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Asymmetric Price and Wage Rigidity in Brazil: Estimation of a DSGE Model via Particle Filter

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Abstract

The aim of this paper is to check whether price and wage rigidity are asymmetric in the Brazilian economy, i.e., whether economic agents adjust them downward or upward. In addition, the effects of monetary and fiscal policies on the dynamics of the economy are assessed. To do that, a nonlinear dynamic stochastic general equilibrium (DSGE) model with asymmetry in price and wage adjustment costs is used, following Aruoba, Bocola, and Schorfheide (2013). This model can produce downward (or upward) price or wage rigidity, which could give rise to strong nonlinearities. Therefore, the model is solved using a nonlinear method and its parameters are estimated by a particle filter. Results indicate that both nominal prices and wages are stickier downward and asymmetric rigidity has an impact on the dynamics of the economy whenever monetary and fiscal policy shocks are present. If the Central Bank implements the monetary policy without considering the effects of asymmetric rigidity, the policy will be suboptimal.

Keywords: DSGE. Particle filter. Monetary and fiscal policy. Nonlinear solution method. Asymmetric adjustment costs.

JEL codes: C11, C15, E52, E62

1. Introduction

The so-called Lucas (1976) critique was a milestone in the improvement of modeling and analysis of macroeconomic models in the 1970s. The assessment of economic policies through econometric models might be detrimental as the parameters of these models are not structural, i.e., they are not invariant to policy regime shifts. In other words, the relationships between aggregate variables would tend to change whenever macroeconomic policies were altered. Thus, in response to the Lucas critique, several macroeconomic models were built upon microfoundations (DSGE).

In the past 20 years, huge improvements have been made in macroeconomic modeling and DSGE models have gained momentum both among scholars and economists. Many central banks around the globe have developed their own DSGE models to assess economic policies and macro variables movements. Given the growing importance of these models for quantitative analyses, their estimation has become increasingly popular. The usual method to estimate the structural parameters of these models consists of Bayesian techniques and likelihood-based inference in order to take the information from the dataset to the model economy, as pointed out by An & Schorfheide (2007). First, the DSGE model is submitted to some solution method and represented in state space and, then, filtering methods are used to estimate likelihood. However, most dynamic models do not have a likelihood function that can be calculated analytically or numerically.

As a way to circumvent this problem, most papers on dynamic economies focus on log-linearized equilibrium conditions, eventually turning the Kalman filter into an essential tool for the simulation of unobservable variables and for the estimation of likelihood of models that describe the behavior of the economy of interest, allowing parameters to be estimated by Bayesian techniques. Nonetheless, the Kalman filter is somewhat restrictive –

with linear state-space representation and Gaussian perturbations – thus limiting the analysis of nonlinear phenomena likely to be observed in the data. Fasolo (2012) mentions some examples of nonlinearities usually observed in the data: the influence of risk aversion and of precautionary saving on aggregate variables, such as consumption and investment; the so-called "fat-tails" often seen in economic shocks and fiscal and monetary policy regime shifts;[1] all of which cannot be properly described by a linear model. Although the economics literature has not reached a consensus about the advantages of the estimation of nonlinear approximations in DSGE models, Fernández-Villaverde, Rubio-Ramírez & Santos (2006) theoretically demonstrate that nonlinear approximations in the DSGE model investigated by them led to a more accurate estimation of the "peak" of the likelihood function, which would consequently have some impact on the estimation of structural parameters of the economy at issue. The authors point out that second-order errors in policy functions cause first-order errors in the likelihood function that arises from the process, which could consequently produce disastrous results for linear estimators, as first-order errors in the likelihood function yield biased parameter estimates. Moreover, the authors show that errors in the approximate likelihood function would build up with the increase in sample size. In other words, the approximation errors associated with the linear representation of DSGE models may lead to significant errors in the corresponding likelihood functions[2] and, therefore, as a result, the approximation of likelihood functions based on a model solved via a linear solution method, may differ from an exact likelihood estimation.

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[1]There are two examples of a monetary policy regime shift in the Brazilian economy: the first one refers to when Gustavo Franco, the then-president of the Central Bank of Brazil, was replaced with Armínio Fraga in 1999, who took over and had his mind set on adopting the inflation-targeting regime, which he eventually did in July 1999, under Resolution 2.615, issued by the National Monetary Council (Giambiagi & Villela, 2005). The second case was when Alexandre Tombini – the current president of the Central Bank – replaced Henrique Meirelles in 2011, adopting a more tolerant stance on inflation, reducing the Selic rate to its historical minimum of 7.25 pp and keeping it at low levels even when inflation is above the midpoint of the target range (4.5%).

[2] Alves (2011) performs a simulation using a comprehensive set of artificial data based on a nonlinear solution of a DSGE model and estimates the structural parameters via log-linear approximations around steady state conditions. The author finds a significant bias in Calvo (1983) stickiness parameter estimates – price rigidity and elasticity of labor supply.

By taking these problems into account and attempting to solve them, Fernández-Villaverde & Rubio-Ramírez (2005) propose the use of a particle filter to estimate the likelihood of the neoclassical growth model and gather empirical evidence about the superiority of nonlinear DSGE estimators to linear ones. However, the empirical evidence provided by the authors cannot be generalized, as it is valid only for the simple model used. Therefore, to put forward stronger arguments in favor of nonlinear methods, Villaverde & Rubio-Ramírez (2007) estimate an extended version of the neoclassical growth model and add essential nonlinearities using the argument that the shocks that underlie the model are subject to stochastic volatility, and thus linear approximations would eventually cancel out the effects of these shocks, rendering this type of approximation inefficient, consequently making nonlinear approximations grow in popularity.

Accordingly, the use of likelihood-based inference becomes important for some reasons. First, following Monfort (1996), from an empirical perspective, this type of inference is a simple way to deal with misspecified models, which is the case of dynamic steady-state economies, which are false by construction, making likelihood-based inference attractive due to its asymptotic properties and to its good behavior in small samples, even when models are misspecified, as argued by Fernández-Villaverde & Rubio-Ramírez (2005). From a theoretical viewpoint, it is assumed that any empirical evidence obtained from data should be included in the likelihood function, as highlighted by Berger & Wolpert (1988). Given this background information, it seems plausible to say that the closer researchers get to the likelihood of the analyzed model, that is, the closer they get to the actual likelihood, the more they will be able to extract all the necessary and available information from the data. Therefore, the appropriate selection of the approximation method is crucial to researchers, as this will enormously influence the likelihood function of the model to be studied.

First of all, after having a look at the previous arguments, the selection of a nonlinear approximation method seems to be highly recommendable to solve a dynamic steady-state model. Notwithstanding, one should bear in mind that models which do not include some kind of nonlinearity and are submitted to nonlinear approximation methods are conducive to a steady state that is quite similar to the that found in linear estimations. Hence, the use of nonlinear solution methods is only useful when the model presents some kind of nonlinearity.[3] In the present paper, the use of nonlinear solution methods is necessary because of the model used. The focus is on the estimation of a new Keynesian model with asymmetric price and wage adjustment costs, following Kim & Ruge-Murcia (2009) and Aruoba, Bocola & Shorfheide (2013). This model can produce downward (or upward) price and wage rigidity. By allowing asymmetry in adjustment costs, the economic agents' decision-making rules may become strongly nonlinear. The key feature of the model is the introduction of asymmetry in price and wage adjustment costs. In other words, besides taking into account price and wage rigidity, the model also adds asymmetry in rigidity. So, depending on the sign and size of the parameters associated with asymmetry, prices and wages in the Brazilian economy can be stickier downward or upward. These asymmetries may be caused by the Brazilian labor market framework, due to the high percentage of informal jobs and turnover, as well as to the large number of military jobs and civil servants, among whom wages are stickier. Moreover, these asymmetries may influence prices at the firm level, since most of the costs Brazilian firms have to cover are associated with the payment of their employees' wages; in addition,

[3] According to Aruoba, Bocola & Shorfheide (2013), there are two types of nonlinearities that may be seen in nonlinear DSGE models. The first ones are known as approximately smooth nonlinearities, in which decision rules contain slopes and, possibly, asymmetries as those which are generated by cost functions or asymmetric loss functions. The other type refers to "kinks" in decision rules, such as those generated by the zero lower bound in nominal interest rates. Also, the authors mention that nonlinear characteristics may be endogenous or exogenous. Slopes in utility functions, in adjustment cost functions, and in production functions may endogenously give rise to nonlinear decision rules households and firms abide by. On the other hand, an example of an exogenous linearity is the stochastic volatility to which exogenous shocks are subjected, which causes business cycle movements.

(downward or upward) rigidity is also expected to influence the prices of goods, which may be even stickier (either upward or downward) than wages, thereby causing more harmful inflationary effects on the economy as a whole. Thus, in a more rigid economy where prices and wages are cut, expansionary monetary policy shocks, by means of a decrease in interests or of fiscal shocks, such as tax reduction or increase in government spending, could force the economy into longer periods of high inflation, bringing about economic imbalance, which would take longer to be overcome.

All that being said, the present paper seeks to answer the following questions: is price and wage rigidity in the Brazilian economy asymmetric, i.e., do economic agents act nonlinearly, being more reluctant to adjust prices and wages downward instead of upward? If so, how do these asymmetries eventually influence economic behavior when the economy faces a temporary monetary and fiscal policy shock? So, the present paper aims to estimate the structural parameters of Brazilian economy – especially those which have to do with asymmetric rigidity - and to assess the behavior of the major macroeconomic variables towards monetary and fiscal policy movements. A DSGE model, proposed by Kim & Ruge-Murcia (2009) and extended by Aruoba, Bocola & Shorfheide (2013), is then used with asymmetric price and wage adjustment costs, which could lead to nonlinearities in economic agents' behaviors. The model is solved using perturbation methods and is estimated with Brazilian data via a particle filter, in order to construct the likelihood. It is important to stress that the particle filter can be used to estimate models other than the nonlinear ones. Empirically, it proved to be superior to the Kalman filter in DSGE models. Several authors reported superior performance of their estimates made from the nonlinear model estimated by sequential Monte Carlo methods. Among these authors are An & Schorfheide (2007), Fernández-Villaverde & Rubio-Ramírez (2007) and Amisano & Tristani (2010), who compare their findings using a model solved by a nonlinear method, and hence estimated by a

particle filter, and the same model in linearized form estimated by the Kalman filter. Even in the work of Fernández-Villaverde & Rubio-Ramírez (2005), in which the authors estimate a quasilinear model (neoclassical growth model), the particle filter outperformed the Kalman filter.

The paper is organized into four sections, in addition to the introduction. Section 2 introduces the nonlinear DSGE model with asymmetric price and wage adjustment costs and provides a brief analysis of the solution method used. Section 3 describes the estimation method, which focuses on the particle filter. Section 4 presents and comments on the results of the empirical analysis. Section 5 concludes.

2. The Theoretical Model

2.1. Evolution of DSGE Models and the New Keynesian Approach

The attempt to understand and analyze how fluctuations occur in macroeconomic variables – such as output, inflation, unemployment, among others – prompted many economic researchers to develop several models, from the mid-20th century onwards, to explain these phenomena. Improvements in macroeconomic theory, along with the development of econometric techniques, allowed constructing more robust models based on microeconomic foundations, with good performance in the adjustment of data and forecasts. These models are known as DSGE (dynamic stochastic general equilibrium). “Dynamic” because time matters, i.e., the past influences the present, and the future (through expectations) also influences the present. “Stochastic” because there are structural shocks, which are accountable for cycles and fluctuations in several macroeconomic variables. And finally, “General Equilibrium” because funds in the economy are allocated via markets, i.e., there are several interdependent markets that interact with each other in a given time period.

Kydland & Prescott (1982) provided the necessary tools to assess the behavior of economic movements. These authors built a model assuming a perfectly competitive market where utility maximizing agents would be subject to budget and technological constraints. According to Romer (2012), what these real business cycle models seek is the construction of a microfounded general equilibrium model and the specification of the shocks observed in the main macroeconomic movements. In addition, according to Rebelo (2005), there were three revolutionary ideas in the seminal work published in 1982. The first one, based on the previous work of Lucas & Prescott (1971), claims that business cycles can be analyzed by dynamic general equilibrium models. The second one shows the possibility to combine business cycles with the growth theory, highlighting that business cycles must be consistent with empirical regularities in long-term growth. Finally, the third idea proposes going way beyond the qualitative comparison of the model's properties with stylized facts that were predominant in theoretical studies until 1982.

The main advantage of the real business cycle model is that it is not subject to Sims[4] and Lucas[5] critiques, as these models are hinged upon microeconomic foundations. In other words, the restrictions imposed on the variables would not occur on an ad hoc basis, since they would be based on the description of behaviors of consumers, firms, and government. Furthermore, the parameters of these models are deemed to be structural, as they remain unchanged when faced with monetary policy regime shifts since they are closely related to preferences and technology, which tend to be stable in the short and medium run. On the other hand, the downside of the model is that it is not checked in practice due to the lack of a

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[4] Sims (1980) argues that the restrictions imposed on structured models aimed at making them identifiable were too strong and then undermined the efficiency and quality of forecasts. The author introduced the vector autoregressive (VAR) model, which was not theoretical and did not require that structured models be identifiable.

[5] Lucas (1976) argues that the use of econometric models aimed at the formulation of economic policies could be harmful, as the parameters of these models would not be structural; i.e., they would not be invariant to the economic policy and would thus be susceptible to variations whenever changes occurred in the economic scenario.

monetary sector. This sector is not necessary because the presence of perfect competition and totally flexible prices render money superneutral, not affecting real economic variables, i.e., the monetary policy in this type of model is irrelevant. However, in empirical studies as those of Christiano, Eichenbaum & Evans (1999), superneutrality was not observed in real data, prompting criticisms against real business cycles for their lack of empirical evidence.

Owing to the harsh criticisms against real business cycles regarding empirical evidence that the forecasts made by the model about monetary disturbances were unrealistic, some assumptions had to be reconsidered. According to Galí (2008), departing from the real business cycle, the change in some assumptions about the original model and the introduction of some frictions gave rise to the new Keynesian model, which contemplates a monopolistic competition across firms and the introduction of nominal rigidity – implying nonneutrality of money in the short run. Most of current DSGE models assume that prices and/or wages are sticky in nominal terms, i.e., they are not adjusted perfectly in each period. The two pricing mechanisms most widely used in the macroeconomic literature are those by Calvo (1983) and by Rotemberg (1982). The former one assumes that prices are gradually adjusted at random intervals and that this sort of rigidity is investigated in studies like those of Christiano, Eichenbaum & Evans (2005) and Smets & Wouters (2004). In the latter mechanism, proposed by Rotemberg, prices are adjusted more slowly than what would be ideal, but identically by all firms, implying convex costs. This type of rigidity was used by Schmitt-Grohé & Uribe (2004) and by An & Schorfheide (2007).

2.2. DSGE Model with Asymmetric Price and Wage Adjustment Costs

The model dealt with herein consists of a single final good firm and of a continuum of intermediate good producing firms used as input for the former. Moreover, a representative household that maximizes its intertemporal utility, subject to budget constraint, is included.

Finally, in the last economic sector, there is a monetary and fiscal authority. The model is based on Kim & Ruge-Murcia (2009) and Aruoba, Bocola & Shorfheide (2013), in which the authors replace the adjustment cost functions proposed by Rotemberg (1982) with “linex” adjustment cost functions, which can capture downward (or upward) price and wage rigidity. The model leaves capital accumulation aside and deals with a closed economy, for the sake of simplicity. In the present paper, the aim of using these asymmetric adjustment cost functions is to try to describe the behavior of price setters in a more realistic fashion, thus seeking to elucidate wage and price movements. Apparently, both methods proposed by Calvo (1983) and Rotemberg (1982) to produce wage and price rigidity do not lead to a complete rigidity mechanism. These methods regard rigidity as symmetrical, i.e., they do not take into consideration that economic agents might cause prices to be stickier downward than upward, resulting in asymmetric rigidity. In other words, it seems plausible to believe that employers are much stricter about reducing wages than about raising them; additionally, in the case of firms, they are stricter about reducing the prices of their goods than about increasing them, especially in a monopolistically competitive market, where firms exercise some monopoly. The use of these functions is also justified by the empirical evidence that their introduction produce nonlinearities in the DSGE model that can explain the nonlinearities observed in U.S. data, as shown by Aruoba, Bocola & Shorfheide (2013). Nevertheless, the authors underscore that the nonlinear dynamics observed in inflation and in wages do not produce nonlinearities in GDP growth or in the interest rate. So, the nonlinearities seen in the dynamics of wages and prices do not spread explicitly across the other U.S. economic variables. The equations for the model proposed by Aruoba, Bocola & Shorfheide (2013) are presented in what follows.

2.2.1 Firms

In the proposed model, a country produces a single final good and a set of intermediate goods indexed by $j \in [0,1]$. The firms that manufacture final goods are perfectly competitive and the good is consumed by households. In their turn, intermediate good firms manufacture differentiated goods in a monopolistically competitive market.

Final good firms are perfectly competitive and combine intermediate goods indexed by $j \in [0,1]$ using the following production function:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\lambda_{p,t}} dj\right)^{\frac{1}{1-\lambda_{p,t}}}. \tag{1}$$

Note that $1/\lambda_{p,t} > 1$ represents elasticity of demand for each intermediate good that embodies the technology. Given that final good firms are in a perfectly competitive market, they maximize their profits according to production function (1) using the prices of all intermediate goods $P_t(j)$ and the price of final goods P_t. The maximization problem is then expressed as:

$$max_{Y_t(j)} P_t Y_t - \int_0^1 P_t(j) Y_t(j)\, dj. \tag{2}$$

By solving the firm's problem, one obtains the input demand function, which is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-1/\lambda_{p,t}} Y_t. \tag{3}$$

In equation (3), the demand for intermediate goods depends on price $P_t\ (j)$ of the intermediate good relative to price P_t of the final good. By using aggregate demand Y_t together with the

zero-profit condition of firm $P_t Y_t = \int_0^1 P_t(j) Y_t(j)\, dj$, it is possible to establish the relationship between the prices of intermediate and final goods:

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\lambda_{p,t-1}}{\lambda_{p,t}}} dj \right)^{\frac{\lambda_{p,t}}{\lambda_{p,t-1}}}. \tag{4}$$

Each intermediate good j is produced by a firm that competes in a monopolistic market and that uses labor force services H_t, through the following constant-returns-to-scale technology:

$$Y_t(j) = A_t H_t(j), \tag{5}$$

where A_t is an exogenous productivity process, common to all intermediate good firms. These firms rent labor force services $H_t\ (j)$ for nominal price W_t. Since the goods produced by each firm are differentiated, the firms enjoy some power in the monopolistically competitive market; thus, prices are a variable of choice. However, the adjustment of nominal prices by the firms is deemed costly. In other words, these firms face nominal rigidity in terms of price adjustment costs. These costs, expressed as a fraction of firms' revenues (see equation (7)), take the shape of a linex function (introduced by the first time by Varian, 1974) and are defined as follows:

$$\Phi_p \left(\frac{P_t(j)}{P_{t-1}(j)} \right) = \varphi_p \left[\frac{exp\left(-\Psi_p \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi \right) \right) + \Psi_p \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi \right) - 1}{\Psi_p^2} \right], \tag{6}$$

where π is the steady-state inflation rate associated with the final good. Parameter φ_p defines the level of price rigidity, whereas parameter Ψ_p controls for the asymmetric adjustment cost,

i.e., asymmetric price rigidity. Therefore, based on nominal wages, the price of final goods, the demand for intermediate goods, and technological constraint, each firm j chooses its labor input $H_t\ (j)$ and its price $P_t(j)$ by maximizing the current value of future profits:

$$E_t\left\{\sum_{s=0}^{\infty}\beta^s Q_{t+s|t}\left[\frac{P_{t+s}(j)}{P_{t+s}}\left(1-\phi_p\left(\frac{P_{t+s}(j)}{P_{t+s-1}}\right)\right)Y_{t+s}(j)-\frac{1}{P_{t+s}}W_{t+s}H_{t+s}(j)\right]\right\}. \tag{7}$$

Here, $Q_{t+s|t}$ is the value at t of a consumption unit at $t+s$ for the household, which is held to be exogenous by the firm.

Hence, firm j chooses its labor input $H_t(j)$ and price $P_t(j)$ in order to maximize the current value of future profits. By replacing $Y_t(j)$ with the production function given by (5) in the maximization equation for the current value of future profits given by (7), the objective function of the intermediate firm is written as:

$$E_t\left\{\sum_{s=0}^{\infty}\beta^s Q_{t+s|t}\left[\frac{P_{t+s}(j)}{P_{t+s}}\left(1-\phi_p\left(\frac{P_{t+s}(j)}{P_{t+s-1}}\right)\right)A_{t+s}H_{t+s}(j)-\frac{1}{P_{t+s}}W_{t+s}H_{t+s}(j)\right]\right\}. \tag{8}$$

This objective function is then maximized relative to $H_t(j)$ and $P_t(j)$ subject to:

$$A_{t+s}H_{t+s}(j)=\left(\frac{P_t(j)}{P_t}\right)^{-1/\lambda_{p,t}}Y_{t+s}. \tag{9}$$

Note that $\mu_{t+s}\beta^s Q_{t+s|t}$ is used to denote the Lagrange multiplier associated with the constraint above. By letting $Q_{t|t}=1$, the first-order condition relative to $H_t(j)$ and $P_t(j)$ is given, respectively, by:

$$\frac{W_t}{P_t} = \frac{P_t(j)}{P_t}\left[1 - \phi_p\left(\frac{P_t(j)}{P_{t-1}(j)}\right)\right]A_t - \mu_t A_t. \tag{10}$$

$$\frac{1}{P_t}\left[1 - \phi_p\left(\frac{P_t(j)}{P_{t-1}(j)}\right)\right]A_t H_t(j) - \frac{P_t(j)}{P_t P_{t-1}(j)}\phi_p'\left(\frac{P_t(j)}{P_{t-1}(j)}\right)A_t H_t(j) - \frac{\mu_t}{\lambda_{p,t} P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\frac{1}{\lambda_{p,t}}-1} Y_t +$$

$$\beta E_t\left\{Q_{t+1|t}\frac{P_{t+1}^2(j)}{P_{t+1}P_t^2(j)}\phi_p'\left(\frac{P_{t+1}(j)}{P_t(j)}\right)A_{t+1}H_{t+1}(j)\right\} = 0. \tag{11}$$

2.2.2. Labor Input

The labor force services used by intermediate good firms are supplied by a group of perfectly competitive trade unions. The union aggregates the different types of services supplied by the households according to the following technology:

$$H_t = \left(\int_0^1 H_t(k)^{1-\lambda_w} dk\right)^{\frac{1}{1-\lambda_w}}. \tag{12}$$

The union chooses the demand for each type of labor in order to maximize its profit, based on $W_t(k)$ and W_t. So, optimal labor demand is given by:

$$H_t(k) = \left(\frac{W_t(k)}{W_t}\right)^{\frac{-1}{\lambda_w}} H_t. \tag{13}$$

As the labor market is perfectly competitive, labor cost W_t and the nominal wages paid to workers are expressed as follows:

$$W_t = \left(\int_0^1 W_t(k)^{\frac{\lambda_w - 1}{\lambda_w}} dk\right)^{\frac{\lambda_w}{\lambda_w - 1}}. \tag{14}$$

2.2.3. Households

In the model, each household contains a continuum of members indexed by k. Household members self-insure, thus equating their marginal utilities in each time period. A member of household k is given utility by consumption $C_t(k)$ relative to a consumption habit. It is assumed that this habit is given by a level of technology A_t. This assumption guarantees that the economy dealt with here evolves along a steady-state growth path. In addition, the household member derives disutility from hours worked $H_t(k)$. Thus, households maximize their intertemporal utility according to the following function:

$$E_t\left\{\sum_{s=0}^{\infty}\beta^s\left[\frac{(c_{t+s}(k)/A_{t+s})^{1-\tau}-1}{1-\tau}-\chi_H\frac{H_{t+s}^{1+1/v}(k)}{1+1/v}\right]\right\}, \tag{15}$$

where β is the intertemporal discount factor, $1/\tau$ is the intertemporal elasticity of substitution, or risk aversion coefficient. Moreover, χ_H is the scale factor that determines hours worked in the steady state and, finally, v stands for the Frisch elasticity of labor supply.

The household member is monopolistic in his/her supply of labor force. As a monopolist, he/she chooses the nominal wage, taking the unions' demand as given. Market frictions are assumed to induce an adjustment in nominal wage costs. These adjustment costs are paid as a fraction of labor earnings (see equation (17)) and these costs have the same structure as that assumed for prices:

$$\phi_w\left(\frac{W_t(k)}{W_{t-1}(k)}\right)=\varphi_w\left[\frac{exp\left(-\Psi_w\left(\frac{W_t(k)}{W_{t-1}(k)}-\gamma\pi\right)\right)+\Psi_w\left(\frac{W_t(k)}{W_{t-1}(k)}-\gamma\pi\right)-1}{\Psi_w^2}\right], \tag{16}$$

where $\gamma\pi$ is the growth rate of nominal wages and γ is the average growth rate for technology, which will be determined in the upcoming subsections. The functional form of price and wage adjustment cost functions is attractive because: first, the cost depends both on the sign (whether it was a raise or decrease in wage) and magnitude (whether the raise or decrease was small or large) of wage adjustment at t compared to $t-1$. Consider, for example, the case where $\Psi_w > 0$. As wages at t are raised compared to $t-1$, the linear term in the function prevails and, consequently, the costs associated with wage increase linearly. On the other hand, as wage at t decreases, being lower than the wage at $t-1$, it is the exponential term that prevails and, thus, the cost associated with wage reduction increases exponentially. As a result, nominal wage reductions incur in higher cost than do wage increases, even in cases in which both magnitudes of change are identical. The opposite is true if $\Psi_w < 0$. Second, this function "nests" the quadratic form observed in the special case where $\Psi_w \to 0$[6] (i.e., the quadratic cost function proposed by Rotemberg, 1982) and wage adjustment costs are symmetric. This allows the comparison between the model with asymmetric costs and the one with quadratic costs. Third, the "linex" function is differentiable at all points and strictly convex for every $\varphi_w > 0$. Finally, this function does not oppose to nominal wage cuts which, albeit relatively rare, are very likely.

In addition, household members have a trade-off between consumption and savings. They have access to the domestic securities market, where government nominal bonds $B_t\ (k)$ are paid at an interest rate R_t. Moreover, they receive real profits $D_t(k)$ from the firms, which they own and, finally, they have to pay lump-sum taxes T_t in order for the fiscal authority to finance their expenditures. Therefore, the households' intertemporal budget constraint takes the following form:

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[6]To verify that, determine the limit $\phi(.)$ according to $\Psi \to 0$ by applying the l'Hôpital rule twice.

$$P_t C_t(k) + B_t(k) + T_t = W_t(k) H_t(k) \left(1 - \phi_w \left(\frac{W_t(k)}{W_{t-1}(k)}\right)\right) + R_{t-1} B_{t-1}(k) + P_t D_t(k) + P_t SC_t, \quad (17)$$

where SC_t is the net flow the household gets from a set of the state of contingency. The Lagrange multiplier associated with the budget constraint is denoted by λ_t. The usual transversality condition on asset accumulation is applied, which rules out Ponzi schemes.

Therefore, in the case of households in this model, the first-order condition related to intertemporal utility maximization (15) through consumption subject to the budget constraint in equation (17) is given by:

$$P_t \lambda_t = \left(\frac{C_t(k)}{A_t}\right)^{-\tau} \frac{1}{A_t}. \quad (18)$$

In addition, let:

$$Q_{t+1|t} = \frac{P_{t+1}\lambda_{t+1}}{P_t \lambda_t}. \quad (19)$$

Thus, by using the definition above, the first-order condition for investments in government bonds becomes:

$$1 = \beta E_t \left\{ Q_{t+1|t} \frac{R_t}{\pi_{t+1}} \right\}. \quad (20)$$

As to the labor market, each member k of household is a monopolistic competitor in relation to his/her choice of wage. By substituting the optimal labor demand given by equation (13)

into the intertemporal utility function given by (15) and into budget constraint at $t+s$ given by (17), we obtain the following expressions:

$$E_t\left\{\sum_{s=0}^{\infty}\beta^s\left[\frac{(c_{t+s}(k)/A_{t+s})^{1-\tau}-1}{1-\tau}-\chi_H\frac{1}{1+\frac{1}{\nu}}\left(\frac{W_{t+s}(k)}{W_{t+s}}\right)^{-\frac{\left(1+\frac{1}{\nu}\right)}{\lambda_w}}H_t^{1+\frac{1}{\nu}}\right]\right\}. \tag{21}$$

$$P_{t+s}C_{t+s}(k)+B_{t+s}(k)+T_{t+s}=W_{t+s}(k)\left(\frac{W_{t+s}(k)}{W_{t+s}}\right)^{-\frac{1}{\lambda_w}}H_{t+s}(k)\left(1-\phi_w\left(\frac{W_{t+s}(k)}{W_{t+s-1}(k)}\right)\right)+$$

$$R_{t+s-1}B_{t+s-1}(k)+P_{t+s}D_{t+s}(k)+P_{t+s}SC_{t+s}. \tag{22}$$

Based on these expressions and taking the demand for services related to aggregate labor H_{t+s} as given, the first-order condition for the maximization of (21) subject to (22) in relation to the following expression $W_t(k)$ is:

$$\frac{\chi_H}{\lambda_w W_t}\left(\frac{W_t(k)}{W_t}\right)^{-\frac{\left(1+\frac{1}{\nu}\right)}{\lambda_w}-1}H_t^{1+\frac{1}{\nu}}+\lambda_t\left(\frac{W_t(k)}{W_t}\right)^{-\frac{1}{\lambda_w}}H_t\left(1-\phi_w\left(\frac{W_t(k)}{W_{t-1}(k)}\right)\right)-$$

$$\frac{\lambda_t}{\lambda_w}\frac{W_t(k)}{W_t}\left(\frac{W_t(k)}{W_t}\right)^{-\frac{1}{\lambda_w}-1}H_t\left(1-\phi_w\left(\frac{W_t(k)}{W_{t-1}(k)}\right)\right)-\lambda_t\frac{W_t(k)}{W_{t-1}(k)}\left(\frac{W_t(k)}{W_t}\right)^{-\frac{1}{\lambda_w}}H_t\phi_w'\left(\frac{W_t(k)}{W_{t-1}(k)}\right)+$$

$$\beta E_t\left\{\lambda_{t+1}\frac{W_{t+1}^2(k)}{W_t^2(k)}\left(\frac{W_{t+1}(k)}{W_{t+1}}\right)^{-\frac{1}{\lambda_w}}H_{t+1}\phi_w'\left(\frac{W_{t+1}(k)}{W_t(k)}\right)\right\}=0. \tag{23}$$

2.2.4. Government: Monetary and Fiscal Authority

The monetary authority uses an inflation-targeting regime and determines the interest rate based on a Taylor rule with interest rate smoothing. According to the specification of the model, the Central Bank reacts when inflation deviates from its target and when output

growth deviates from its steady-state equilibrium. Therefore, the monetary policy rule adopted by the Central Bank is given as follows:

$$R_t = R_{t-1}^{\rho_R}\left[r\pi^*\left(\frac{\pi_t}{\pi^*}\right)^{\Psi_1}\left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\Psi_2}\right]^{1-\rho_R} exp(\varepsilon_{R,t}), \tag{24}$$

where r is the steady-state real interest rate, π_t is the inflation rate defined as $\pi_t = P_t/P_{t-1}$ and π^* is the inflation target, which coincides with the equilibrium steady-state interest rate.

On the other hand, the fiscal authority consumes a fraction of aggregate output Y_t, where $\varsigma_t \in [0,1]$ following an exogenous process. The government applies a fixed tax to finance any deficit in its revenue. The government's budget constraint is given by:

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t, \tag{25}$$

where $G_t = \varsigma_t Y_t$.

2.2.5. Exogenous Processes

The model is disturbed by four exogenous processes. Aggregate productivity evolves according to:

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln a_t, \tag{26}$$

where $\ln a_t = \rho_a \ln a_{t-1} + \varepsilon_{a,t}$. Then, on average, technology grows at rate γ and a_t captures the exogenous movements of the normal technology growth rate. One defines $g_t = 1/(1-\varsigma_t)$. It is assumed that:

$$\ln g_t = \left(1 - \rho_g\right) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t}. \tag{27}$$

The inverse elasticity of demand for intermediate goods evolves according to a logged first-order autoregressive process:

$$\ln \lambda_{p,t} = \left(1 - \rho_p\right) \ln \lambda_{p,t-1} + \varepsilon_{p,t}. \tag{28}$$

Finally, the monetary policy shock $\varepsilon_{R,t}$ is assumed to be serially uncorrelated. All these four processes are independent and normally distributed with zero mean and standard deviation σ_z, σ_g, σ_p and σ_R.

2.3. Solution of the Model

The nonlinear nature often associated with DSGE models does not allow for their closed analytical solution and, consequently, this nonlinearity implies a likelihood function[7] that cannot be calculated analytically or numerically. To solve this problem, most of the literature on dynamic economies has focused on approximate likelihood obtained from the log-linearized version of the original model. When this approach is used, it is possible to use the Kalman filter for constructing the likelihood function and its estimation. However, linearization depends both on the accurate approximation of the model by a linear relationship and on the assumption that economic shocks are normally distributed; but both hypotheses are problematic. Firstly, the impact of linearization is stronger than it looks like. Fernández-Villaverde, Rubio-Ramírez & Santos (2006) demonstrate that second-order errors in policy

[7]Recall that, as highlighted by An & Schorfheide (2007), likelihood-based inference is a useful tool that can take dynamic steady-state models to real economic data.

functions may have first-order effects on the resulting likelihood function. Moreover, they show that likelihood errors get worse as sample size increases; i.e., small errors in policy functions accumulate at the same rate as sample size increases. Fernández-Villaverde & Rubio-Ramírez (2005), Amisano & Tristani (2010), An & Schorfheide (2007), and Andreasen (2011) use nonlinear approximations in their respective models and report that such approximations provide a more appropriate fit of the data. Secondly, the hypothesis of normal shocks hinders the investigation of models with time-varying volatility. Fernández-Villaverde & Rubio-Ramírez (2007) use a model in which shocks are subject to stochastic volatility and this induces both fundamental nonlinearities in the model and nonnormal distributions; in addition, the authors argue that the use of a linear approximation method would eliminate the effects of these shocks, not managing to explore this mechanism, requiring linear approximation for the analysis of the model proposed by them. A similar case is observed in the model proposed by Kim & Ruge-Murcia (2009) and extended by Aruoba, Bocola & Shorfheide (2013), which was used in this paper. A linearized version of the model would eliminate asymmetries in price and wage adjustment costs, which would not allow analyzing the effects on the economy caused by the introduction of these rigidity mechanisms[8]. Therefore, the use of a nonlinear approximation method to maintain the nonlinear structure of the model is advisable.

So, before directing our attention to the estimation of the model, it is necessary to first define the linear solution method to be used. As the solution method provides the policy functions that will be used to calculate likelihood, which will strongly influence the estimation, it is important to choose a method that is as accurate as possible. Nevertheless, besides the fact that the method should be accurate, it is also important that it be quick, since

[8] Aruoba, Bocola & Shorfheide (2013) demonstrated that the introduction of these asymmetries in price and wage adjustment costs in the DSGE model explain very well the nonlinearities observed in prices and wages in the U.S. economy in the sample selected by them.

the likelihood function will be calculated several times for different sets of parameter values. There is a wide range of methods for the linear approximation of a DSGE model, each of them with advantages and disadvantages, as pointed out by Judd (1998). Additionally, the nonlinear solution method can be based on local or global approximation. The perturbation method is the one which is most widely used for the solution of dynamic models, which builds a Taylor series expansion of the agents' policy functions around the steady state and a perturbation parameter. However, as argued by Aruoba, Fernández-Villaverde & Rubio-Ramírez (2006), because it is a local approximation method, it is only satisfactory around the steady state, i.e., this could be a problem in cases in which the economy is subject to large shocks or is in crisis (e.g., the 1929 crisis or the 2008 financial crisis). On the other hand, global methods are considered more robust, among which the most famous are the projection methods, in which the policy functions corresponding to the model's solution are represented as a linear combination of previously known basis functions. As examples, we have the finite element method, value function interaction, and spectral methods based on Chebyshev[9] polynomials.

In brief, the nonlinear solution method chosen for the study is a second-order perturbation method, accurate and quick, as reported by Aruoba, Fernández-Villaverde & Rubio-Ramírez (2006). In addition, Moura (2010) tests several nonlinear approximation methods using a simple real business cycle model and argues that global methods seem to be the best approximation method, but the author warns that its implementation is way too complicated and slower than perturbation methods. Hence, according to the author, perturbation methods appear to yield a good trade-off between programming, computing time,

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[9]Judd (1998), Miranda & Fackler (2002), DeJong & Dave (2007), and Aruoba, Fernández-Villaverde & Rubio-Ramírez (2006) are good references on these methods. For further reading on perturbation methods, see Judd & Guu (1997), Kim, Kim, Schumburg & Sims (2003), Schimitt-Grohé & Uribe (2004), and Gomme & Klein (2008).

and approximation accuracy. Therefore, although second-order perturbation methods do not yield the best accuracy when the nonlinearity of the model is higher, this method is seemingly the most attractive one, as it is the simplest and quickest to implement, which is important in the case of DSGE models, in which for each set of parameter values, the model must be solved as many times as necessary until it converts.

Then, following Schmitt-Grohé & Uribe (2004), the set of equilibrium conditions of most DSGE models can be denoted as:

$$E_t g(Y_{t+1}, Y_t, S_{t+1}, x_t) = 0, \tag{29}$$

where E_t is the expectation operator conditional on the information available at t. The vector of predetermined variables x_t has size $n_x \times 1$ and the vector of the variables that are not predetermined, i.e., y_t, has size $n_y \times 1$. One should define $n = n_x + n_y$. Function g maps $\mathbb{R}^{n_y} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_x}$ in $\mathbb{R}^n$. Moreover, the state vector x_t can be partitioned as:

$$x_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}. \tag{30}$$

where $x_{1,t}$ denotes the predetermined endogenous state variables and vector $x_{2,t}$ denotes the exogenous state variables, i.e., economic shocks. According to the authors, the solution in equation (29) is given as follows:

$$y_t = h_k(x_t, \sigma), \tag{31}$$

$$x_{t+1} = f_k(x_t, \sigma) + \aleph \sigma \varepsilon_{t+1}, \tag{32}$$

where h_k maps $\mathbb{R}^{n_x} \times \mathbb{R}^+$ in $\mathbb{R}^{n_y}$ and f_k maps $\mathbb{R}^{n_x} \times \mathbb{R}^+$ in $\mathbb{R}^{n_x}$. Matrix $\aleph$ is of order $n_x \times n_\varepsilon$ and is given by $\aleph = [\emptyset \quad \tilde{\aleph}]'$. The goal is then to find a second-order approximation for functions h_k and f_k around the nonstochastic steady state, $x_t = \bar{x}$ e $\sigma = 0$.[10]

3. The Nonlinear Estimation Method

Given the nonlinearity generated by the DSGE model presented in the previous section, together with a nonlinear solution method, the use of the traditional Kalman filter is no longer possible. Thus, it is necessary to use a robust statistical tool that can estimate the nonlinear DSGE model. The present study proposes the use of sequential Monte Carlo methods which, albeit harder to implement and computationally expensive, can estimate models that generate nonlinearities in the data, i.e., they allow the researcher to deal with more realistic models.

In addition, empirical studies show that nonlinear models provide more accurate estimates of structural parameters in DSGE models. Some authors reported a higher performance of nonlinear model estimations compared with that of linear models. Some examples include Fernández-Villaverde & Rubio-Ramírez (2005), who estimate a neoclassical growth model; An & Shorfheide (2007), who estimate a new Keynesian model with quadratic price adjustment costs; and Amisano & Tristani (2010), who estimate a model based on Christiano, Eichenbaum & Evans (2005). All of these authors use the traditional particle filter or some extension of it. Therefore, sequential Monte Carlo methods will be discussed herein.

[10]For a full exposition, see Schmitt-Grohé & Uribe (2004).

3.1. Bayesian Estimation

Bayesian methods are good for estimating dynamic state problems. This approach attempts to construct the state's probability density function (pdf) by taking into consideration all the information available up to the time of estimation. For a more specific problem, as is the case of linear Gaussian estimation, the pdf remains Gaussian for each filter interaction, and the Kalman filter reproduces and updates the mean and covariance of the distribution. On the other hand, for nonlinear and/or non-Gaussian problems, there is no general analytical expression (closed form) for the necessary pdf.

To compute the Bayesian state estimator, this study is based on the work by Gordon, Salmond & Smith (1993), which is concerned with the discrete-time estimation problem. The authors consider that the state vector $x_k \in \mathbb{R}^n$ moves according to the following model:[11]

$$x_{k+1} = f_k(x_k, w_k), \tag{33}$$

where $f_k: \mathbb{R}^n x \mathbb{R}^m \to \mathbb{R}^n$ is the transition function and $w_k \in \mathbb{R}^m$ is an independent white noise sequence with zero mean of past and current states. The pdf of w_k is assumed to be known. In addition, in discrete time, the measures $y_k \in \mathbb{R}^p$ become available. These measures are related to the state vector through the measurement equation given by:

$$y_k = h_k(x_k, v_k), \tag{34}$$

where $h_k: \mathbb{R}^n x \mathbb{R}^r \to \mathbb{R}^p$ is a measurement function and $v_k \in \mathbb{R}^r$ is another independent white noise sequence with zero mean of past and current states with known pdf. The initial pdf of

[11] Note that the system formed by equations (31) and (32) in the previous section can be converted to its general form, given by equations (33) and (34) in this section.

state vector $p(x_1|D_0) \equiv p(x_1)$ is assumed to be available, as well as the functional forms f_i and h_i for $i = 1, \dots, k$. The information available up to period k is the set of measures $D_k = \{y_i : i = 1, \dots, k\}$.

The objective of the Bayesian estimator is to construct the pdf of state vector x_k given the whole set of available information, i.e., $p(x_k|D_k)$. A priori, this pdf can be obtained recursively in two stages: forecasting and updating. It is supposed that the pdf $p(x_{k-1}|D_{k-1})$ necessary in period $k-1$ is available. If that is true, then it is possible to obtain the pdf a *priori* from the state in period k by using the system formed by equations (33) and (34).

$$p(x_k|D_{k-1}) = \int p(x_k|x_{k-1}) p(x_{k-1}|D_{k-1})\, dx_{k-1}. \tag{35}$$

Here, the probabilistic model of the evolution of state, $p(x_k|x_{k-1})$, which is a Markov model, is defined by the system of equations and by the known statistics of w_{k-1}.

$$p(x_k|x_{k-1}) = \int p(x_k|x_{k-1}, w_{k-1}) p(w_{k-1}|x_{k-1})\, dw_{k-1}.$$

Since, by assumption, $p(w_{k-1}|x_{k-1}) = p(w_{k-1})$, we have:

$$p(x_k|x_{k-1}) = \int \delta\big(x_k - f_{k-1}(x_{k-1}, w_{k-1})\big)\, x\, p(w_{k-1})\, dw_{k-1}, \tag{36}$$

where $\delta(.)$ is the Dirac delta function. This delta function appears because if x_{k-1} and w_{k-1} are known, then x_k is obtained from a purely deterministic relationship - as is the case of equation (33). Thus, in period k, a measure y_k becomes available and can be used to update the Bayes rule a *posteriori*:

$$p(x_k|D_k) = \frac{p(y_k|x_k)p(x_k|D_{k-1})}{p(y_k|D_{k-1})}, \tag{37}$$

where the normalized vector is given by:

$$p(y_k|D_{k-1}) = \int p(y_k|x_k)p(x_k|D_{k-1})\, dx_k. \tag{38}$$

The conditional pdf of y_k considering x_k, $p(y_k|x_k)$, is defined by the measurement equation and by the known statistics of v_k

$$p(y_k|x_k) = \int \delta\big(y_k - h_k(x_k, v_k)\big)p\,(v_k)\, dv_k. \tag{39}$$

In the updating equation, equation (37), the measure y_k is used to change the prior predicted by previous periods, and to obtain the required posterior distribution of the state.

Recurrent relationships in equations (35) and (37) are the formal solution to the recursive Bayesian estimation problem. However, Gordon, Salmond & Smith (1993) warn that an analytical solution to this problem is only possible for a relatively small and limited choice of measurement system and models, the most important of which is the Kalman filter,[12] which assumes that functional forms f_k and h_k of state x_k are linear, demanding that both w_k and v_k be additive, independent, and Gaussian with known variances. Nevertheless, as pointed out by the authors, these assumptions are not very realistic and thus are not reasonable for several applications in the real world. Therefore, it is necessary to construct a more sophisticated statistical method that admits its use in nonlinear systems, thereby allowing models to be built in a more realistic fashion.

[12] Hamilton (1989) gives a detailed description of the Kalman filter.

There exist a few alternatives to deal with this problem, including two improvements of the traditional Kalman filter, in an attempt to make the estimation of nonlinear systems better, namely the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF). According to Simon (2006), the EKF is the state estimation algorithm most widely used in nonlinear systems. Anyway, the author warns that this filter may be hard to adjust and, more often than not, yields unreliable estimates in cases of severe nonlinearity. That occurs because the filter relies on linearization to propagate state means and covariance. On the other hand, the UKF provides remarkable improvements in the accuracy of estimates when compared with the EKF; but the UKF is only an approximate nonlinear estimator. In other words, the EKF calculates the mean of a nonlinear system with first-order accuracy, whereas the UKF improves that by yielding an estimate with higher-order accuracy. Notwithstanding, Simon (2006) comments that the use of these filters simply delays the inevitable disparity that is observed when the nonlinear measurement system is too severe.

Hence, the method used herein, i.e., the particle filter (fully nonlinear state estimator), introduced in what follows, is proposed with the aim of improving the estimation of DSGE models, thus bringing these theoretical models closer to the reality of economic data.

3.2. Particle filter

The particle filter – also known as sequential importance sampling method or simply as sequential Monte Carlo method, is introduced here. It is a fully nonlinear numerical state estimator that may be used to estimate any model in state-space form. According to Simon (2006), the particle filter, or Monte Carlo filter, is a “brute force” statistical estimation method that often works well with problems that are not easily dealt with by the conventional Kalman filter, that is, highly nonlinear systems.

The particle filter was originally introduced by Metropolis & Ulam (1949), who described the mathematical treatment of physical phenomena. According to those authors, problems involving just some particles, through the analysis of ordinary differential equation systems, were investigated in classic mechanics. However, as argued by the authors, a totally different technique is necessary for the description of systems with a large number of particles, the so-called method of statistical mechanics, which focuses on the analysis of the properties of a set of particles rather than on the observation of individual particles. Note that the particle filter is an improvement of the SIS (sequential importance sampling) algorithm. It is the foundation on which the literature on the subsequent sequential Monte Carlo methods rests. According to Arulampalam *et al*. (2002), the SIS algorithm suffers from a degeneracy problem, in which, after some interactions, only one of all initial particles has a non-negligible weight and therefore all Monte Carlo estimations of the integrals are made using a sample of size 1. Conversely, the particle filter proposed by Gordon, Salmond & Smith (1993) and Kitagawa (1996) is considered to be the standard Monte Carlo method. This filter, which Arulampalam *et al*. (2002) later called generic particle filter, adds a resampling step to the algorithm, which helps reduce the degeneracy observed in the SIS algorithm. Despite its broad use in several other areas of research,[13] just recently has the particle filter begun to be used as a statistical tool to calculate likelihood in nonlinear models in economics. For instance, Kim, Shephard & Chib (1998) and Pit & Shepard (1999) applied this method to stochastic volatility models. However, pioneering research into the use of particle filters in DSGE models was conducted by Fernández-Villaverde & Rubio-Ramírez (2005). By blazing the trail for the use of particle filter to estimate macroeconomic models, other similar works were introduced into the literature on DSGE models involving sequential Monte Carlo methods: An (2007), An & Schorfheide (2007), Fernández-Villaverde & Rubio-Ramírez

[13]See Cox (1996) for an application of the particle filter to mobile robot localization.

(2007), Flury & Shephard (2011), Amisano & Tristani (2010), DeJong *et al*. (2009), Strid (2006), Andreasen (2011), Fasolo (2012), and Aruoba, Bocola & Shorfheide (2013).

Intuition and equations derived from the particle filter are based on Simon (2006). At the beginning of each estimation step, a given number N of state vectors is randomly generated based on the initial probability density function (pdf), $p(x_0)$, which is assumed to be known beforehand. These state vectors are known as particles and are denoted by $x_{0,i}^{+}(i = 1, \dots, N)$. In each period $k = 1, 2, \dots$, the particles are propagated to the next step using equation $f(.)$:

$$x_{k,i}^{-} = f_{k-1}\left(x_{k-1,i}^{+}, w_{k-1}^{i}\right)(i = 1, \dots, N), \tag{40}$$

where each error vector w_{k-1}^{i} is randomly generated based on the previously known pdf of w_{k-1}. After the measurement at k, the corresponding conditional likelihood of each particle $x_{k,i}^{-}$ is calculated, i.e., the pdf $p\left(y_k | x_{k,i}^{-}\right)$ is assessed. This can be done when the nonlinear measurement equation and the pdf of error measurement are known. For example, if an m-dimensional measurement equation is given by $y_k = h(x_k) + v_k$ with $v_k \sim N(0, R)$; then, a corresponding likelihood $q_{\ i}$ that measures it is equal to a specific measurement y^*, assuming that x_k is equal to particle $x_{k,i}^{-}$, it can be written as:

$$q_i = P\left[(y_k = y^*)\left(x_k = x_{k,i}^{-}\right)\right]$$

$$= P\left[v_k = y^* - h\left(x_{k,i}^{-}\right)\right] \sim \frac{1}{(2\pi)^{m/2}|R|^{1/2}} exp\left(\frac{-\left[y^* - h\left(x_{k,i}^{-}\right)\right]^T R^{-1}\left[y^* - h\left(x_{k,i}^{-}\right)\right]}{2}\right). \tag{41}$$

The symbol$\sim$ in the equation above means that the probability is not actually given by the right-hand side expression, but that the probability is directly proportional to what is

found on the right hand side of the equation. Thus, if this equation is used for all particles $x_{k,i}^{-}(i = 1, \dots, N)$, the corresponding likelihoods that the state is equal in each particle will hold. Now, the corresponding likelihoods obtained in equation (41) are normalized as follows:

$$q_i = \frac{q_i}{\sum_{j=1}^{N} q_j}. \tag{42}$$

This way, the sum of all likelihoods will equal 1. The following step consists in conducting a new sampling of the particles based on the estimated likelihoods. In other words, a new set of particles $x_{k,i}^{+}$ is calculated, which is generated randomly from corresponding likelihoods q_i. This can be attained in different ways. A simpler method, albeit not necessarily efficient, is shown next: For $i = 1, \dots, N$, perform the following two steps:

1. Generate a random number r that is evenly distributed on the interval $[0, 1]$.
2. Add likelihoods q_i, one at a time, until the total sum is larger than r. That is, $\sum_{m=1}^{j-1} q_m < r$; however, $\sum_{m=1}^{j} q_m \geq r$. The new particle $x_{k,i}^{+}$ is then calculated by the old particle $x_{k,i}^{-}$.

This resampling approach is formally advocated by Smith & Gelfand (1992), who show that the joint pdf of new particles $x_{k,i}^{+}$ tends towards pdf $p(x_k|y_k)$ when the number of samples N approaches ∞.

Now, one has a set of particles $x_{k,i}^{+}$, which are distributed according to the pdf $p(x_k|y_k)$. It is possible to calculate any statistical measure of this pdf. For example, to calculate the value of $E(x_k|y_k)$, its approximation can be obtained as the algebraic average of the particles:

$$E(x_k|y_k) \approx \frac{1}{N}\sum_{i=1}^{N} x_{k,i}^{+}. \tag{43}$$

Finally, the particle filter can be briefly described by the following steps:

(a) Consider the following system in state-space form:

$$\begin{aligned} x_{k+1} &= f_k(x_k, w_k) \\ y_k &= h_k(x_k, v_k), \end{aligned} \tag{44}$$

where $f_k(.)$ and $h_k(.)$ are the equations that form the nonlinear system, k is a time index, x_k is the state vector, w_k is the vector of errors of the process equation, y_k is the vector of measurement and v_k is the vector of errors of the measurement equation. In addition, w_k and v_k are independent white noise processes with known pdfs.

(b) By assuming that the pdf of the initial state $p(x_0)$ is known, N initial particles are randomly generated based on pdf $p(x_0)$. These particles are denoted by $x_{0,i}^{+}(i = 1, \dots, N)$. The parameter N is chosen by the researcher, who is faced with a trade-off between computational effort and accuracy of the estimation.

(c) For periods $k = 1, 2, \dots$ the following procedures are adopted:

a. Perform the time propagation step to obtain prior particles $x_{k,i}^{-}$ using the known process equation and the known pdf of the noise process:

$$x_{k,i}^{-} = f_{k-1}\left(x_{k-1,i}^{+}, w_{k-1}^{i}\right)(i = 1, \dots, N), \tag{45}$$

where each vector of errors w_{k-1}^{i} is generated randomly based on the known pdf of w_{k-1}.

b. Calculate likelihood q_i of each particle $x_{k,i}^{-}$ conditioned on the vector of measurement y_k. That is done by assessing the pdf $p\left(y_k | x_{k,i}^{-}\right)$ based on the nonlinear measurement equation and on the pdf of the measurement equation errors.

c. Normalize the corresponding likelihoods obtained in the previous steps as follows:

$$q_i = \frac{q_i}{\sum_{j=1}^{N} q_j}. \tag{46}$$

Now the sum of all likelihoods equals 1.

d. A set of posterior particles $x_{k,i}^{+}$ is generated based on corresponding likelihoods q_i. This is the resampling step.

Now that a set of particles $x_{k,i}^{+}$ was obtained, distributed according to pdf $p(x_k | y_k)$, the researcher can calculate the statistical measures (often mean and covariance) of this pdf.

Finally, the rationale behind the particle filter is that the particles generated from distribution $p(x_0)$ which do not contribute to characterizing the state vector in each time

period should be eliminated, leaving only those particles that have a larger weight on the distribution. Therefore, the filter includes only the particles around the relevant regions of the state space. Hence, the sampling step is the key element of the particle filter, based on a slight modification in the standard of the SIS method. In the case of the standard importance sampling method, each particle generated in the projection step would be sampled with the same probability. However, it is known from the literature on the sequential Monte Carlo method that when $t \to \infty$, there is a degeneracy problem, in which, except for a single specific particle, all the other weights converge to zero. Moreover, even the single particle with weight equal to 1 does not necessarily provide the best description of the state vector. Accordingly, the modification proposed and later used by Fernández-Villaverde & Rubio-Ramírez (2005) is of great importance to the proper operation of the particle filter.

3.3. Problems involving the particle filter and its alternatives

The literature on the sequential Monte Carlo methods underscores that the standard particle filter suffers from the so-called "sample impoverishment,"[14] which may be regarded as a milder degeneracy problem, i.e., less harmful to the method. Note that the sample of a very large number of particles will not degenerate as occurs with the SIS algorithm; however, this number will decrease for only some distinct particle values. Sample impoverishment occurs mainly when there are outliers in the data or in situations in which the measurement equation gives a lot of information about the states. Pitt & Shephard (1999) comment that the presence of outliers in the data often requires higher values for the number of particles in

[14]To calculate the inefficiency of the particle filter (how severe sample impoverishment is) and to calculate the quality of the approximation to the state density with the particle filter, Arulampalam *et al*. (2002) suggest estimating the so-called effective sample size. The idea is to use the variance of importance weights as a tool to assess the quality of the distribution. This statistic is given by:

$$N^{eff} = \frac{1}{\sum_{i=1}^{N} (q_t^i)^2}.$$

order to generate a good approximation to the density. Moreover, as commented by Moura (2010), the importance sampler used to approximate the integrals in the filtering process only includes the information available in the observed variables at $t-1$, not using the information at t. Accordingly, some extensions to the standard particle filter were proposed in order to include information available at t, making it more efficient, for instance, the auxiliary particle filter proposed by Pitt & Shephard (1999) and the conditional particle filter introduced by Amisano & Tristani (2010). However, it should be highlighted that the consistency of the particle filter is based mainly on the number of particles used to approximate the probability density function to the state variables in each time period. Nevertheless, there is a trade-off between the accuracy of the filter and the time necessary to perform the procedure. Fasolo (2012) shows that the log likelihood values become more accurate as the number of particles increases. Additionally, the author compares the performance of the standard particle filter and of the auxiliary one and concludes that, in spite of the latter having a good performance when the number of observations is large and the number of particles is relatively small, the standard particle filter is significantly less computationally demanding and, therefore, this filter tends to have a better performance with a large number of particles in comparison with the auxiliary filter.

Therefore, there are some extensions to the standard particle filter aimed at improving its performance. Nonetheless, the particle filter proposed by Fernández-Villaverde & Rubio-Ramírez (2005 and 2007) is still a robust statistical tool with better results than the traditional Kalman filter, as demonstrated by Fernández-Villaverde & Rubio-Ramírez (2005), An & Schorfheide (2007), and Amisano & Tristani (2010).

3.4. Posterior distribution simulations

After obtaining the likelihood, it may be used in a Bayesian algorithm that simulates the posteriors of the parameters. In the present paper, the algorithm known as Random-Walk Metropolis (RWM) introduced in Chib (2001) and in An & Schorfheide (2007) is used. Briefly put, sequential Monte Carlo methods are used to calculate the likelihood of the DSGE model, and then this likelihood is plugged into a Monte Carlo Markov Chain (MCMC) process.

So as to carry out this process successfully, first one estimates the linearized version of the DSGE model introduced in Section 2 using the RWM algorithm. By using the same covariance matrix for the distribution proposed to generate the particles, as in the case of the linearized DSGE model, we then run the RWM algorithm based on the likelihood function associated with the second-order approximation of the DSGE model. Note that the covariance matrix of the proposed distribution is defined such that the RWM algorithm will have an acceptance rate around 50%. A total of 100,000 particles are used to approximate the likelihood function of the nonlinear DSGE model, whereas the variance of measurement errors is defined as 10% of the variance of the observation sample. Finally, 275,000 simulations of the posterior distribution of the nonlinear DSGE model are obtained. It should be underscored that the first 75,000 simulations are eliminated and that only the statistical results obtained from the remaining simulations are reported.

Also, the number of particles chosen in the present paper is much larger than that observed in previous studies. For example, Fernández-Villaverde & Rubio-Ramírez (2005) use 60,000 particles in the neoclassical growth model. Afterwards, the same authors use an extended version of this model with 80,000 particles. On the other hand, An & Schorfheide (2007) estimate a new Keynesian model with 40,000 particles. Finally, Aruoba, Bocola & Shorfheide (2013) use 80,000 particles to calculate the likelihood function of a DSGE model

with asymmetric adjustment costs for both prices and wages. Therefore, in this paper, we used the “brute force” approach to tackle the afore-mentioned impoverishment problem. Despite the higher computational costs, the increase in the number of particles helps minimize the impoverishment problem.

4. Estimation and Results

After the discussion on the methods for solution of the DSGE model and also on the estimation methods, the present section deals with the empirical results of the proposed model. The model shown in Section 2 is solved using a nonlinear method and its structural parameters are estimated by the particle filter. The estimation results obtained through the Kalman filter for the linearized version of the model are briefly described. However, no comparison is made between the performance of the two filters, as in other studies such as An & Schorfheide (2007) and Amisano & Tristani (2010). Therefore, the first part of this section shows the data and prior distributions. The second part is concerned with posterior distributions, comments on the results, and compares the parameters with other literature data. Finally, the section concludes by analyzing the impulse response functions aiming to assess the dynamics of the economy in its linear version (symmetric rigidity) and in its linear version (asymmetric rigidity) in the presence of a transient shock to the monetary and fiscal policies.

4.1. Data and prior distributions

This paper uses quarterly data from 2000:1 to 2014:4, totaling 60 observations. Note that the selection of this period is due to a regime shift, with the introduction of the inflation-targeting regime in 1999. Four observable macroeconomic variables were utilized: output, inflation, interest rate, and wages. The GDP, IPCA (broad consumer price index) and the Selic interest rate data published by the Brazilian Institute of Geography and Statistics (IBGE)

were used. Average earnings, a labor market variable disclosed by DIEESE, were used. This indicator uses the behavior of the metropolitan area of São Paulo as proxy for the national dynamics. The decision to adopt this series rather than that published by the IBGE was based on the structural break in this series in 2002 caused by a methodological change.

The priors were estimated following economic studies conducted in Brazil, and in the case of parameters that had not been calculated for the Brazilian economy, the same prior of the original model was used. The estimation of parameter β was based on Shorfheide (2000). The mean value for parameter v was set as 2. We chose the value 1.30 for τ, which is employed in the SAMBA model by Castro *et al.* (2011). The values of monetary authority parameters (ρ_r, Ψ_2 and Ψ_2) were the same as those of the SAMBA model. As to shock persistence parameters (ρ_a, ρ_g and ρ_p), the prior was equal to 0.85. The values of rigidity parameters are the same described in Aruoba, Bocola & Shorfheide (2013), since there is no empirical evidence regarding these parameters in the Brazilian economy. Finally, the inverse gamma distribution was chosen for all standard deviations. Tables 4.1 and 4.2 summarize the priors used for the parameters.

4.2. Estimation results: linear and nonlinear models

First of all, this subsection briefly presents the results for the estimation made using the Kalman filter. The first step consisted in finding the solution to the model shown in Section 2 by a linear approximation via Taylor expansions or logarithmic approximations. After linearization, the solution to the model is written as deviation of the values from the steady state and the model is given in difference equation form with rational expectations. The next step consisted in writing the solution in state-space form, assessing the likelihood function via the Kalman filter. The parameters estimated by this method will not be analyzed thoroughly as the focus is on the result of the nonlinear model. Nevertheless, as demonstrated

in Table 4.1, the estimated parameters appear to be in line with the results obtained for the Brazilian economy, except for the interest rate smoothing parameter, whose value was apparently way below that observed by Castro *et al*. (2011) and Sin & Gaglianone (2006). On the other hand, the value obtained for β was smaller than the usual one often calculated for Brazil (around 0.989).

However, the most important task to be accomplished in this section concerns the analysis of the results obtained for the estimation made using the particle filter. Unlike previous findings, the DSGE model is solved by using a second-order perturbation method, which results in a nonlinear representation in state-space form. In this case, it is no longer possible to use the Kalman filter to assess the likelihood function. Table 4.2 shows the results for the estimation made using the particle filter.

Table 4.1 – Priors and Posteriors of the Linearized Model Estimated via the Kalman Filter

	Prior			*Posterior*		
Parameter	Mean	Standard deviation	Distribution	Mean	90% confidence interval	
β	0,9930	0.0200	Beta	0.9796	0.9763	0.9827
τ	1.7000	0.1000	Normal	1.8494	1.6742	2.0316
v	2.0000	0.7500	Normal	0.2457	0.1801	0.3223
$k(\varphi_p)$[15]	0.3000	0.2000	Gamma	0.0177	0.0082	0.0277
φ_w	15.000	7.5000	Gamma	3.6561	2.2331	6.3084
Ψ_p	-	-	-	-	-	-
Ψ_w	-	-	-	-	-	-
Ψ_1	1.7000	0.1000	Normal	1.7767	1.5835	1.9567
Ψ_2	0.2000	0.1000	Gamma	0.2312	0.1701	0.2935
ρ_R	0.5000	0.2000	Beta	0.3930	0.3006	0.4825
ρ_g	0.8500	0.1000	Beta	0.9835	0.9479	0.9901
ρ_a	0.8500	0.1000	Beta	0.8063	0.7832	0.8396
ρ_p	0.8500	0.4000	Beta	0.5618	0.4262	0.7150

Source: Data compiled by the authors

Table 4.2 summarizes the parameter estimates for the nonlinear model. Note that, in general, the estimated parameters do not differ considerably from the values commonly found

[15]Note that in the case of the log-linearized version of the DSGE model, the parameter φ_p cannot be identified; however, it is implied in $k(\varphi_p) = \tau \frac{1-\lambda_{p,t}}{\lambda_{p,t}\pi^2\varphi_p}$ which provides the slope of the new Keynesian Phillips curve in the linearized version of the model.

in the empirical literature with Brazilian data. The first part of Table 4.2 shows the statistics for household preferences. The intertemporal discount factor, β, was equal to 0.9951, smaller than the values commonly calibrated (0.989) for the Brazilian economy described by Castro *et al*. (2011) and Vasconcelos & Divino (2012). Conversely, parameter τ – which measures the inverse intertemporal elasticity of substitution and is also known as risk aversion coefficient – yielded an estimate of 1.8480, higher than that obtained by Portugal & Silva (2011) and Castro *et al.* (2011). Interestingly, empirical models, such as those published by the Central Bank of Brazil in its quarterly inflation reports, show that the interest rate eventually produces a small effect on inflation compared with the effects observed in DSGE models seen in impulse response functions. Finally, the elasticity of labor supply, v, yielded a value of 0.1692, indicating wage rigidity in the specification of the model after the introduction of asymmetric wage adjustment costs.

As to the monetary policy coefficients, the Central Bank of Brazil seemingly attaches more weight to the deviations in inflation than in output. The estimated parameter that responds to inflation, Ψ_1, yielded 1.6220. This value was smaller than that observed by Casto *et al*. (2011), i.e., 2.43. However, it was higher than that obtained by Sin & Gaglianone (2006), 1.33, and Santos & Kanczuk (2011), 1.50. The value of parameter Ψ_2, which indicates the importance given by the Central Bank to the deviation from output, was 0.3905, much higher than that observed by the afore-mentioned authors – 0.16, 0.13, and 0.16, respectively. The estimated parameters demonstrate that, although the Central Bank attaches more weight to inflation than to output, in the case of the model used here, the monetary authority gives considerable attention to deviations from output, which occurs in a lesser degree in most empirical studies on Brazil. Nonetheless, according to this finding, the monetary authority reacts to output when it is above its potential so as to prevent higher inflation rates caused by excess demand. Finally, the interest rate smoothing coefficient, ρ_r, corresponded to 0.7419,

which is lower than that reported by Castro *et al.* (2011) and by Sin & Gaglione (2006), who obtained 0.79 and 0.8402, respectively. This indicates that the monetary authority takes into account the previous interest rate level in order to make decisions about its increase. In other words, the Central Bank usually keeps the interest rate stable over time instead of changing it abruptly. In sum, the results found here indicate that 74% of the current interest rate is determined by the past interest rate value; so, 26% of the deviations in inflation and output are adjusted in each period.

Table 4.2 – Priors and Posteriors of the Nonlinear Model Estimated via the Particle Filter

	Prior			*Posterior*		
Parameter	Mean	Standard deviation	Distribution	Mean	90% confidence interval	
β	0.9930	0.0200	Beta	0.9951	0.9928	0.9972
τ	1.7000	0.1000	Normal	1.8480	1.6828	2.0202
v	2.0000	0.7500	Normal	0.1692	0.1069	0.2392
φ_p	15.000	7.5000	Gamma	7.6928	5.3382	10.2143
φ_w	15.000	7.5000	Gamma	26.6619	20.2389	36.3142
Ψ_p	-300.00*	300.00*	Uniform	253.3584	212.6989	290.5934
Ψ_w	-200.00*	200.00*	Uniform	19.5157	3.2506	40.7773
Ψ_1	1.7000	0.1000	Normal	1.6220	1.4588	1.7920
Ψ_2	0.2000	0.1000	Gamma	0.3905	0.3194	0.4628
ρ_R	0.5000	0.2000	Beta	0.7419	0.6495	0.8314
ρ_g	0.8500	0.1000	Beta	0.7219	0.6863	0.7685
ρ_a	0.8500	0.1000	Beta	0.9357	0.9126	0.9690
ρ_p	0.8500	0.4000	Beta	0.7272	0.5916	0.8804

*Upper and lower bounds of the uniform distribution

Source: Data compiled by the authors

The technology growth shock is the most persistent one among the exogenous shocks of the model, ρ_a is approximately 0.94. However, the government spending shock, which

represents a generic demand shock, is given by $\rho_g = 0.7219$. Finally, the persistence of the shock to the inverse elasticity of demand for intermediate goods, ρ_p, is equal to 0.73.

Finally, the rigidity and asymmetry parameters are analyzed in the price and wage adjustment cost functions. The parameters that govern price and wage rigidity, $\varphi_p = 7.6928$ and $\varphi_w = 26.6619$, indicate both price and wage rigidity in the Brazilian economy. Moreover, the parameters show that wage rigidity is higher than price rigidity. Notwithstanding, this finding can be explained by the values obtained for other rigidity parameters – those which provide evidence of asymmetric rigidity – which will be discussed in what follows. Therefore, the most interesting and important results are those related to asymmetry parameters for price and wage adjustment cost functions, as these parameters have not been estimated for the Brazilian economy yet. For parameters Ψ_p and Ψ_w the estimates were 253.3584 and 19.5157, respectively. By comparing these results with those obtained by Aruoba, Bocola & Shorfheide (2013), one has a higher value for asymmetric price ($\Psi_p = 165$) and a lower one than that observed for asymmetric wage ($\Psi_w = 59$). Also, by comparing the findings with those described by Kim & Ruge-Murcia (2009), the wage parameter is way below the values obtained by other authors ($\Psi_w = 901.4$); however, in that study, only wage asymmetry is assumed to exist, which might have influenced the results, in addition to the fact that the authors estimated their DSGE model using the method of simulated moments. By and large, the positive estimates of $\Psi_p = 253.3584$ and $\Psi_w = 19.5157$ imply that it is harder to lower prices and wages than to raise them; in addition, price asymmetry is more pronounced than wage asymmetry. This result may have influenced the value of parameters φ_p and φ_w mentioned earlier. Note that the value of φ_p was much lower than φ_w; on the other hand, the value of Ψ_p was much higher than Ψ_w. What this result shows is that the firm has a real cost (e.g., in terms of goods or hours worked) when it changes its price, which is identified by parameter φ_p, but because of the small value of this parameter,

the firms start with a low cost and then bring it closer to market efficiency when they adjust their prices. Nevertheless, the high value of Ψ_p tells us that firms adjust their prices at a higher cost when they need to lower rather than to increase their prices. The opposite is true for wages, i.e., wage rigidity is high, but wage asymmetry is low although it indicates that the cost associated with lowering them is higher. Thus, we gathered evidence that both prices and wages are sticky in the Brazilian economy; furthermore, they are stickier downward than upward. This may be so mainly due to the following factors. In the case of wages, as pointed out by Camargo (2009), the Brazilian labor market has some peculiarities, with a comprehensive body of laws governing its operation, in addition to being complex and giving meticulous attention to detail, which eventually hinders the appropriate "drafting" of labor contracts, with different characteristics from those established by law, rendering the labor contract framework legally binding and barely negotiable. Hence, by drawing up contracts with more flexible clauses than those required by law, firms risk having these contracts declared void by the labor court, resulting in high costs. Therefore, wages become sticky. On the other hand, the presence of strong unions in the domestic economy may produce downward wage rigidity, more often than not, not allowing for cuts in wages based on workers' productivity. In the case of prices, two factors could explain rigidity. The first one stems from wage rigidity itself, which ends up having an impact on firms' costs. Since wages are stickier downward, firms eventually do not adjust their prices efficiently (setting wages according to productivity). Moreover, as highlighted previously, firms risk having to cover high costs by trying to negotiate contracts that are more flexible than what the Brazilian law stipulates, and are at risk of having these contracts deemed void, giving rise to expensive lawsuits. The second situation is concerned with the fact that firms often operate in a market that is not fully competitive, i.e., most firms have some monopoly over their products since such products are minimally different from those marketed by their competitors. However, in

order to offer differentiated products in the market, firms eventually bear additional costs involving advertising and product development, which lead to larger costs and have an impact on the firms' overall costs, prompting them to operate at higher costs.

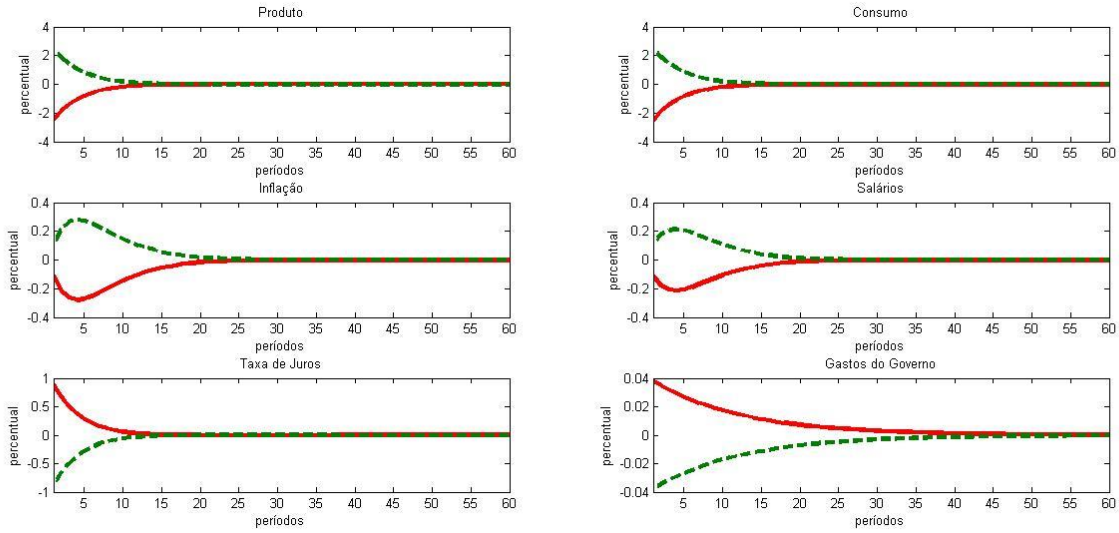
4.3. Impulse Response Analysis

After estimating the structural parameters of the model in the previous subsection, the focus now is on analyzing the dynamics of shock propagation with and without asymmetries in price and wage adjustment costs. In other words, we are going to have a look at the dynamics of the economy in the linear (symmetric) and nonlinear (asymmetric) versions. Therefore, we seek to analyze how monetary policy shocks and government spending shocks propagate by using impulse response functions and comparing them with the steady-state equilibrium scenario. Hence, this section examines how the economy responds to shocks. Starting with the steady state, the economy is subject to an unexpected temporary shock and then the reactions concerning consumption, output, interest rate, government spending, nominal wages, and inflation are plotted as a function of time. Note that the monetary policy shock corresponds to a reduction or increase in the nominal interest rate whereas a government spending shock should be viewed as a generic demand shock.

Before analyzing the impulse response functions, it is necessary to comment on some important aspects. As pointed out by Gallant *et al*. (1993) and Koop *et al.* (1996), in the case of linear models, the reactions to a shock of size ϵ correspond to 50% of those of a shock of size 2ϵ and are also mirrored in a shock of size $-\epsilon$. Thus, any appropriate normalization – for example, $\epsilon = 1$ – eventually summarizes all the relevant information about the dynamics imposed by the model, as shown in graphs 4.1 and 4.3. However, as demonstrated by Kim & Ruge-Murcia (2009), in nonlinear models, as the one introduced in Section 2, the responses of the variables depend on both the sign and the magnitude of the shock. Bearing that in mind,

the focus will be on the nonlinear model, making only brief comments on the dynamics of the linearized model and making brief comments on the behavior of both models.

Graph 4.1 – symmetric Model: One-Standard-Deviation Shock to the Interest Rate

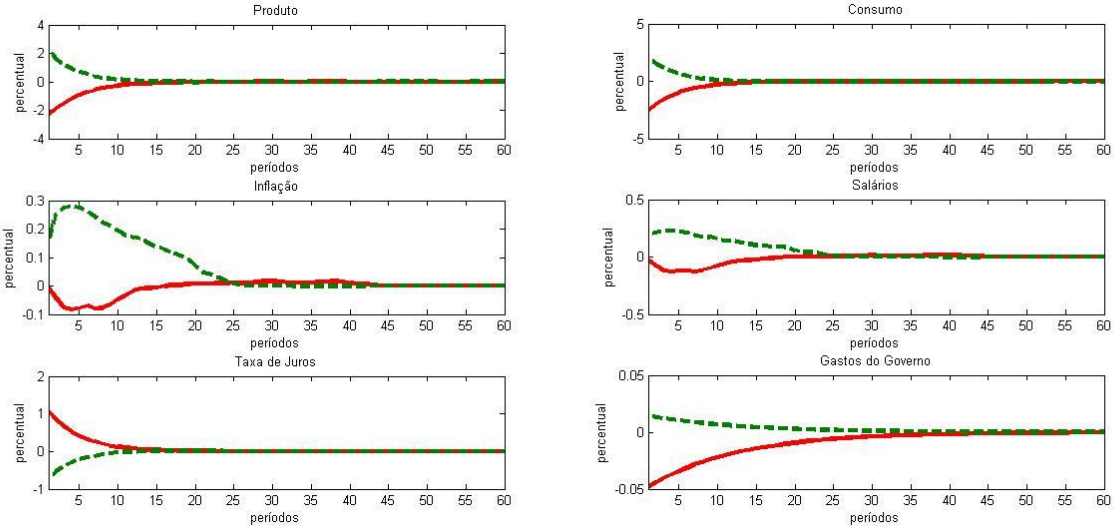


Source: Data compiled by the authors

Graph 4.1 shows the dynamic response of some selected variables to a shock to an expansionary monetary policy (reducing nominal interests) and a contractionary policy (increasing nominal interests). The solid red line denotes an increase in the nominal interest rate, whereas the dashed green line refers to an accommodative monetary policy.[16] With the increase in interests, private consumption decreases, thereby reducing aggregate output. In addition, both prices and wages have a quite similar reduction, returning to their steady state after 20 periods. On the other hand, the government increases its spending in order to prompt the economy back into equilibrium.

[16]The magnitude of initial innovation in the Central Bank's nominal interest rate shock corresponds to 1% and -1%.

Graph 4.2 – Asymmetric Model: One-Standard-Deviation Shock to the Interest Rate



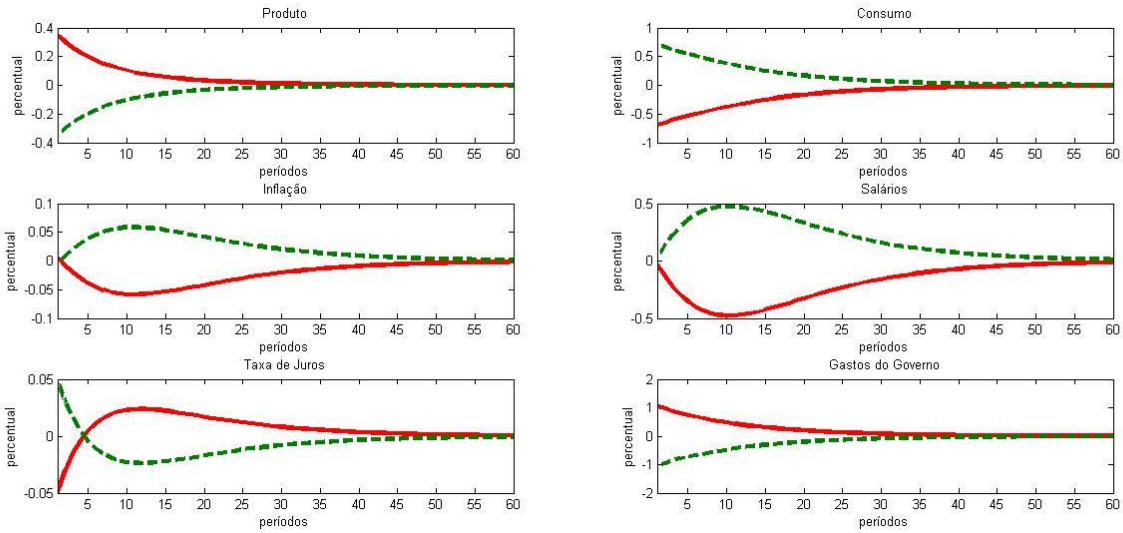
Source: Data compiled by the authors

Graph 4.2 illustrates the dynamics of the asymmetric model after a monetary policy shock. The interest rates increase while private consumption decreases, since households have access to the domestic securities market, leading to a household saving behavior as the consumption of goods is postponed to future periods. Output decreases due to the decline in consumption and this decrease is further enhanced as a result of the reduction in government spending in the current period. In a slower economy, nominal wages have a slight reduction relative to their steady state, which eventually helps reduce the general prices, thus deflating the economy as a whole. On the other hand, an accommodative monetary policy boosts consumption, causing an increase in output, being also influenced by a slight increase in government spending. In a heated economy, wages increase markedly, and this ends up radically affecting the price level. Notably, the most interesting results are those associated with the behavior of wage and inflation variables, owing to the introduction of parameters related to asymmetries in price and wage adjustment costs. Note that an accommodative policy causes a much larger increase in inflation and in wages than a reduction in them in a

contractionary monetary policy. This occurs because the values of parameters Ψ_w and Ψ_p are greater than zero, indicating that economic agents are stricter about reducing wages and prices.

Furthermore, in the case of prices and wages, the return to the respective steady state values take longer when the Central Bank decides to invigorate the economy, but in the case of output and consumption, the return to the steady state occurs virtually in the same period. This means that in an expansionary scenario, output and consumption go up, but that happens at the cost of a higher and more persistent inflation rate. Conversely, after an increase in prices and wages under expansionary conditions, economic agents take longer to adjust prices and wages, since they have an asymmetric behavior and are stickier downward. Therefore, in an economy in which prices and wages are stickier downward, the Central Bank needs to be more careful when adopting expansionary policies, as prices and wages could be higher than those the economy can tolerate, leading to periods in which inflation is much higher and the wages are higher than the workers' productivity.

Graph 4.3 – symmetric Model: One-Standard-Deviation Shock to Government Spending

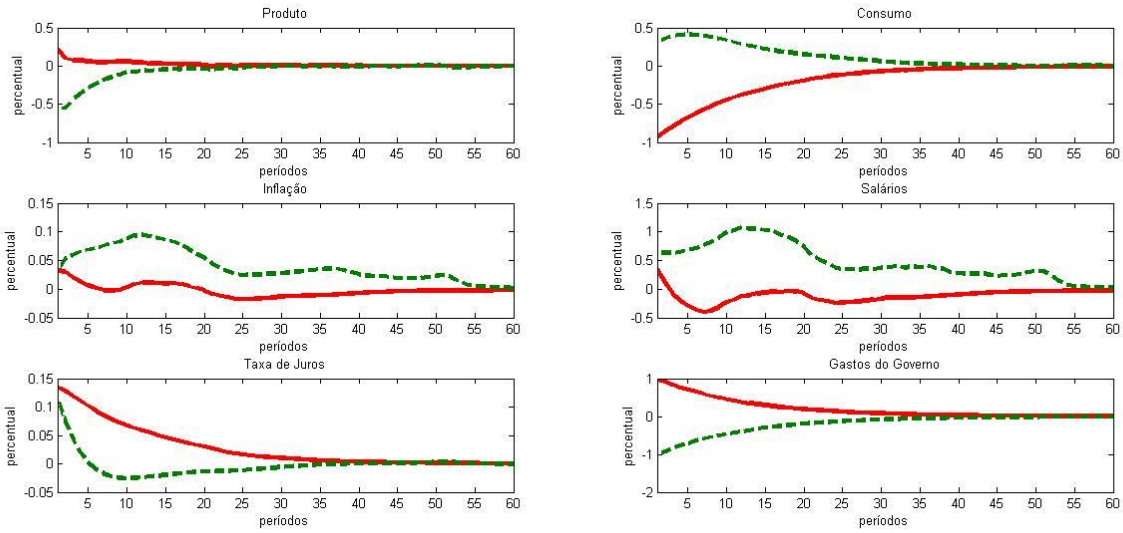


Source: Data compiled by the authors

Graph 4.3 shows the dynamic response of some variables to an expansionary (higher government spending) and a contractionary (lower government spending) fiscal policy shock. The solid red line indicates an increase in spending whereas the dashed green line refers to a reduction in spending.[17] An increase in spending causes a decline in consumption, but output reacts positively to fiscal stimuli. Owing to lower consumption, wages eventually go down, which causes a reduction in general prices, while the monetary authority reduces interest rates in order to stimulate consumption.

[17]The magnitude of initial innovation in government spending shocks corresponds to 1% and -1%.

Graph 4.4 – Asymmetric Model: One-Standard-Deviation Shock to Government Spending



Source: Data compiled by the authors

In Graph 4.4, when spending increases, private consumption decreases considerably, since households in this economy are Ricardian; consequently, there is a large crowding out effect, in which an increase in government spending reduces private consumption. Output increases slightly in the first quarters even with the abrupt reduction in household consumption, but it goes back quickly to the initial steady state. This increase can be explained by a boost in demand caused by higher government spending. Regarding nominal wages, they are increased slightly as a result of the initial fiscal incentive. Nonetheless, as private consumption declines and aggregate output returns to its steady state rather quickly, wages go back to a lower level in subsequent quarters. The increase in wages eventually affects inflation and thus the Central Bank intervenes by driving interests up in order to counteract the inflationary effects in the short run. On the other hand, a contractionary fiscal policy brings about a small increase in household consumption whereas aggregate output shows a remarkable reduction. Due to burgeoning demand, resulting from an increase in

private consumption, nominal wages go up sharply and remain way above the point of equilibrium for a long time. With higher wages, firms opt to increase the prices of their products so as to adjust to household income, thus maximizing their profit, at least in the short run. As shown in Graph 4.4, wage and price movements are similar, indicating downward rigidity, caused by the introduction of asymmetric adjustment costs in the model. When an inflationary period is observed, the monetary authority increases the interest rates; however, the increase in interest rates is smaller than the previous one. Apparently, the Central Bank decides to fight inflationary effects, but as aggregate output decreases, the increase in interest rates does not take place abruptly so as not to push aggregate output further down. This behavior is quite similar to that observed in the past four years, in which the Central Bank was more tolerant of inflation to prevent output from being affected by higher interest rates. In summary, as shown in Graph 4.4, fiscal expansion contributes little to aggregate output movement whereas fiscal contraction causes a decrease in output. The effects on wages and on inflation may be severe both in an expansionary and contractionary policy environment.

Finally, by comparing the behavior of the symmetric model with that of the asymmetric one, when there is a monetary policy shock, an expansionary policy causes a similar wage movement in both models. However, prices increase more abruptly in the asymmetric model and stay off the equilibrium path for a longer time (around five quarters more than in the linearized model), indicating higher persistence of inflationary effects on the Brazilian economy when the agents behave asymmetrically towards prices and wages. In the case of consumption and output, the responses are very similar in both models, showing that nonlinearities in prices and wages do not drastically affect the behavior of these variables. On the other hand, in the presence of a fiscal policy shock, output and consumption behave differently in each model. In the case of prices and wages, a temporary fiscal imbalance eventually leads to higher disequilibrium in both models, taking around 55 periods to return to

the steady state, five quarters more than in the asymmetric model. In brief, the comparison of the dynamics of both models in the presence of a monetary or fiscal policy shock shows a much larger effect on prices and wages in the asymmetric case. This finding was expected since the estimated parameters associated with the level of asymmetry in the nonlinear model provide evidence that the economy is stickier downward. Accordingly, in an economy in which price and wage setters are stickier downward, both the Central Bank and the fiscal authority should proceed with caution, as an accommodative policy or an imbalance in public accounts eventually has a stronger impact on wages and on inflation and, consequently, the economy will need a longer time to adjust. Thus, at election time, when the incumbent government often creates economic incentives, an imbalance in public accounts or a reduction in interest rates by the Central Bank in order to boost credit and consumption can lead to higher and more persistent inflation rates, preventing the economy from returning as fast as possible to its steady state.

5. Conclusion

The use of linearized DSGE models turned the Kalman filter into the standard tool for likelihood estimation, helping with the estimation of these economic models. However, this filter often has a very restrictive framework, thus limiting the analysis of nonlinear phenomena observed in the real world. Therefore, the pioneering work by Fernández-Villaverde & Rubio-Ramírez (2005) opened up a new avenue to the study of nonlinear macroeconomic models by developing a sequential Monte Carlo algorithm that allows estimating the likelihood function of a model solved by a nonlinear method. From then on, several studies on the estimation of nonlinear models with the use of sequential Monte Carlo methods have been conducted, including, among others, An & Schorfheide (2007),

Fernández-Villaverde & Rubio-Ramírez (2007), Amisano & Tristani (2010), Andreasen (2011), and Fasolo (2012).

Among these methods, the particle filter is the most famous one. It is a numerical state estimator that is fully nonlinear and that can be used to estimate any model in state-space form. According to Simon (2006), the particle filter, or Monte Carlo filter, is a "brute force" statistical method that often works well with problems that are not easily solved by the conventional Kalman filter, i.e., highly nonlinear systems.

Thus, having at hand a statistical tool that allows using nonlinear DSGE models, which generates a better representation of the economy, it is possible to use a model that generates fundamental nonlinearities. Taking this into consideration, a DSGE model with asymmetric price and wage adjustment costs, proposed by Kim & Ruge-Murcia (2009) and expanded by Aruoba, Bocola & Shorfheide (2013), was used, producing nonlinearity in the economic agents' behavior. This model allows checking whether prices and wages are stickier upward or downward. In other words, the aim of this paper was to verify whether price and wage rigidity in the Brazilian economy is asymmetric and how it affects the dynamics of the economy. Therefore, the present study estimated the structural parameters of the Brazilian economy and assessed the behavior of the major macroeconomic variables towards monetary and fiscal policy movements. The model is solved by using second-order perturbation methods based on Schmitt-Grohé & Uribe (2004) and estimated with Brazilian data, using the particle filter in order to construct the likelihood.

The results provide evidence that wages and prices in the Brazilian economy are stickier downward, as the parameters associated with asymmetric rigidity are greater than zero. In other words, there is asymmetry – which is weaker in the case of wages and stronger in the case of prices – making economic agents be stricter when it comes to reducing rather than increasing wages and prices. These asymmetries, in the case of prices, may be caused by

some monopoly on the part of firms or result from the downward wage rigidity, since most of the costs defrayed by firms are associated with their payrolls. Asymmetry in wages may be attributed to the characteristics of the Brazilian labor market, due to a very complex body of laws governing the drafting of contracts. Moreover, there is high turnover and a large number of informal job contracts, which may result in remarkable asymmetries.

Based on the impulse response analysis, it is possible to conclude that in the presence of asymmetries in price and wage rigidity, the Central Bank should be more careful about the formulation of an expansionary monetary policy, given that prices and wages are stickier downward and monetary easing could thus lead to longer periods of high inflation. Likewise, fiscal policy movements (be they expansionary or contractionary) eventually have a negative impact, especially on inflation, which means that it is better to keep the fiscal policy in its steady state, i.e., to have a neutral fiscal policy. Additionally, by comparing the dynamics of asymmetric economy (solved by a nonlinear method and estimated by a particle filter) with symmetric economy (linearized version in which likelihood is estimated by the Kalman filter), it was observed that nonlinear economy takes longer to return to its steady state after a monetary or fiscal shock and that nonlinearities are more pronounced in prices and wages than in any other macroeconomic variables.

Finally, several elements can be added and improved in a future study. It would be interesting, for instance, to investigate the underlying causes of asymmetries in price and wage rigidity. In addition, another suggestion is to use a DSGE model for an open economy and include capital accumulation in order to bring the theoretical model as closest as possible to the real economy. Regarding the particle filter, it would be interesting to use extensions of this filter, such as the conditional particle filter (Amisano & Tristani, 2010) or the particle filter proposed by Andreasen (2011) in an attempt to further improve its estimates.

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