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Steady-State Analysis of an Open Economy General Equilibrium Model for Brazil[*]

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Abstract

The aim of the present research is to build an open economy recursive general equilibrium model for the Brazilian economy in order to numerically assess the corresponding steady state equilibrium. This characterization allows us to numerically compute the endogenously determined steady state key relationship, namely the primary surplus aggregate output as well as the debt-product ratio among other variables, as functions of the monetary and fiscal policy parameters chosen by the government of the model economy.

The adopted model introduces a transaction technology, which allows us to obtain a monetary equilibrium at steady state. This economy differs from the one used by Ljungqvist and Sargent (2000) as it considers an open economy with accumulation and production.

The main result has shown that under the adopted parameterization the steady state of the model economy can be numerically characterized by a debt output ratio of 0.3387. The numerical simulations show alternative steady states attainable by the government of the model economy. In order to finance higher expenses the government is bounded by the trade-off of higher interest rates (low inflation or high return on real money balances) for lower operational surpluses due to the higher debt output ratio at the long run equilibrium.

JEL Classification: E52, E62
Keywords: Stochastic dynamic general equilibrium model, steady state equilibrium

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Introduction

The aim of the present research is to build an open economy recursive general equilibrium model for the Brazilian economy in order to numerically assess the corresponding steady state equilibrium. This characterization allows us to numerically compute the endogenously determined steady state key relationship, namely the primary surplus aggregate output as well as the debt-product ratio among other variables, as functions of the monetary and fiscal policy parameters chosen by the government of the model economy.

The monetary policy is captured by the choice of the nominal interest rate and the fiscal policy, in turn, is modeled as the choice of labor and capital income tax rates by the government of the artificial economy.

The adopted model is similar to the one proposed by Ljungqvist and Sargent (2000) for obtaining a monetary equilibrium at steady state by means of the introduction of a transaction technology[1]. Moreover, the present model economy extends this basic model to an open economy with capital accumulation and production processes as well.

The main result has shown that under the adopted parameterization the steady state of the model economy can be numerically characterized by an aggregate debt output ratio of 0.5568, consistent at the long run equilibrium with a debt service to aggregate output ratio of 3.42% and a tax burden of 11.69%. It is important to note that the analysis is one of a (long-run) steady state of an artificial economy without any kind of frictions, in particular without any risk of default. Therefore these results can be supported in equilibrium.

The performed sensitivity analysis has shown a high dependency of the obtained results on the physical as well as domestic financial assets accumulation processes as it is modeled in the benchmark model.

[1] See Ljungqvist and Sargent (2000) for more details.

Given the choice of fiscal policy, the government of this artificial economy could obtain the volume of resources needed by means of determining the nominal gross interest rate at steady state. But the trade-off becomes apparent: the higher the chosen interest rate, the higher the share of outstanding aggregate debt in aggregate product – which it turn induces a higher operational deficit at the long run equilibrium.

On the other hand, the optimal choice of consumption, investment and time allocation among labor and transaction determines the real money balances demanded in equilibrium as an increasing function of the former and the later. Given the arbitrage condition and the inter-temporal budget constraint faced by the families and the government, the trade-off among the different sources of public expenses financing means becomes apparent. For a given choice of tax parameters, if the government chooses to increase its participation in aggregate output, in order to finance higher expenses the government is bounded by the trade-off of higher interest rates (low inflation or high return on real money balances) for low operational surpluses due to the higher debt output ratio, hence a higher debt service at the long run equilibrium.

The paper is organized as follows. Section 1 presents the set up of the benchmark model. The corresponding analytical definition and characterization of the model's steady state is introduced in Section 2. The parameterization of this benchmark economy is in turn described in Section 3. Section 4 explains the main results obtained with our numerical simulations by considering alternative fiscal and monetary policy choices adopted by the government in the artificial economy.

1. Model Economy

The model economy consists of a continuum of unitary mass households, a perfectly competitive productive sector and government or central agency acting in an open economy. Sub-section 1 below describes the problem of the representative household. Sub-section 2 characterizes the problem faced by the productive sector. Sub-section 3 in turn introduces the roll of the government/central agency that sets the monetary and fiscal policy instruments of the economy, given a budget constraint. Sub-section 4 describes the equilibrium balance of payments condition, which has to be satisfied at the steady state that we are seeking to achieve.

1.1. The Characterization of the Households

The households of the model economy consist of a continuum of identical unitary mass families whose problem is to maximize the flow of discounted utility derived from consumption, c_t, and leisure, l_t, choosing the optimal sequences of consumption, labor, money holdings, investment and domestic public bond holdings, $\{c_t, h_t, \frac{m_t}{P_t}, i_t, b^d{}_{t+1}\}$, subject to per period budget constraint, expression (2) below.

Hence the problem of this representative agent can be summarized as:

$$\max_{\{c_t, h_t, m_{t+1}, i_t, b^d_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{1}$$

subject to

$$\frac{b^d_{t+1}}{R_t} + c_t + i_t + \frac{m_{t+1}}{P_t} = q_t \qquad \forall\, t \geq 0 \tag{2}$$

where the per period disposable income for the representative household is given by the following expression,

$$q_t = (1-\tau)[w_t h_t + (r_t - \delta)k_t] + b^d_t + \frac{m_t}{P_t} \tag{3}$$

Moreover, in order to obtain a monetary equilibrium, the following transaction technology is introduced into the model economy. The household has a unit time endowment at every period, which can be allocated into leisure, work and transaction, i.e. $1 = l_t + h_t + s(c_t, m_{t+1}/P_t)$. The transaction technology[2] is assumed to be such that $\partial s/\partial c, \partial^2 s/\partial c^2, \partial s/\partial(m'/P), \partial^2 s/\partial(m'/P)^2 \geq 0;\quad \partial s/\partial(m'/P), \partial^2 s/\partial c \partial m'/P \leq 0$. In particular, $s(c_t, m_{t+1}/P_t) = c_t\,(1+ m_{t+1}/P_t)^{-1}$.

In the above set up, τ represents the proportional tax rate on labor as well as capital income, R_t the real return on domestic bonds holdings and, P_t time t price level, such that the budget constraint (2) above is expressed in terms of time t unit of consumption good. The law of motion for capital formation is assumed to be linear, i.e. $k_{t+1} = (1-\delta)k_t + i_t$, as well as initial conditions $(k_0, m_0, b_0) > 0$ given.

[2] For more details refer to Ljungqvist and Sargent (2000), chapter 17.

In particular, it is assumed that the instantaneous utility function of the representative household is a function defined as follows:

$$u(c_t) = \gamma \ln(c_t) + (1-\gamma)\ln(l_t) \tag{4}$$

such that $l_t=1 - h_t - s(c_t, m_{t+1}/P_t)$, and $s(c_t, m_{t+1}/P_t) = c_t\,(1+ m_{t+1}/P_t)^{-1}$.

1.2. The Productive sector

The large number of competitive and identical firms acting in this model economy, using a constant return to scale technology, enables us to use an aggregate production function specified as a Cobb-Douglas production function.

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha}$$

$$\frac{Y_t}{H_t} = A\left(\frac{K_t}{H_t}\right)^{\alpha} = Ak_t^{\alpha} \tag{5}$$

$$r_t = A\alpha\, k_t^{\alpha-1}\,,\; w_t = Ak_t^{\alpha}(1-\alpha)$$

where the last two expressions of (5) above refer to the market clearing condition applied to the representative firm's first order condition for profit maximization.

Alternatively, the above-assumed technology available to the competitive firms can be expressed in intensive form as follows.

$$y_t = f(k_t) = Ak_t^{\alpha} \tag{6}$$

The production of this economy is allocated into the domestic (d) as well as external (f) markets, noticing that the later constitutes the exports of the economy, i.e.

$$\begin{array}{l} y_t = y_t^d + y_t^f \\ y_t^d = c_t^d + i_t + g_t \quad ; \quad y_t^f = \varphi\, y_t\,; \quad g_t = \vartheta\, y_t\,; \quad c_t^d = c_t - cm_t \end{array} \tag{7}$$

where, for computational simplicity and given our aim of studying the long run equilibrium, exports, y^f, imports of consumption goods, cm, and public expenses, g, at every period t, are assumed to be a fixed proportion of aggregate output y, captured by parameters φ, ϕ and ϑ respectively.

1.3. The Government

The government of this economy determines the fiscal and monetary policy of the economy, and also is the sole agent in this economy who can issue external debt. Therefore, this sector collects proportional labor and capital income taxes (8), fixes the amount of seignorage (9), issues public domestic and external bonds (10) and (11) respectively, in order to finance its current expenses *Gc* and current debt servicing *Gs* (12), i.e.

$$T_t = \tau\,(w_t h_t + (\tau_t - \delta)k_t) \tag{8}$$

$$\frac{M_{t+1} - M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t} - \frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}} Rm_t - \frac{M_t}{P_t}\,; \quad Rm_t = \frac{P_{t+1}}{P_t} \tag{9}$$

$$\frac{B^d_{t+1} - B^d_t}{P_t} \tag{10}$$

$$\frac{(B^f_{t+1} - B^f_t)}{P^*_t} \tag{11}$$

$$G_t = Gc_t + Gs_t \qquad Gs_t = r^b_t \frac{B^d_t}{P_t} + r^f_t \frac{B^f_t}{P^*_t} \tag{12}$$

Therefore, the following budget constraint will be faced at every period by the government.

$$\tau[w_t + (r_t - \delta)k_t] + \frac{M_{t+1} - M_t}{P_t} + \frac{B^d_{t+1} - B^d_t}{P_t} +$$

$$+ \frac{(B^f_{t+1} - B^f_t)}{P^*_t} = g_t + r^b_t \frac{B^d_t}{P_t} + r^f_t \frac{B^f_t}{P^*_t} \tag{13}$$

1.4. Balance of Payments

By definition, the Balance of Payments (*BP*) consists of the Current Account (*CA*) and the Capital Account (*CapAcc)*, given by expression (14) below. The *CA* in turn adds the trade balance (*TB*) and the net debt service payments abroad, herein consisting of the services due to the foreign debt position of the model economy, as stated in (15). The *CapAcc* in turn captures the net foreign savings inflow into the economy as expressed in (16).

$$BP_t = CA_t + Cap.Acc_t \tag{14}$$

$$CA_t = TB_t - r_t^f \frac{B_t^f}{P_t^*} \tag{15}$$

where, $TB_t = X_t - M_t = y_t^f - \frac{c_t^f}{P_t^*}$ and,

$$Cap.Acc_t = \frac{(B_{t+1}^f - B_t^f)}{P_t^*} \tag{16}$$

The nominal exchange rate will be determined according to the Purchasing Parity Condition, i.e. $e=P^*/P$, and the net position of foreign assets at steady state, B^f_t, according to the Balance of Payment equilibrium condition $BP = 0$, namely, in the long run equilibrium we have:

$$CA_t = TB_t - r_t^f \frac{B_t^f}{P_t^*} = 0 \tag{16′}$$

Summing up, the above model economy describes an economic environment where a representative household chooses optimally sequences $\{c_t, \frac{m_{t+1}}{P_t}, k_{t+1}, b^d{}_{t+1}\}$, given foreign prices, P^*, factor markets clearing prices, w and r, and the law of motion of the system, $k_{t+1} = (1-\delta)k_t + i_t$. In equilibrium, the aggregate consistency condition is satisfied, i.e. $y_t = c_t + g_t + i_t + TB_t$.

1.5. Competitive General Equilibrium – CGE

Given the above set up of the model economy, the definition of the CGE can be expressed as follows.

Definition: A CGE of the above model economy consists of sequences of $\{c_t\}_0^\infty, \{h_t\}_0^\infty, \{m_{t+1}/P_t\}_0^\infty, \{b_{t+1}^d\}_0^\infty and \{i_t\}_0^\infty$, such that given

(i) exogenous sequences for $\{y_t^f\}_0^\infty, \{r_t^f\}_0^\infty, \{P_t^*\}_0^\infty$; fiscal policy parameters (proportional income tax rate and public expenses share in output) and monetary policy parameters, i.e. τ, ϑ, Rm,

(ii) initial conditions $(B/P)_0 = (b/P)_0 \geq 0, k_0 > 0, m_0 > 0$,

(iii) the law of motion for asset (physical and financial) accumulation, and

(iv) the transaction technology, i.e. , $s(c_t, m_{t+1}/P_t) = c_t (1+ m_{t+1}/P_t)^{-1}$,

(1) the sequences $\{c_t\}_0^\infty$, $\{m_{t+1}/P_t\}_0^\infty$, $\{b_{t+1}\}_0^\infty$ *and* $\{k_{t+1}\}_0^\infty$ solve the representative agent's (consumer's) problem (1),

(2) the sequence $\{k_t\}_1^\infty$ solves the representative firm problem (6),

(3) goods and inputs markets clear, satisfying aggregate consistency, i.e.:

$y_t = y_t^d + y_t^f = c_t + g_t + i_t + TB_t = wh + rk$ and,

external bonds and trade markets clear, i.e. $CA_t = TB_t - r_t^f \dfrac{B_t^f}{P_t^*} = 0$ at steady state, assuming Purchasing Power Parity holds at every period.

2. Steady State Characterization

In this section the steady state solutions for the endogenous variables are algebraically derived. To this end, subsection 2.1 presents the representative household problem expressed as a dynamic programming problem and, the corresponding necessary conditions for optimality. Subsection 2.2 introduces the corresponding competitive firms' problem and factor markets clearing conditions. Finally, Subsection 2.3 explicitly derives analytically the steady state solutions.

2.1. The Households Problem as a Dynamic Programming Problem

Introducing the assumed functional form for the instantaneous utility function (4) and substituting the law of motion for capital formation into the representative household intertemporal problem (1) the initial intertemporal problem can be expressed as:

$$\max_{\{c_t, h_t, m_{t+1}, b_{t+1}^d, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t (\gamma \ln(c_t) + (1-\gamma)\ln(l_t)) \tag{17}$$

subject to

$$\begin{aligned} &\frac{b_{t+1}^d}{R_t} + \frac{m_{t+1}}{P_t} + c_t + k_{t+1} - (1-\delta)k_t = \\ &\qquad = (1-\tau)[w_t h_t + (r_t - \delta)k_t] + b_t^d + \frac{m_t}{P_t} \end{aligned} \tag{18}$$

and $l_t = 1 - h_t - s(c_t, m_{t+1}/P_t)$, (19)

such that $s(c_t, m_{t+1}/P_t) = c_t(1 + m_{t+1}/P_t)^{-1}$, and $c_t \geq 0, m_{t+1} \geq 0, b_{t+1}^d \geq 0, k_{t+1} \geq 0, \quad \forall t \geq 0$

The corresponding Bellman Equation, applying maximum principle, can be therefore expressed as:

$$V\left(k, b^d, m\right) = \max_{\left\{k', b^{d}{}', m', h\right\}} \left\{\gamma \ln(c) + (1-\gamma)\ln(l) + \beta V\left(k', b^{d}{}', m'\right)\right\} \tag{20}$$

where:

$$\begin{aligned} c = (1-\tau)[wh + (r-\delta)k] + b^d + \frac{m}{P} - \\ -\left(\frac{b^{d}{}'}{R} + \frac{m'}{P} + k' - (1-\delta)k\right) \end{aligned} \tag{21a}$$

and $\quad l=1 - h - s(c, m'/P),\ s(c, m'/P) = c\,(1+ m'/P)^{-1}$ (21b)

given $(k_0, m_0, b_0) > 0$.

Substituting budget constraint (22a) and (22b) into objective (21), first order conditions of the right hand problem in equation (21) become:

$$k': \frac{\partial u}{\partial c}\frac{\partial c}{\partial k'} + \frac{\partial u}{\partial l}\frac{\partial l}{\partial s}\frac{\partial s}{\partial c}\frac{\partial c}{\partial k'} = \beta \frac{\partial V\left(k', b^{d}{}', m'\right)}{\partial k'} \tag{22}$$

$$b^{d}{}': \frac{\partial u}{\partial c}\frac{\partial c}{\partial b'} + \frac{\partial u}{\partial l}\frac{\partial l}{\partial s}\frac{\partial s}{\partial c}\frac{\partial c}{\partial b'} = \beta \frac{\partial V\left(k', b^{d}{}', m'\right)}{\partial b^{d}{}'} \tag{23}$$

$$m': \frac{\partial u}{\partial c}\frac{\partial c}{\partial m'} + \frac{\partial u}{\partial l}\frac{\partial l}{\partial s}\frac{\partial s}{\partial m'} = \beta \frac{\partial V\left(k', b^{d}{}', m'\right)}{\partial m'} \tag{24}$$

$$h: \left(\frac{\gamma}{c} - \frac{(1-\gamma)}{(m'/P)(1-h) - c}\right)(1-\tau)w = \frac{1-\gamma}{1-h-s(c, m'/P)} \tag{25}$$

Then, optimal choices require:

(i) From (22) the foregone marginal utility of consumption, which represents the opportunity cost of accumulating physical capital for next period, to be equal to the discounted marginal value obtained next period through accumulation.

(ii) From (23) and (24) the opportunity cost of holding assets (bonds and money) in real terms must equal the discounted marginal value attainable next period with those asset holdings,

(iii) From (25) the marginal utility of leisure must equal the opportunity cost of acquiring it, net of the cost derived from the assumed transaction technology.

Observing the (partial) derivatives of the value function with the respect to state variables z = k, b^d, m , i.e.

$$\frac{\partial V\left(k, b^d, m\right)}{\partial k} = \left[(1-\tau)(r-\delta) + (1-\delta)\right]\left(\frac{\partial u}{\partial c} - \frac{\partial u}{\partial l}\frac{\partial l}{\partial s}\frac{\partial s}{\partial c}\right) \tag{26}$$

$$\frac{\partial V\left(k,b^{d},m\right)}{\partial b^{d}}=\frac{\partial u}{\partial c}-\frac{\partial u}{\partial l}\frac{\partial l}{\partial s}\frac{\partial s}{\partial c} \tag{27}$$

$$\frac{\partial V\left(k,b^{d},m\right)}{\partial m}=\frac{\partial u}{\partial c}\frac{\partial c}{\partial m}+\frac{\partial u}{\partial l}\frac{\partial l}{\partial s}\frac{\partial s}{\partial m} \tag{28}$$

and given the steady state property, namely $\frac{\partial V'(\cdot)}{\partial z'}\equiv\frac{\partial V(\cdot)}{\partial z}$ for all $z=k,b^{d},m$, we can substitute (26) ~ (28) into (22) ~ (25) to obtain the corresponding Euler Equations, as below.

Labor Euler Equation: from (25) above, the optimal choice of time allocated to work needs to satisfy:

$$\frac{\gamma}{c}(1-\tau)w=\frac{(1-\gamma)}{1-h-s\left(c,\frac{m'}{P}\right)} \tag{29}$$

Capital Euler Equation: from equations (22) and (26) above, the optimal choice of physical capital accumulation for next period, at steady state, has to meet the following criteria.

$$\frac{\gamma}{c}-\frac{(1-\gamma)}{l}\frac{1}{\left(1+\frac{m'}{P}\right)}=\beta\left[(1-\delta)(r-\delta)+(1-\delta)\right]\left(\frac{\gamma}{c}-\frac{(1-\gamma)}{l}\frac{1}{\left(1+\frac{m'}{P}\right)}\right) \tag{30}$$

Therefore, at steady state we have the following relationship between the discount factor and the real return of capital,

$$\beta^{-1}=(1-\tau)(r-\delta)+(1-\delta) \tag{31}$$

Real Money Balances' Euler Equation: from (24) and (28) above, the optimal real money holding at steady state is required to satisfy:

$$-\frac{\gamma}{c}\frac{1}{P}+\frac{(1-\gamma)}{l}\frac{\left[\frac{1}{P}\left(1+\frac{m'}{P}\right)+\frac{c}{P}\right]}{\left(1+\frac{m'}{P}\right)^{2}}+\beta\frac{\gamma}{c}\frac{1}{P}=0 \tag{32}$$

Domestic Public Bonds' Euler Equation: from (23) and (27) above, at steady state, optimal domestic public bonds holding is required to meet:

$$\frac{1}{R}\left[\frac{\gamma}{c}-\frac{(1-\gamma)}{l}\frac{1}{\left(1+\frac{m'}{P}\right)}\right]=\beta\left[\frac{\gamma}{c}-\frac{(1-\gamma)}{l}\frac{1}{\left(1+\frac{m'}{P}\right)}\right] \quad (33)$$

Therefore, at steady state the discount factor will equal the inverse of the real return on bonds, i.e. $\beta = R^{-1}$.

2.2. Adding the Competitive Firms Problem

The first order condition for profit maximization together with the factors markets clearing condition, given the Cobb-Douglas constant return to scale technology, give us equilibrium factor prices as:

$$r = A\alpha k^{\alpha-1} \quad (34)$$

$$w = Ak^{\alpha}(1-\alpha) \quad (35)$$

2.3. Steady State Equilibrium

Given the definition of equilibrium given in Subsection 1.5, using the above Euler Equations and equilibrium factor prices, the steady state equilibrium values of the endogenous variables can be determined as follows:

(a) Per worker capital stock, k^{ss}

Substituting into the capital Euler Equation (30) the equilibrium capital rental price (34) we obtain per worker capital stock at steady state k^{ss} solving the following equation.

$$\frac{1}{\beta}=(1-\tau)(A\alpha k^{ss^{\alpha-1}}-\delta)+(1-\delta) \quad (36)$$

$$\Rightarrow \quad k^{ss}=\left[\frac{\beta^{-1}-(1-\delta)+(1-\tau)\delta}{(1-\tau)A\alpha}\right]^{\frac{1}{1-\alpha}} \quad (37)$$

(b) Per worker aggregate output, y^{ss}

Substituting the expression of per worker capital stock into the intensive form of production technology one can obtain the analytical expression for aggregate output as:

$$y^{ss} = Ak^{ss^{\alpha}} \quad (38)$$

Since the (domestic) production is allocated between foreign consumption, y^f, (exogenous) and domestic consumption, c, government expenses, g, and investment i, domestically allocated production can be residually obtained, i.e.

$$
\begin{aligned}
& y^{ss} = y^{d,ss} + y^{f,ss} \\
& y^{ss} = c^{d,ss} + i^{ss} + g^{ss} \\
& y^{f,ss} = \varphi \ y^{ss} \\
& g^{ss} = \vartheta \, y^{ss} \\
& c^{d,ss} = c^{ss} - cm^{ss} \\
& cm^{ss} = \phi \, c^{ss}
\end{aligned}
\tag{39}
$$

(c) Investment at steady state

Given the linear law of motion for capital formation, investment in steady state is in turn given by:

$$i^{ss} = \delta k^{ss} \tag{40}$$

where k^{ss} is given by expression (33) above.

(d) Consumption of domestically produced goods at steady state, $c^{d,ss}$

Substituting (38) and (40) into (39) we obtain the consumption of domestically produced goods at steady state as:

$$c^{d,ss} = Ak^{ss^{\alpha}} - y^f - \delta k^{ss} - cm^{ss} \tag{41}$$

(e) Real money holding at steady state,

Using the Real Money Balance Euler Equation (32), the money holdings at steady state is given as the solution to the second degree polynomial for real money demand function, equation (30) above,

$$a_1(m'/P)^2 + a_2(m'/P) + a_3 = 0 \tag{42}$$

where ; $a_1 = 1 - \beta \ Rm$; $a_2 = -(1-\tau)w^{ss}$ and $a_3 = -(1-\tau)w^{ss}c^{ss}$. Moreover, at steady state, the condition $R = (1-\tau_k)(r-\delta) + (1\text{-}\delta)$ precludes the rate of return dominance of financial versus physical assets and $Rm \le R$ (non-negative gross interest rate) states the arbitrage condition. Also observe that, if $Rm = R = \beta^{1}$, then, real money holdings will equal the equilibrium consumption level, i.e. $(m'/P)^{ss} = c^{ss}$.

(g) Real outstanding debt (bond holding) at steady state, B^{ss}

Using the expression for $c^{d,ss}$ given by (41), *(m'/P)*^{ss} by (42) and k^{ss} by (37) into the government budget constraint (13) taken at steady state, we can find an expression for real domestic bonds holdings as:

$$B^{d,ss} = (R/(R-1))\left[(1-Rm)\left(\frac{m'}{P}\right)^{ss} - (\vartheta-\tau)y^{ss} - B^{f,ss}\right] \tag{43}$$

where external outstanding debt $B^{f,ss}$ can be derived from the Balance of Payment equilibrium condition at steady state: the Trade Balance must be just enough to cover the external debt services, such that $B^{f,ss}=(1/r^f)(y^{f,ss}-cm^{ss})$.

2.4. Aggregate Consistency Condition at Steady State

Aggregate consistency requires that both the households budget constraint and the government budget equations together must be consistent with the aggregate available resources.

Taking the government budget constraint (13) in real terms at steady state, and the budget constraint of the households at steady state (18), the equilibrium assets market for domestic bonds induces that the outstanding real domestic debt of the government must equal the steady state saving decision of the households in domestic financial assets, i.e. $\left(\frac{B^d}{P}\right)^{ss} = \left(\frac{b^d}{P}\right)^{ss}$. Finally, adding the external account steady state condition gives, in turn, the known National Account identity in intensive form:

$$y^{f,ss} - cm^{ss} + c^{d,ss} + g + \delta k^{ss} = w^{ss} + rk^{ss} = y^{ss} \tag{44}$$

Section 3. Parameterization of the Model Economy

The model economy and the corresponding steady state analytical solutions for the endogenous variables presented above will be numerically computed by means of solving for the steady state equilibrium values. The corresponding parameter values are introduced in this section.

To this end, Subsection 1 introduces the list of behavioral parameters needed to compute the steady state Euler equation solutions. Subsection 2, in turn, presents the set of technological parameters, Subsection 3 the set of fiscal policy parameters, Subsection

4 the ones corresponding to monetary policy and, finally, Subsection 5 the long run exogenous relationships, which are assumed as given in steady state for the model economy at hand.

3.1. Behavioral (B) Parameter Values

B1) γ : elasticity of substitution between consumption and leisure, estimated as (1-(1/(1+λ)), where λ= 0.31 computed taking into consideration that, on average, 31% of available time is allocated by the household to market activities.

Source: IBGE

B2) β : intertemporal discount factor set to 0.9

Source: Ellery, Gomes and Bugarin (2001)

3.2. Technological (T) Parameters

T1) $\delta = 0.05$: depreciation rate.

T2) $\alpha = 0.35$, factor share parameter.

Source: Ellery, Gomes and Bugarin (2001)

T3) A = 1, productivity parameter normalized to one at steady state.

3.3. Fiscal Policy (FP) Parameter Values

FP1) τ : proportional income tax rates on labor and capital income.

Computed as equivalent tax rates from observed tax share.

$\tau = 0.2$.

Source: Bugarin (1998), Varsano (1997, 1998) and Secretaria da Receita Federal (1996).

FP2) $\vartheta = 0.17$: steady state government spending (goods and services) participation in aggregate output.

Source: IBGE, System of National Accounts. Mean, 1947-1998.

3.4. Long Run Exogenous Variables

EV1) φ: export share in GDP and ϕ import share in total consumption.

$\varphi = 0.079$, $\phi = 0.09$, thus exports and imports at steady state are computed as $y^f = \varphi y^{ss}$ and $cm^{ss} = \phi y^{ss}$.

Source: IBGE, System of National Accounts. Mean, 1947-1998.

EV2) r^f : foreign real interest rate

$r^f = 2.75\%$

Source: Lam, J.P. and Tkacz, G. (2004)

EV3) $k = K/Y = 2.7$: capital output steady state ratio,

Source: Ellery (2002), or alternatively calibrated from capital Euler Equation.

Table 1 below summarizes the parameter values adopted for the steady state analysis.

Table 1. Parameter Values

Parameters	**Values**
Preferences	$\gamma = 0.6$; $\beta = 0.9$;
Technology	$\delta = 0.05$; $\alpha = 0.35$; $A = 1$;
Fiscal and Monetary Policy	$\tau = 0.2$; , $\vartheta = 0.17$;
Long run relationships	$\varphi = 0.079$; $\phi = 0.09$; K/Y = 1.73
Foreign Variables	$r^f = 2.75\%$; $P^* = 1$

Note: numerical results obtained using opent.m Matlab script file available at Depep/Bacen.

Section 4. Steady State Analysis

This Section briefly describes the main preliminary numerical results obtained in our simulations. Sub-section 4.1. shows the steady state key aggregate relationships obtained at the steady state of the above general equilibrium model economy, whereas Sub-section 4.2. presents the comparative steady state results in terms of key aggregate variables relationships. The sensitivity of alternative steady state characterization is analyzed in terms of the aggregate debt output ratio, operational debt output ratio, domestic debt output ratio, as well as the trade off among seignorage revenue output, aggregate debt output and operational deficit output ratios.

4.1. Steady State Results

The key aggregate variables relationships computed at steady state, with the calibration presented above, are summarized in Table 2 below. It was assumed along this steady state computation that the monetary authority chooses a constant price level at the long run equilibrium. Hence a nominal interest rate of 4.17% is compatible at steady state with this assumption[3].

Table 2. Steady State Key Variables Relationships

Relationship	Code Name	Value at Steady State
Domestic Debt / Aggregate Output	Bd^{ss}/y^{ss}	0.2262
External Debt / Aggregate Output	Bf^{ss}/y^{ss}	0.3306
Aggregate Debt /Aggregate Output	$DY = Bd^{ss}/y^{ss} + Bf^{ss}/y^{ss}$	0.5568
Debt Service / Aggregate Output	$(r^f Bf^{ss} + r^b B^d/P) / y^{ss}$	0.0342
Trade Balance	$TB = y^f - c^f$	0.0046
Terms of Trade	$TT = cm/yf$	0.9091
Tax Share / Output	$TXY = \tau(w + (r-\delta)k)/y$	0.1169

Observation: numerical computation implemented using Matlab code openMAY.m available at Depep/Bacen.

The above Table shows that if the monetary authority chooses a constant price level to prevail at steady state, a gross nominal interest rate of 4.17% has to be taken. Parameterizing the model economy with the Brazilian long run key aggregate relationships, keeping the government share in aggregate output at 17% and the tax rate at 20%, the corresponding steady state shows an aggregate debt output ratio of 55.68%, with an outstanding domestic debt output ratio of 22.62% and external debt output ratio of 33.06%. These figures are compatible with a tax share of 11.60% and a debt service accounting for 3.42% of aggregate output at the long run equilibrium. Moreover, the term of trade turned out to reach 0.9091.

4.2 Alternative Steady States Comparative Analysis

(a) Alternative Government Consumption Share in Aggregate Output and Monetary Policy

This sub-section presents the main results of alternative steady state numerical simulations. This exercise is based on varying the share of government expenses in aggregate output simultaneously with the alternative long run gross nominal interest rate chosen by the monetary authority.

[3] Recall the return on domestic bonds, R, equals at steady state the inverse of the discount factor, β, and the ratio of former to the real return on money balances $Rm=(P/P')$ is, by construction, the gross nominal interest rate, $1+i$. Therefore, if $P'=P$ at steady state, implies $i=(\beta^{1}-1)$.

In this exercise, real variable decisions are unaffected at the steady state equilibrium. The values for these variables are introduced in Table 3.

Table 3. Steady State Real Variables

Variable	**Value at Long Run Equilibrium**
Capital Stock, k	0.1387
Aggregate Product, y	0.5009
Private Investment/Aggregate Output	0.1385
Real Wage	0.3256
Capital Real Rental Price	0.2639

For our numerical simulations, the government share in aggregate output was taken from 0.17% to 20%, keeping the tax rate on labor and capital income at 20%. The alternative nominal interest rates were taken from a close interval of 4.1% to 4.4%[4]. Taking a rather fine grid $\Delta g \text{ x } \Delta i = 201 \text{x } 1001 = 201{,}201$ steady states were computed.

Figures 1 to 4 show the apparent trade-off between different alternative policy choices' equilibrium result at steady state. The policy choices induce different steady state characterizations by means of the following mechanism.

The increase of the government share in aggregate output reduces the consumption goods available at equilibrium. This lower consumption level in turn reduces the long run equilibrium demand for money balances. On the other side, the government can increase the return on money balances by choosing a higher nominal interest rate to be held at steady state. But this choice implies facing a higher share of outstanding aggregate debt in aggregate product, hence a higher operational debt output ratio at the long run equilibrium.

[4] This rather fine interval for the interest rate is taken due to the unstable results obtained outside this range mainly through the resolution of the second order polynomial demand for real money balances.

Figure 1. Aggregate Debt Output Ratios at Alternative Steady States

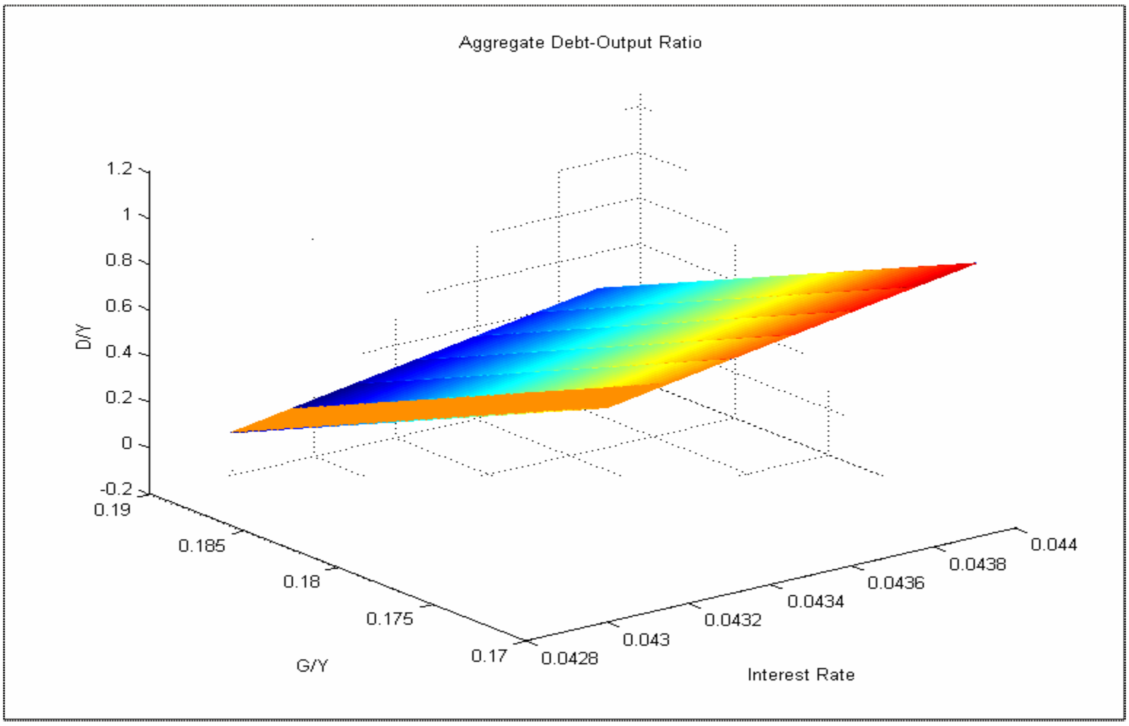
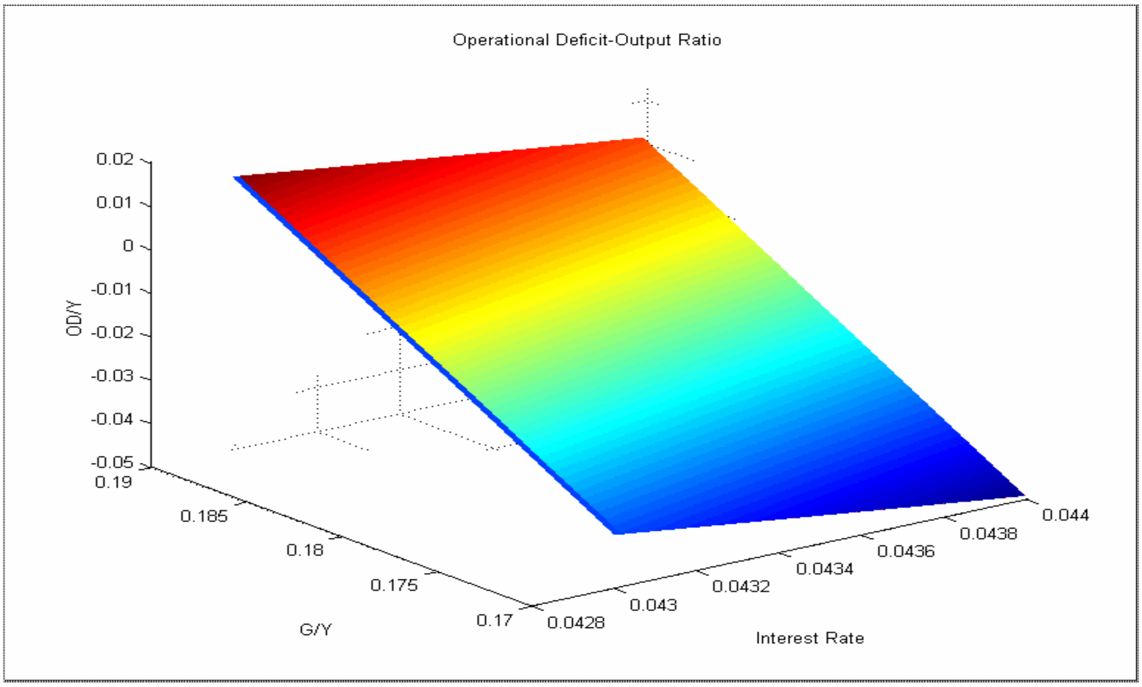


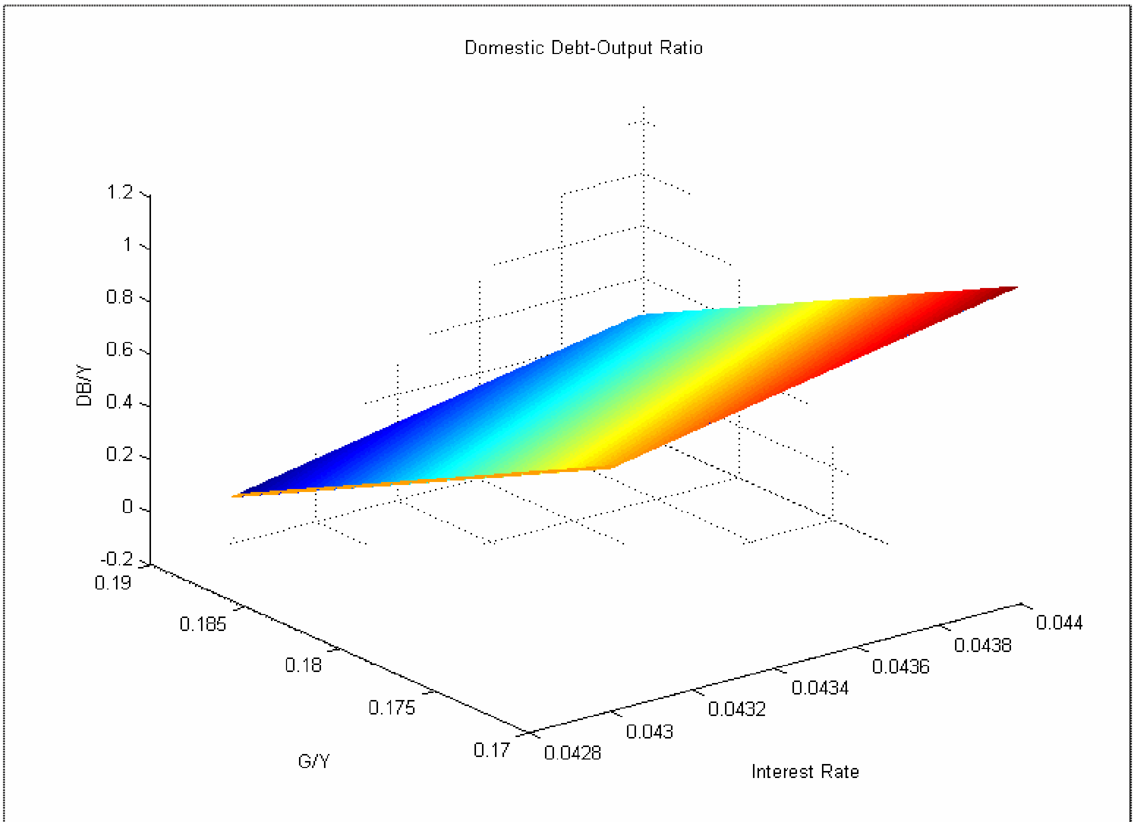
Figure 2. Operational Deficit Output Ratio at Alternative Steady States



Figures 1 and 2 above illustrate the numerical results of alternative steady states. For instance, when the government aims to keep the long run equilibrium at a share of 19% of aggregate output if the selected interest rate is 4.4%, the corresponding aggregate debt output ratio at steady state reaches about 25% of aggregate product with an operational surplus of approximately 1.3% of aggregate output. Comparing both figures, it is apparent that the higher the chosen interest rate and the government share in aggregate output at steady state, the higher the aggregate debt-output ratio, as well as the operational debt participation in aggregate output needed to support it at the long run equilibrium.

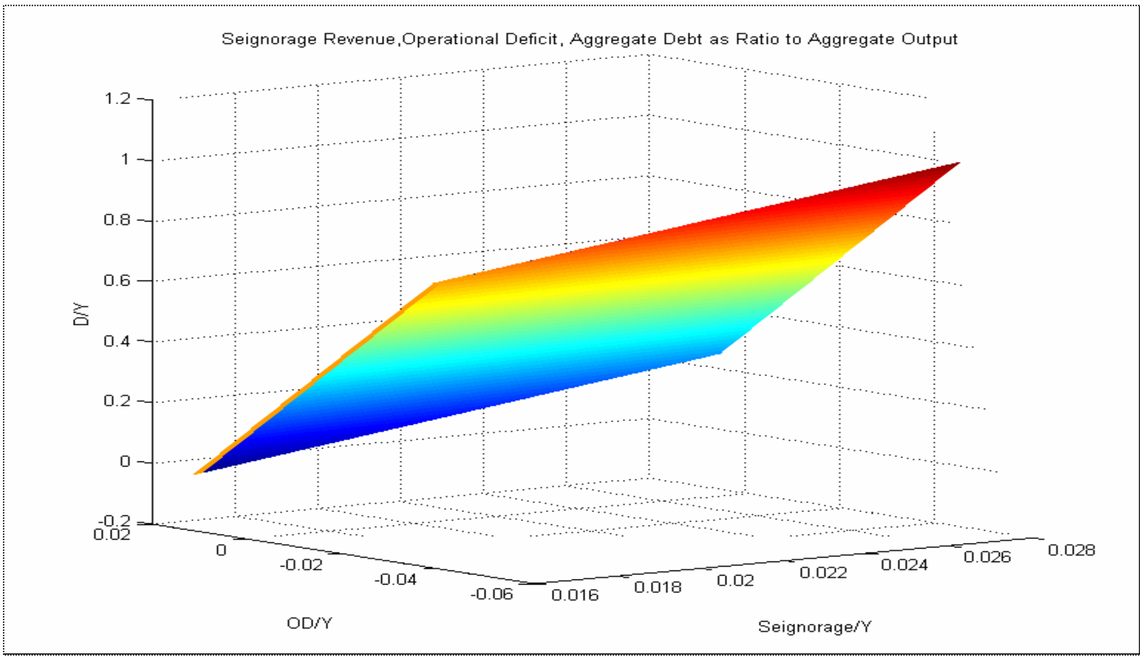
Moreover, as can be seen in Figure 3 below, when a small increase of only 2.33% in the interest rate at steady state, from 4.3% to 4.4%, is considered, the participation of the domestic debt into aggregate output also increases sharply from 5% to 30%, even keeping the government share at 19%.

Figure 3. Domestic Debt Output Ratio at Alternative Steady States



Once the impact of the relationship among fiscal (government share in aggregate product) and monetary (interest rate) policy choices at steady state equilibrium were considered in Figures 1 to 3 above, Figure 4 shows in turn the corresponding different steady state characterizations in terms of the seignorage revenue, aggregate outstanding debt, and operational deficit participation in aggregate product.

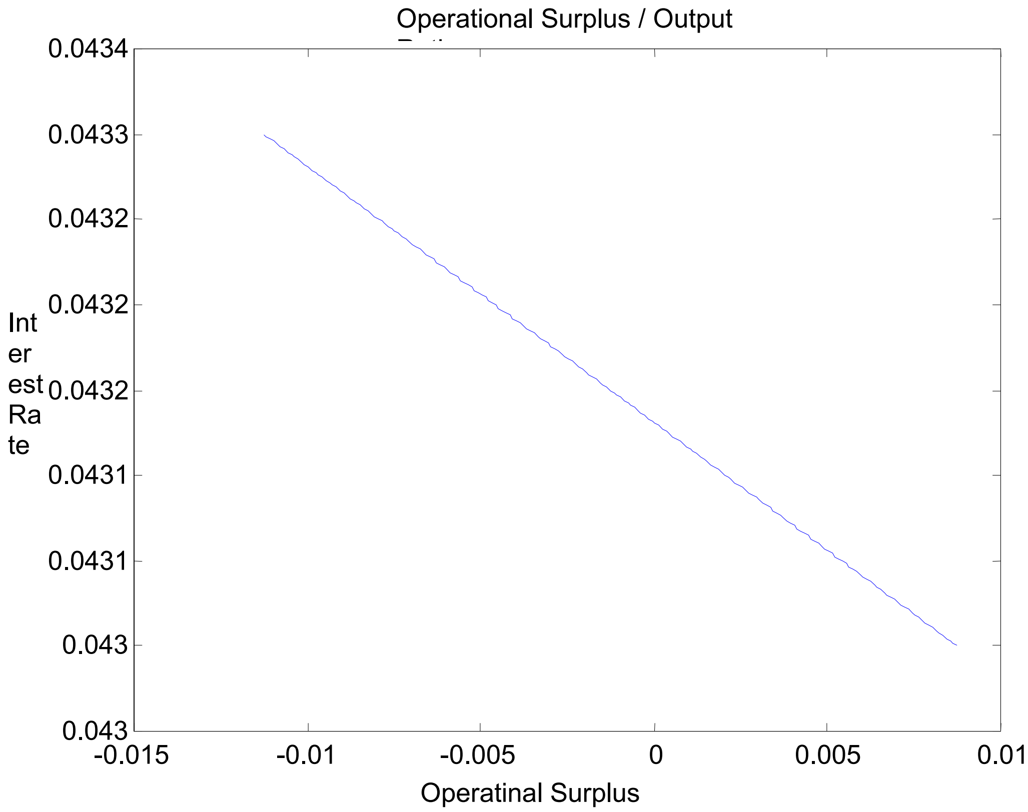
Figure 4. Seignorage Revenue, Operational Deficit and Aggregate Debt as Ratios to Aggregate Output at Alternative Steady States



The plotted alternative steady states clearly show that the higher the outstanding debt output ratio at steady state, the higher the operational deficit and the higher the seignorage revenue financing needed to support those alternative steady states.

Finally, Figure 5 shows the corresponding steady state values of the interest rate policy choice and the resulting operational surplus participation in aggregate output. Clearly, the higher the chosen interest rate at steady state, the lower the consistent operational surplus supported at the long run equilibrium.

Figure 5. Operational Surplus Aggregate Output Ratio and Interest Rates at Alternative Steady States



(b) Alternative Income Tax Rate and Monetary Policy

This sub-section presents, in turn, different steady state equilibrium solutions obtained by varying the proportional income tax rate from 18% to 20% while changing also the monetary policy choices on steady state inflation rate (inverse of the return on money balances). The corresponding alternative nominal interest rates were taken from a close interval of 4.35% to 4.4%, always keeping the observed long run government consumption share in aggregate output of 17%. Taking a rather fine grid $\Delta\tau$ x Δi = 2001x 51= 102.051 steady states were computed.

It is important to note that in this case, the steady state real variables' values change as we vary the income tax rate, through the distortion induced in the capital accumulation process.

Figures 6 (a) to (d) below present the alternative numerical solutions obtained at steady state in terms of main aggregate variables under those alternative policy choices. The lower the interest and the income tax rates chosen respectively by the monetary authority and the fiscal authority, the higher the capital stock at steady state, hence the higher the long run equilibrium aggregate output. Even though a higher tax rate depresses long run aggregate product, aggregate consumption response is positive due to the fact that lower interest rates increase the capital stock and therefore labor productivity. Finally, given a government consumption share in aggregate output of 17%, the lower the inflation rate (higher return on money balances) chosen at steady state by the monetary authority, which is reflected by construction through a higher gross nominal interest rate, the lower is the compatible primary deficit at steady state.

Figure 6. Main Variables at Steady State

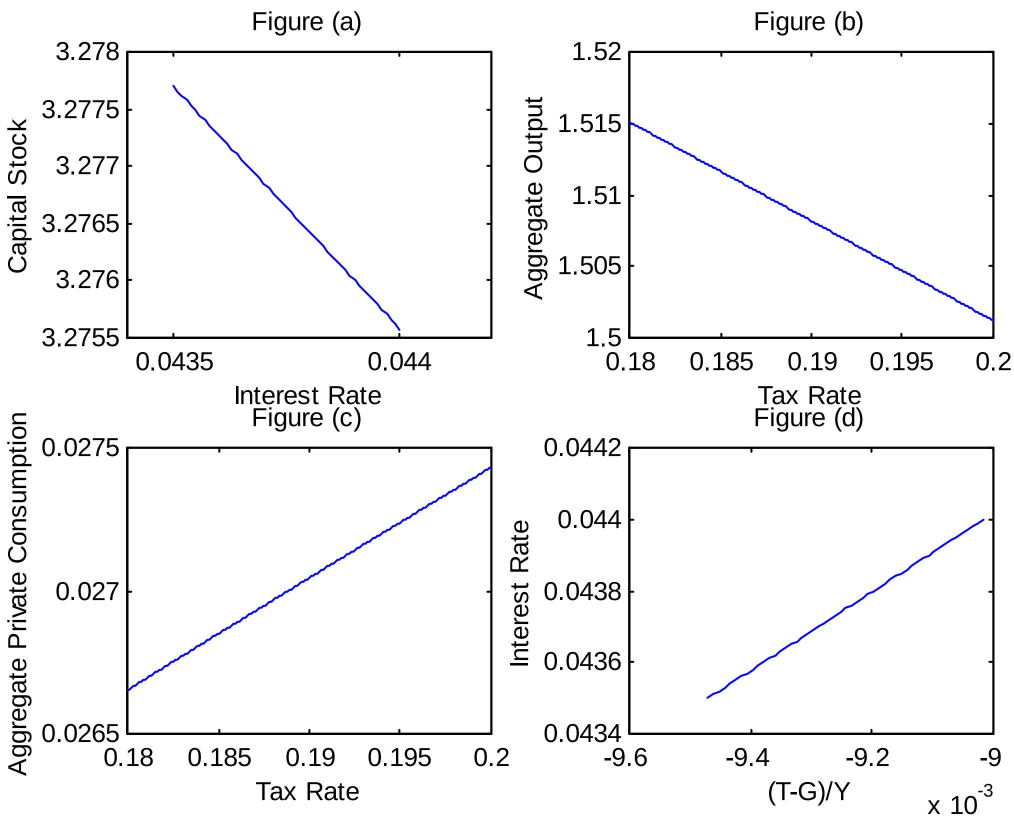
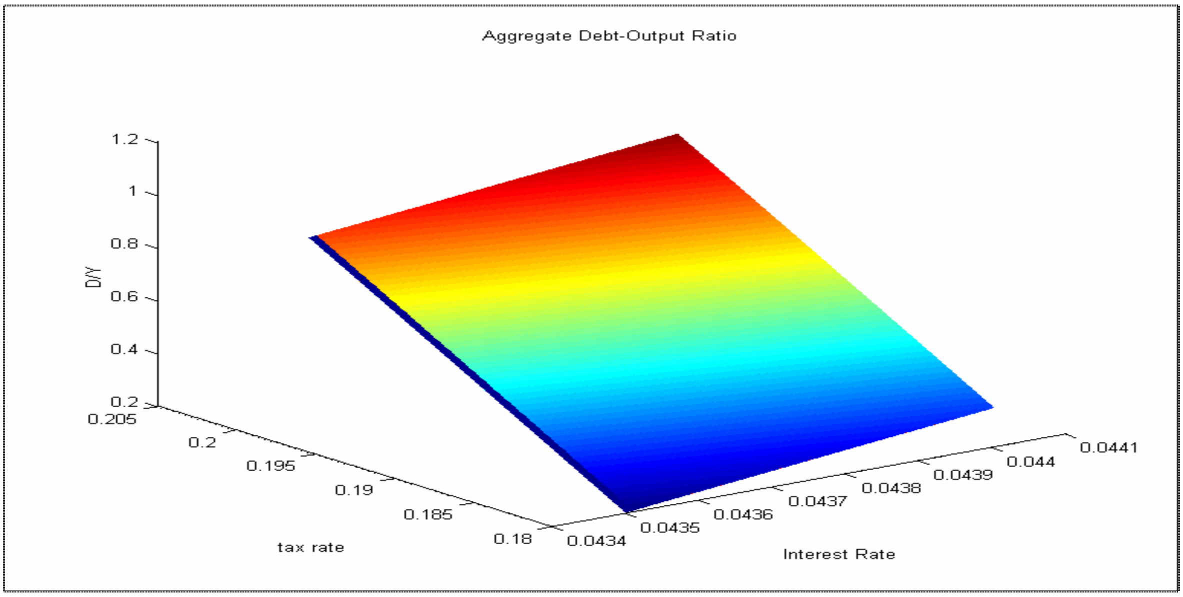


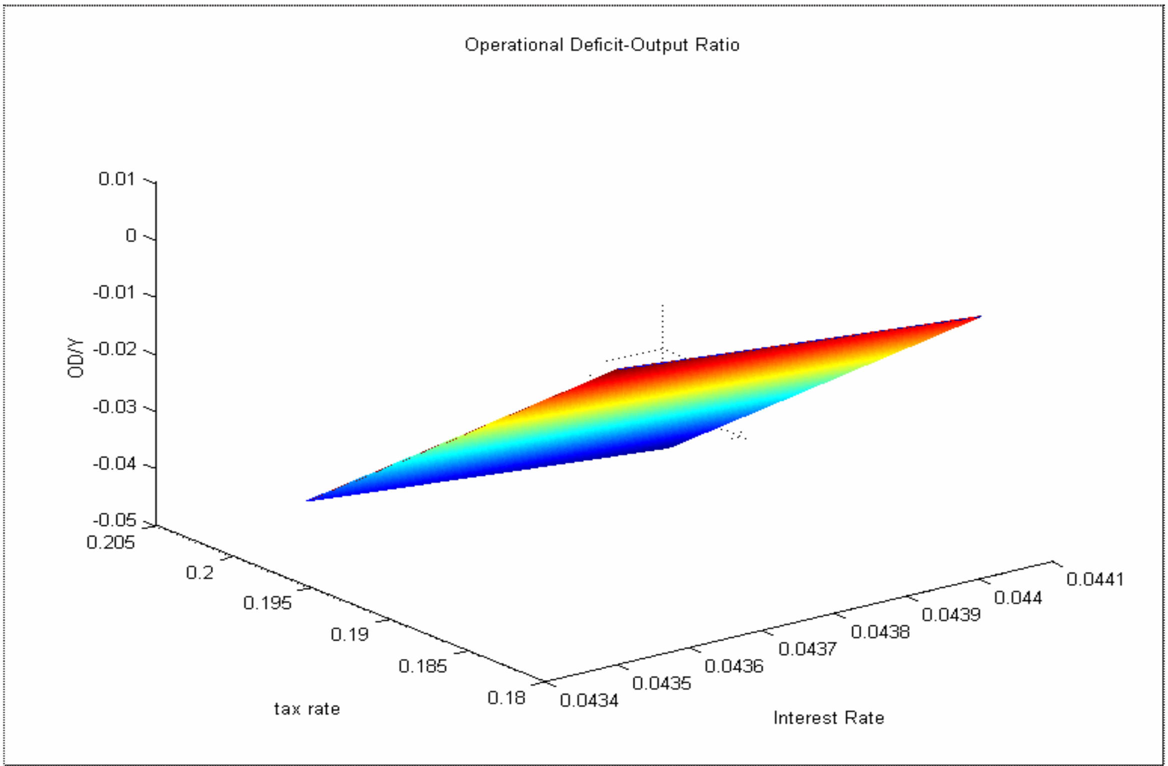
Figure 7 shows the aggregate debt output ratio obtained in our numerical simulation. As can be seen, in this case, the lower the gross nominal interest and the tax rates, set by the authorities to prevail at steady state, the lower the aggregate debt output ratio in the long run equilibrium; i.e. setting them at 4.35% and 18%, respectively, the model economy can achieve a 20% debt output ratio in the long run.

Figure 7



Alternatively, Figure 8 below introduces the effect of the chosen policy parameters in terms of the ratio between the operational deficit and aggregate output at steady state.

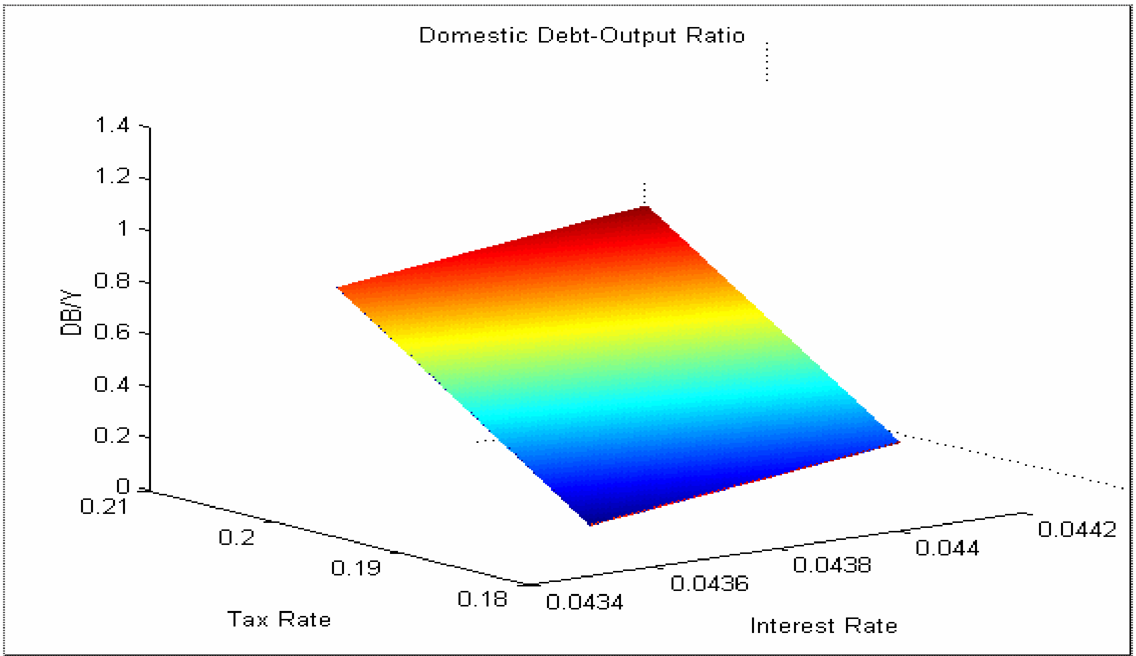
Figure 8



As the figure above shows, all alternatively considered policy choices induce a steady state which is characterized by an operational deficit of at most 4.5% of aggregate product. This is to say that, the generated primary surplus in any of those considered cases is enough for offsetting the resources needed for the outstanding aggregate debt servicing.

Figure 9 presents the outstanding domestic debt as a ratio to the aggregate product at those alternative steady states. For instance, we can see that if the authorities choose to set a steady state inflation rate of 4.24%[5] and a tax rate of approximately 21%, the outstanding domestic debt aggregate output ratio reaches more than 60% of aggregate product.

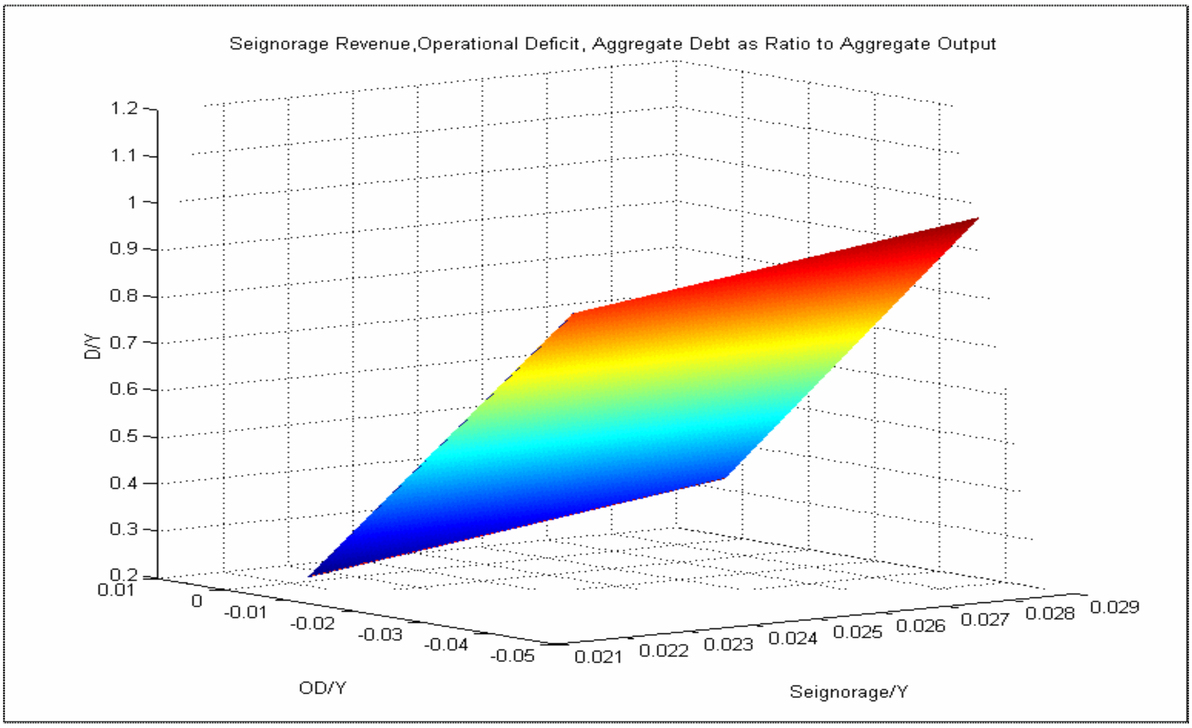
Figure 9



Finally, Figure 10 shows the main fiscal relationship at the alternative considered steady states in terms of the share in aggregate output. The trade-off is apparent, for a steady state characterized by a high outstanding aggregate debt is consistent with a higher operational deficit as well as a higher seignorage revenue requirement.

[5] Recall that at the long run equilibrium the ratio between the return on domestic bonds (inverse of discount factor) and the return on real money balances (inverse of inflation rate) equals the gross nominal interest rate, i.e. $(1/\beta)/(P/P')=i$, therefore if $1/\beta=1.0417$ and $i=4.42\%$, $(P'/P)=4.24\%$.

Figure 10



Conclusion

The above scenario for an open economy, parameterized to reproduce the main long run relationships for the Brazilian economy, provides some interesting insights about the long run behavior of the model economy.

The model is one of the simplest artificial economies one could think of. The only friction in the economy is introduced by the existence of a tax policy affecting the optimal intertemporal allocations of the households. Moreover, it is assumed that the representative households face a transaction technology such that the unitary time endowment is to be spent in equilibrium among leisure, labor and transaction. This transaction technology allowed us to derive a monetary equilibrium in the long run.

The government of the model economy, on the other hand, can choose the participation in aggregate output, as well as the real return on money balances that it would like to support at the steady state.

The numerical simulations give us some insights about the long run characterization of such an economy. With the adopted parameterization compatible with the long run

Brazilian economic evidence, if the monetary authority sets to have a constant price level at steady state (zero inflation rate), the outstanding aggregate debt output ratio reaches 0.3378, and 1.4% of aggregate output must be allocated to service it with a tax effort of 17.87% of aggregate product.

Moreover, the performed exercises clearly show the trade-off between outstanding debt and the primary surplus needed at steady state.

This simulation has also shed light on the direction to be pursed in the present research agenda. The most natural extension should consider alternative steady state characterizations changing the tax parameters that affect the intertemporal allocation of resources, and alternative interest rate at steady state.

Finally, the steady state results naturally call for a study on the transitional dynamics towards the steady state, which can be obtained by adding some more frictions into the model economy, as for instance the presence of a probability of default of the debtor economy.

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