theoretical model is based on Dubey, Geanakoplos and Shubik (2005), who extended the basic Arrow-Debreu general equilibrium model with incomplete markets to allow for either total or partial default with direct punishment in terms of utility. The punishment can be thought as financial loss, restriction to new credit, a fee or any other event that may lead to a reduction of the defaulter’s utility. The reason for choosing that model is that default emerges in equilibrium, differently from Alvarez and Jermann (2000) and Kehoe and Levine (1993, 2001), where there is endogenous solvency constraints but no default in equilibrium.

According to Zame (1993), default plays an important role in the economy, as it opens the markets for individuals that have a high (but not sure) probability of not honoring their debt. With no possibility of default, those individuals would not be part of the market. An alternative form of punish-
ment for default is proposed by Geanakoplos e Zame (2000). In this case, a warranty, called collateral, is required from the borrower. In the event of default, the collateral is taken by the lender. Maldonado and Orrillo (2007) show that, under some circumstances, these two modeling strategies are equivalent. However the collateral model will not be considered since our data set is composed of unsecured loans.

The model by Dubey, Geanakoplos and Shubik (2005) is used to derive theoretical relationships among interest rates and the probability of default. Specifically, we show that there is a positive relationship between the probability of default and the loan real interest rate and, surprisingly, a negative relationship between the probability of default and the economy basic real interest rate. By decreasing the basic real interest rate, via easing monetary policy, there will be incentives for banks to become less restrictive on credit analysis, lending money to borrowers with worse credit history. The entrance of riskier borrowers into the financial market causes the probability of default to rise. This is particularly critical in periods of economic crisis, like the international financial turmoil started in US late 2007. Lower basic real interest rates implied by expansionist monetary policies reduce financial earnings and make banks to diminish credit barriers for new borrowers, increasing lend for bad payers. This process contributes to raise the probability of default in the economy.

The theoretical findings are confirmed by the empirical evidence for the Brazilian economy. We applied survival analysis to estimate the probability of default under the influence of macroeconomic conditions. This technique allows for the inclusion of both censored data and time-varying macroeconomic variables. In fact, estimation of survival models has been recently pursued by several authors, including Bellotti and Crook (2007), Banasik et al (1999), Tang et al. (2007), Stepanova and Thomas (2001, 2002), Andreeva (2006), among others. They have applied survival analysis to predict
the probability of default using distinct data sets of credit information. The attempt to introduce macroeconomic variables to improve the model’s prediction power has also been tracked. Bellotti and Crook (2007), for instance, used a data set of credit card holders to show that inclusion of macroeconomic variables improves the model predictive performance when compared with the benchmark survival model and logistic regression. The novelty of our approach is to apply the framework to the Brazilian financial market and provide both theoretical and empirical foundations for our major findings.

The Cox proportional hazard model with time-varying covariates was estimated for a huge sample of clients from a large Brazilian bank. The results showed that positive variations in the loan real interest rate are followed by raising in the probability of default, while increases in the economy’s basic real interest rate lead to negative variations in the individual’s probability of default. These findings do not imply that Central Banks should raise interest rates during financial crises. They simple suggest that reductions in the basic interest rate should not be followed by credit expansion which eases credit history analysis of potential borrowers. Keeping careful analysis of the borrowers’ credit risk would avoid undesirable increases in the individuals’ probability of default. Notice that the Brazilian evidence agrees with other findings in the literature. Under a distinct framework, Bernanke, Gertler and Gilchrist (1996) reported that lower interest rates may result in banks to lend to borrowers that were regarded in the past as too risky. Recent additional empirical evidence can be found in Jimenez et al. (2008) and Ioannidou et al. (2008), who estimated discrete choice models for Spain and a quasi-natural experiment for Bolivia, respectively.

The paper is organized as follows. The next section describes the general equilibrium with incomplete markets (GEI) economy and derives theoretical relations among the probability of default and interest rates. The third section presents the Cox proportional hazard model with time-varying covari-
ates. The empirical results are reported and discussed in the fourth section. Finally, the fifth section is dedicated to concluding remarks.

2 Economic Model

The model is a simplified version of Dubey, Shubik and Genakoplos (2005), who allow agents to default on their promises subject to a direct penalty in their utility function. The penalty can be thought of as a credit restriction or any other event that reduces the agent’s utility in case of default.

The economy lasts for two periods with uncertainty only in the second one. This uncertainty is modeled by a finite set of states of nature $S = \{1, \ldots, S\}$. Assume that there is only one good in each period and in each state of nature so that the consumption set is taken to be $R_{+}^{S+1}$. Each agent $h \in H = \{1, \ldots, H\}$ is characterized by a utility function $U^h : R_{+}^{S+1} \rightarrow R$, which is assumed to be twice differentiable, strict increasing and concave, and an endowment vector of commodities $\omega^h \in R_{+}^{S+1}$.

The set of financial assets is denoted by $J = \{1, \ldots, J\}$. Each asset $j \in J$ is represented by a vector $r^j \in R_{+}^S$ where $r^1 = (1, \ldots, 1)$ is assumed to be the risk-free asset that pays one unit of the commodity in every state of nature. For the sake of simplicity, we assume that there is only one risky asset on which agents can default. Thus, we have that $J = 2$ assets. The two assets available for trading can be thought as a government bond and a loan subject to credit risk. Payoffs are in terms of the consumption good. We assume that $S > 2$ so that markets are incomplete.

Agent’s problem

Given the market payment rate $t \in [0, 1]^S$, and a price vector $(q, \pi) \in R_2^2$, each agent $h$ chooses a consumption-investment plan $(x; b; (\theta, \varphi)) \in R_{+}^{S+1} \times \ldots$
\( R \times R^2_+ \), and delivery plan \( d \in R^S_+ \) in order to maximize his/her payoff

\[
V = u^h(x) - \sum_s \lambda_s [r_s \varphi - d_s]
\]

subject to the following budget constraints

\[
x_o + qb + \pi \theta \leq \omega_o^h + \pi \varphi
\]

\[
x_s + d_s \leq \omega_s^h + (1 + r)b + t_s r_s \theta, s \in S
\]

\[
0 \leq d_s \leq t_s r_s \varphi, s \in S
\]

The payoff in (1) defines that each agent, as borrower in the risky asset, suffers a penalty which is proportional to the non-paid amount. The default quantity can be either zero (full loan repayment), total debt (zero loan repayment), or any strictly positive amount between those two limits. This is summarized by constraint (4).

Lastly, (2) and (3) are the usual budget constraints. Equation (2) states that consumption and investment in government bonds and in financial assets in the first period are financed by the first-period wealth and borrowing. In each state of nature of the second-period, equation (3) defines that consumption and delivery are financed by wealth and financial return from government bonds and from non-defaulted private loans.

**Equilibrium**

An equilibrium for this economy consists of a market payment rate \( t \in [0, 1]^S \), a price vector \((q, \pi) \in R^2_+ \), consumption-investment plan \((x; b; (\theta, \varphi)) \in R^{S+1}_+ \times R \times R^2_+ \), and a delivery plan \( d \in R^S_+ \) satisfying the following properties:

1. The choices are optimal. That is, for each \( h \in H \) the vector

\[
(x^h; b^h; (\theta^h, \varphi)^h; d^h)
\]

maximizes the payoff \( V \) subject to the budget constraints (2) and (3).
2. Markets clear:

\[ \sum_h x_h^s = \sum_h \omega_h^s, s = 0, 1, \ldots, S. \]

\[ \sum \theta^h = \sum \varphi^h, \sum b = 0. \]

3. The payment rate, given by

\[ t_s = \frac{\sum_h d_h^s}{\sum_h r_s \varphi^h}, \] provided that \( \sum_h \varphi^h > 0 \), is rationally anticipated, and so is the default rate, defined by \( k_s = 1 - t_s \).

**Remark:** Notice that items 2 and 3 imply that commodity markets clear for each state of nature. In equilibrium, we have \( 0 \leq d_s^h \leq r_s \varphi^h, s = 1, 2 \), so that the value function of each agent is

\[ V(t, q, \pi) = u^h(\omega_o^h - q b - \pi \theta + \pi \varphi) + \sum_s p_s v^h(\omega_s^h + t_s r_s \theta^h + (1 + r) b_s^h - d_s^h) \]

\[ - \sum \lambda [r_s \varphi^h - d_s^h] \]

**Theorem 1** For each state \( s \in S \), in equilibrium, the following inequalities hold:

\[ \frac{\partial k_s}{\partial R_s} \geq 0 \] and \( \frac{\partial k_s}{\partial R} < 0 \) (5)

where \( R_s = \frac{c_s}{\pi} \) is the real rate of return of the risky asset and \( R = \frac{1+r}{q} \) is the real rate of return of the risk-free asset.

**Proof:** see appendix A.

If we interpret the risky asset as a risky loan, then the first inequality in (5) states that the market default rate depends positively on the loan interest rate. Similarly, if we interpret the risk-free asset as a government bond, the second inequality in (5) states that the market default rate depends negatively on the government bond interest rate. In the Brazilian case, the
later might be represented by the overnight selic rate.\footnote{Interest rate paid by the Brazilian government bonds.} Given that there is no money in the model and the consumption good is the numerary, both interest rates are expressed in real terms.

### 3 Econometric Model

Survival analysis is used in the model estimation because it allows studying the duration of failure in a given population and facilitates the inclusion of both observations that have not failed, called censored data, and time-dependent macroeconomic variables. When an individual borrows money from a financial institution, he/she is obligated by contract to make frequent payments (usually monthly) until full repayment of the debt. We consider those who have left a certain number of parcels unpaid as defaulters. Thus, we define:

\[
D(I, t) = \begin{cases} 
1, & \text{if borrower } i\text{'s days of delay are } t \geq \alpha \\
0, & \text{if borrower } i\text{'s days of delay are } t < \alpha 
\end{cases} \tag{6}
\]

where \(i\) refers to the \(i\)-th client of a given credit portfolio and \(t\) is the time measured in days. The number of days of delay, \(\alpha\), which characterizes whether a client has defaulted, is determined by the migration matrix to delay. This matrix computes the probability of migration from a given range of delay to a higher range of delay. In fact, it is a conditional probability, where the next state depends exclusively upon the current state. Due to the structure of our sample, the delay interval in the migration matrix is 30 days, which is the frequency of the parcels due date.

It is assumed that the cut-off point defining default is the first range of delay for which the probability of migrating to the next range is greater than 90%. This cut-off point assures that the great majority of clients who achieve
this range of delay will not leave the state of default, resulting in a financial loss for the financial institution. Notice that a client might be in default in a given month and not be in the next one, provided that he/she repays all delayed parcels. However, this change between states happens only in 10% of the cases.

3.1 Survival Analysis

Among the statistical models that can be used to analyze probability of default are non-parametric techniques, probabilistic models, logistic regressions, and survival analysis. We have chosen the latter because it matches the loan default process, allows modeling both probability and time of default, provides forecast as a function of time and enables the inclusion of time-dependent covariates. This latter feature appears as a modification to the original Cox (1972, 1975) regression. Another important advantage is its versatility, given that it does not require any assumption regarding the probability distribution of the data.

The Cox’s model hazard function is given by:

$$h(t) = \lim_{\delta t \to 0} \left( \frac{P(t \leq T < t + \delta t | T \geq t)}{\delta t} \right)$$

and the Cox’s proportional hazard model with time-varying covariates can be written as:

$$h(t, X(t), \beta) = h_\alpha(t) g(X'(t)\beta)$$

where $g(.)$ is a non-negative function that must be specified, such that $g(0) = (1, X(t))$ is a vector of covariates, $\beta$ is a vector of parameters to be estimated, and $h_\alpha(t)$ is a nonparametric baseline hazard function which depends on time but not on the covariates. The function $g(X'(t)\beta)$ is the parametric
component and usually assumes the form:

\[ g(X'(t)\beta) = \exp\{X'(t)\beta\} = \exp\{\beta_1 x_1 + \ldots + \beta_n x_n\} \] (9)

The model is also known as the proportional hazard model because the rate of failure of any two distinct individuals is constant across time. This assumption is broken when the model includes time-dependent covariates. The interpretation of the estimated coefficients, \( \beta \), is also affected. Each estimated coefficient can be interpreted as the logarithm of the hazard ratio when the value of the covariate being analyzed changes by one unit and all other covariates are kept constant. The survival probability up to time \( t \) is:

\[ S(t) = P(t \leq T) = \exp\left(-\int_0^t h(u)du\right) \] (10)

Here, \( S(t) \) represents the probability that the individual does not default until time \( t \). Thus, we can consider:

\[ S(t) = 1 - DP(t) \]

where \( DP(t) \) is the probability of default in time \( t \).

The individuals who did not default during the period are not included in the modeling. These observations are regarded as censured and their life time is the last time that they were observed. Thus,

\[ c_i = \begin{cases} 0, & \text{if the observation is censured} \\ 1, & \text{otherwise} \end{cases} \] (11)

Individuals who have not been fully observed for some reason were also considered censured and received the same treatment.

The estimation of the model parameters is difficult because of the non-parametric component. Cox proposed a partial-maximum likelihood method, which considers the knowledge of past history of failures in the construction
of the maximum likelihood function. The general idea of the method is described below.

Assume that, for a sample of \( n \) individuals, there are \( k \leq n \) distinct failures in time periods \( t_1 < t_2 < \ldots < t_k \). The conditional probability that the \( i \)-th observation will fail at time \( t_i \) given the observations that are under risk at \( t_i \) is:

\[
P[i \in I \text{ defaults at } t_i | \text{ a failure at } t_i \text{ and history up to } t_i] = \frac{h_i(t|X_i)}{\sum_{j \in R(t_i)} h_j(t|X_j)} = \frac{h_o \exp(X_i'(t)\beta)}{\sum_{j \in R(t_i)} h_o \exp(X_j'(t)\beta)}
\] (12)

where \( R(t_i) \) is the set of indexes of observations that are under risk at time \( t_i \), that is, are observations that have not presented failure up to time \( t_i \). Thus, the \( h_o(t) \) term disappears. To estimate the parameters of the model, the maximum likelihood function is given by the product of all terms represented by (12) associated to distinct failure times. That is,

\[
L(\beta) = \prod_{i=1}^{k} \frac{\exp(X_i'(t)\beta)}{\sum_{j \in R(t_i)} \exp(X_j'(t)\beta)} = \prod_{i=1}^{n} \left( \frac{\exp(X_i'(t)\beta)}{\sum_{j \in R(t_i)} \exp(X_j'(t)\beta)} \right)^{c_i}
\] (13)

where \( c_i \) is the censure indicator. To find values of \( \beta \) that maximizes the partial-maximum likelihood function, \( L(\beta) \), one shall resolve the system of equations defined by \( U(\beta) = 0 \), in which \( U(\beta) \) is the vector of scores from \( l(\beta) = \log(L(\beta)) \). Thus,

\[
U(\beta) = \sum c_i \left[ X_i - \frac{\sum_{j \in R(t_i)} X_j \exp(X_j'(t)\beta)}{\sum_{j \in R(t_i)} \exp(X_j'(t)\beta)} \right] = 0
\] (14)

In practice, it might be the case that time of failure is even and also time of failure and censure are even. In these cases, the partial-maximum likelihood
function must be modified, as suggested by Breslow (1972) and Peto (1972). Let \( s_i \) be the vector of \( p \) covariates for an individual who fails at the same time \( t_i \), with \( 1 \leq i \leq k \), and \( d_i \) be the number of failures at that time. The approximation proposed for the partial-maximum likelihood function is:

\[
L(\beta) = \prod_{i=1}^{k} \left[ \frac{\exp(s_i'(t)\beta)}{\sum_{j \in R(t_i)} \exp(X_j'(t)\beta)} \right]^{d_i}
\]

In order to verify the quality of the adjustment, the percentage of successful forecasts made by the estimated model might be analyzed. It is based on the sum of two terms:

(i) Percentage of cases that were forecasted as default in a given time and came up as default in the observed data;

(ii) Percentage of cases that were forecasted as non-default and came up as non-default in the observed data.

The higher the success rate, the better the adjustment of the model.

4 Results

4.1 Data

The data set is comprised of 445,889 individual contracts of a given credit operation from a major Brazilian bank, which will not be revealed to assure confidentiality. The loan interest rate is pre-fixed and known at the moment of the contract signature. There is no collateral. In case of default, the punishment for the borrower is the inclusion of his/her name in a national public list of bad payers. This public list is consulted by financial institutions when evaluating an individual’s credit history. We had access to contracts signed between January 2003 and December 2007. However, the year of 2007
did not entered in the estimation, being used only to analyze the model’s forecasting performance. The time-dependent covariates are also monthly for the same period.

Characteristics of the contract and borrowers included value of the contract, loan interest rate, borrower’s age and dummy variables for the Brazilian state where the contract was signed. The dummies intend to capture whether the state’s level of per capita income affects probability of default. It might be higher in poorer states than in richer ones. Provided that the model has no constant, 27 dummy variables might be included, representing each one of the 26 Brazilian states and the Federal District.

The time-dependent covariates, used to control for macroeconomic conditions, were defined by real interest rates, output gap, unemployment rate, and inflation rate. A step-wise technique, described in the next section, was applied to decide which ones are statistically significant and should remain in the model. The real interest rates were computed as the log-difference between the nominal interest rates and the inflation rate. Inflation was measured by the percentage change in the wide consumer price index (IPCA). This rate is used by the Brazilian Central Bank in the inflation-targeting monetary policy regime. For the government bonds, the interest rate was given by the over selic, which is the basic interest of the economy and instrument of the Brazilian monetary policy. For the loan, the interest rate depends on the contract itself and is fixed a priori.

The output gap is represented by the percentage difference between observed output, measured by real GDP, and potential output, estimated by a linear trend. The unemployment rate is computed by IBGE (Brazilian Institute of Geography and Statistics) as a weighted average of unemployment in the 6 major metropolitan regions of the country. All variables are available at the official site of Ipeadata (www.ipeadata.gov.br). Notice that these monthly variables are time dependent, observed across the duration of
the contract. In the estimation, however, only their values at the moment of default were considered.

4.2 Estimation and Analysis

The Cox proportional hazard model with time-varying covariates was used to estimate the contract’s probability of default across its life. About 52% of the data was censured because either did not achieve default during the period or was no longer available (for instance, contracts paid in full or renegotiated). Given that the proportions of censured and non-censured are so close, there is no need for extracting a balanced sample.

In order to define the number of delayed days characterizing a contract as defaulted, it was computed the migration matrix, reported in Table 1. The cut off point for the definition of default is 90 days, as suggested by the Basileia agreement. About 91% of the contracts that achieve this range of delay migrated to longer delays.

[INSERT TABLE 1 ABOUT HERE]

The survival time is defined as the time it takes to complete at least 90 days of delay between any two repayments of the contract. Given that payments are monthly, one can calculate delay in repayments of 3 or more parcels. Contracts which have not achieved 90 days of delay in the period are treated as censured. The survival time is equal to the number of months it was observed. Analogously, contracts which were no longer observed during the period were also regarded as censured and their survival time was the number of months they were observed. Table 2 reports survival times for the data set.

[INSERT TABLE 2 ABOUT HERE]

The estimation was carried out in the statistical package SAS, which already has the Cox model programmed under the toolbox “proc PHREG”. The model parameters are estimated by partial maximum likelihood, as orig-
inally proposed by Cox. To select the control variables, a step-wise technique was used, where the variables are added into the model one by one and only those which are statistically significant according to a Chi-Square statistics remain in the regression. This procedure demands testing several combinations among the variables. Only variables with a p-value smaller than 0.05 were kept in the model. Finally, to select between two alternative models, the Akaike and Schwartz information criteria were applied. Table 3 reports results for the best estimated model.

\[\text{INSERT TABLE 3 ABOUT HERE}\]

The fourth column in Table 3 displays the risk ratio, computed as:

\[RR = \exp(\hat{\beta})\]

This ratio indicates an increase or decrease in the survival probability of the contract when the analyzed variable is increased by 1 unit and all other variables are kept constant. Thus, the result emerging from the estimated model is the probability that a given contract will enter default during the time it was observed.

In order to verify the fitness rate of the model, the following procedure was implemented:

i) the survival probability of each contract within 12 months was observed;
ii) contracts that presented survival probability less than 65% were considered as potential defaulters at the 12th month;
iii) those contracts were compared with the ones that actually defaulted in the 12th month of life;
iv) the percentage of fitness for this case was 0.64, meaning that 64% of the contracts were correctly classified.

The previous strategy is subject to two types of errors. Type I error is to classify as non-default a contract that ended up achieving default during the specified period of time. Type II error is to classify as default a contract that
did not achieve default during the specified period of time. For the estimated model, the probability of type I error was 31.07% while for the type II error it was 38.9%. This means that 31.07% of the contracts that were previously considered non-defaulters ended up defaulting up to the 12th month of life. On the other hand, 38.9% of the contracts that were expected to default up to the 12th month did not achieve this status. In general, the latter error is more acceptable for a conservative financial institution. However, those contracts still subject to the possibility of achieving default at longer horizons.

The model results might be interpreted by analyzing the estimated parameters. For instance, an estimated coefficient with negative sign indicates that a rise in the corresponding variable, keeping all others unchanged, decreases the probability of default. On the other hand, a positive estimated coefficient would have an opposite effect. This interpretation is possible due to the format of the survival function described in Section 3. It is also possible to obtain estimated magnitudes of impacts on the survival probability.

The last column of Table 3 reports estimates for the effects of selected variables on the survival probability. The first coefficient indicates that a rise of 1 year in the borrower’s age reduces the contract probability of default by 2.1%. Older individuals tend to have higher personal income and so lower probability of default. The estimated coefficient for the “value of the contract” indicates that higher amount contracts show smaller probability of default. That is because, in this case, the lender makes a more careful analysis on the borrower’s capacity of repayment.

It is worthwhile noting that the negative sign on the basic real interest rate corroborates the findings of the theoretical model. Specifically, the estimated coefficient suggests that a decrease of 1% in the economy basic real interest rate increases the probability of default by 36.6%. This is because, under a lower basic real interest rate, implied by easing monetary policy, banks
tend to lend more for borrowers with worse credit history, who carry out a lower probability of repaying contract obligations. This strategy of the banks is usually attributed to an attempt of expanding credit to compensate for financial losses imposed by a lower real interest rate. On the other hand, a 1% increase in the loan real interest rate increases the contract probability of default by 16.7%. Intuitively, a rise in the loan interest rate increases the probability of default because makes it harder for the borrower to honor debt repayments.

Another interesting result is given by the variable unemployment rate. As expected, the probability of default is positively related to the unemployment rate of the economy. Higher unemployment implies lower personal income and thus less capacity of repaying financial obligations. Finally, the dummy variables to control for the Brazilian state where the contract was signed were also statistically significant in the estimated model. In general, the probability of default is higher in the poorer states of the country.

5 Concluding Remarks

This paper investigated, theoretically and empirically, the determinants of the probability of default by individual borrowers, taking the Brazilian economy as a case study. The general equilibrium model for an incomplete market economy where agents are allowed to default was based on Dubey, Shubik and Genakoplos (2005). We showed that the probability of default depends positively on the loan real interest rate and negatively on the economy basic real interest rate. In order to empirically evaluate those findings, we estimated the Cox proportional hazard model with time-varying covariates. Specifically, it was used a sample of 445,889 contracts of a given credit operation for individual borrowers from a major Brazilian bank.

The empirical results indicated that the probability of default is sensitive
to macroeconomic variables and specific characteristics of both contracts and borrowers. The theoretical finding on the opposite effects coming from distinct interest rates to the probability of default was confirmed by the data. The intuition is that a decrease in the basic real interest rate, implied by an expansionist monetary policy, leads banks to easy credit history analysis of borrowers and assume more credit risk. Banks will try to compensate for financial losses due to a lower basic real interest rate by expanding credit operations. This strategy will bring to the financial market borrowers with higher probability of default. On the other hand, a higher loan interest rate increases the probability of default because it reduces the borrower’s capacity of debt repayment.

This evidence is confirmed by the recent developments of the international financial crisis, which battered the US economy late 2007 and spread worldwide. Despite successive decreases in the federal funds rate, it has been observed an upward trend in the default rate within the US economy. This phenomenon has also been observed in other countries affected by the financial turmoil. The theoretical and empirical results here reported suggest that inadvertently expanding domestic credit under an easing monetary policy that lowers basic interest rates to face the financial crisis might lead to undesirable increasing the individual’s probability of default. This side effect shall be accounted for when addressing economic costs of excessively expansionist monetary policies.

References


**Appendix A**

**Proof of Theorem 1:** From the first-order conditions we have:

\[
(u^h)'(x_o^h) = \sum_s p_s (v^h)'(x_s^h) t_s \frac{r_s}{\pi} \tag{A.1}
\]
in relation to $\varphi > 0$

$$(u^h)'(x_o^h) = \sum_s \lambda_s \frac{r_s}{\pi} \quad (A.2)$$

in relation to $d_s \in [0, r_s \varphi[$

$$p_s(v^h)'(x_s^h) = \lambda_s \quad (A.3)$$

in relation to $b$

$$(u^h)'(x_o^h) = \frac{1 + r}{q} \sum_s p_s(v^h)'(x_s^h) \quad (A.4)$$

Equalizing (A.1) and (A.4) one has

$$\sum_s p_s(v^h)'(x_s^h) t_s \frac{r_s}{\pi} = \frac{1 + r}{q} \sum_s p_s(v^h)'(x_s^h) \quad (A.5)$$

Substituting (A.3) into (A.5) we have

$$\sum_s \lambda_s t_s \frac{r_s}{\pi} = \frac{1 + r}{q} \sum_s \lambda_s \quad (A.6)$$

Define the return rate of the risk asset, and the return rate of the risk-free asset respectively to be

$$R_s = \frac{r_s}{\pi} \text{ and } R = \frac{1 + r}{q} \quad (A.7)$$

Substituting (A.7) into (A.6) and assuming equal penalties in each state of nature, (A.6) becomes

$$\sum_s t_s R_s = SR \quad (A.8)$$

Define the function $F : [0, 1]^S \times R^S \times R \to R$ to be

$$F((t_s), (R_s), R) = \sum_s t_s R_s - SR \quad (A.9)$$

Differentiating (A.9) in relation to $t_s$ one has

$$\frac{\partial F}{\partial t_s} = R_s > 0$$
Therefore we can apply the Implicity Function Theorem, since (A.8) is always satisfied, to obtain
\[
\frac{\partial t_s}{\partial R_s} = -\frac{\partial F}{\partial R_s} = -\frac{t_s}{R_s}, \ s \in S \quad (A.10)
\]
\[
\frac{\partial t_s}{\partial R} = -\frac{\partial F}{\partial t_s} = -\frac{S}{R_s} \quad (A.11)
\]

If we define the default rate as being 1 minus the market payment rate we have
\[
k_s = 1 - t_s, \ s \in S
\]

Thus from (A.10) and (A.11) one has
\[
\frac{\partial k_s}{\partial R_s} \geq 0 \text{ and } \frac{\partial k_s}{\partial R} < 0,
\]
so that Theorem 1 follows.
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<th>Delay (in days)</th>
<th>0 to 29</th>
<th>30 to 59</th>
<th>60 to 89</th>
<th>90 to 119</th>
<th>120 to 149</th>
<th>150 to 179</th>
<th>180 to 209</th>
<th>210 to 239</th>
<th>240 to 269</th>
<th>270 and more</th>
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Table 3 - Estimated Cox regression for the probability of default

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<th>Variable</th>
<th>Coefficient</th>
<th>P-value</th>
<th>Risk ratio</th>
<th>Effect on Y</th>
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Note: the dependent variable (Y) is the probability of default.