ISSN 1518-3548

BANCO CENTRAL DO BRASIL

Monitoring Vulnerability and Impact Diffusion in Financial Networks

Thiago Christiano Silva, Sergio Rubens Stancato de Souza and Benjamin Miranda Tabak

July, 2015

Working Papers

392

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	n. 392	July	2015	p. 1-40

Working Paper Series

Edited by Research Department (Depep) – E-mail: workingpaper@bcb.gov.br

Editor: Francisco Marcos Rodrigues Figueiredo – E-mail: francisco-marcos.figueiredo@bcb.gov.br
Editorial Assistant: Jane Sofia Moita – E-mail: jane.sofia@bcb.gov.br
Head of Research Department: Eduardo José Araújo Lima – E-mail: eduardo.lima@bcb.gov.br

The Banco Central do Brasil Working Papers are all evaluated in double blind referee process.

Authorized by Luiz Awazu Pereira da Silva, Deputy Governor for Economic Policy.

General Control of Publications

Banco Central do Brasil

Comun/Dipiv/Coivi

SBS – Quadra 3 – Bloco B – Edifício-Sede – 14º andar

Caixa Postal 8.670

70074-900 Brasília – DF – Brazil

Phones: +55 (61) 3414-3710 and 3414-3565

Fax: +55 (61) 3414-1898

E-mail: editor@bcb.gov.br

Citizen Service Division

Banco Central do Brasil

Deati/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Toll Free: 0800 9792345

Fax: +55 (61) 3414-2553

Internet: <http//www.bcb.gov.br/?CONTACTUS>

Monitoring Vulnerability and Impact Diffusion in Financial Networks

Thiago Christiano Silva[*]

Sergio Rubens Stancato de Souza[**]

Benjamin Miranda Tabak[***]

Abstract

In this paper, we propose novel risk-related network measurements to identify the roles that financial institutions play as potential targets or sources of contagion. We derive theoretical properties and provide a clear systemic risk interpretation for the proposed measures. Devised upon the notion of communicability in networks, we introduce the impact susceptibility index, which indicates whether market participants are locally or remotely vulnerable. We show that this index can be used as a financial stability monitoring tool and apply it to analyze the Brazilian financial market. We find that non-banking institutions are potentially remote vulnerable in certain periods, while banking institutions are not susceptible to indirect impacts. To address the perspective of market participants as sources of contagion, we propose the impact diffusion influence index, which captures the potential influence of financial institutions on propagating impacts in the network. We unveil the presence of a portion of non-large banking institutions that is consistently more influential than large banks in potentially diffusing impacts throughout the network. Regarding financial system stability, regulators should identify the entities that play these two roles, as they can render the system more risky.

Keywords: systemic risk, monitoring, vulnerability, impact propagation, networks.
JEL Classification: G01, G21, G28, C63.

[*]Research Department, Banco Central do Brasil, e-mail: thiago.silva@bcb.gov.br
[**]Research Department, Banco Central do Brasil, e-mail: sergio.souza@bcb.gov.br
[***]Universidade Católica de Brasília, e-mail: benjaminm.tabak@gmail.com

1 Introduction

One of the main lessons from the financial crisis of 2008-2009 is that financial systems are interconnected in a complex way. These interconnections form large financial networks in which several types of financial institutions connect to each other by different financial instruments, such as interbank lending, derivatives and other instruments.[1] Although interconnectedness of financial institutions amplifies the propagation of insolvency, illiquidity and losses through the financial system, which may spillover to the real economy exacerbating or even generating financial crisis, to date, there is no consensus on how we can derive financial connectivity measures that can be useful for financial regulation and surveillance.

A very relevant question is how the financial network topology influences the probability of contagion within the network and systemic risk. Allen and Gale (2000), Allen et al. (2012) and Gai and Kapadia (2010) show that more clustered structures carry a higher component of systemic risk. In contrast, Battiston et al. (2012a) show that the relationship between the probability of default, both individually and systemic, and connectivity is U-shaped. Recent research has suggested a core-periphery structure for financial networks (Lux (2015)). Hojman and Szeidl (2008) develop a model of network formation and show that if the benefits from being connected exhibit decreasing returns to scale and decay with the network distance then one can generate a core-periphery network. In a recent paper, Silva et al. (2015) has shown that the Brazilian financial market has a core-periphery structure that changes over time. Little is known on how these topologies imply different levels of systemic risk. Therefore, determining network topologies that are more prone to systemic risk still remains as an important question in the research agenda.[2]

Minoiu and Reyes (2013) show that connectivity is volatile, which implies that network topology may be changing over time. In an interesting case study, Martínez-Jaramillo et al. (2014) show that interbank exposures have changed for the Mexican case after the collapse of Lehman Brothers. Therefore, one could expect network topology to change substantially over time. In this case, monitoring how the network evolves over time is crucial for the assessment of systemic risk and the likelihood of contagion in different periods. Nonetheless, little is known on what measures would be useful for monitoring these financial networks. This issue is of utmost importance as the failure of financial institutions can have relevant implications for the real economy and hence

[1] A study commissioned from the International Monetary Fund, the Bank for International Settlements, the Financial Stability Board and the G20 has found that size is not the only dimension that has to be evaluated when determining systemic importance of financial institutions, although it is the most important factor. The study finds that interconnectedness is the second most important factor in the determination of the systemic importance (IMF et al. (2009)).

[2] See also Gai et al. (2011) which shows that complexity and concentration in financial networks may amplify financial fragility.

far-reaching consequences.

Our paper contributes to the literature in four main ways. First, we propose two new network measures that have clear systemic risk interpretation. Second, we derive theoretical properties for these network measures. Third, we show how these measures evolve over time using a unique dataset on complete exposures for the Brazilian financial market. Fourth, we show how these measures can be useful for the surveillance of financial networks. Therefore, these measures can be very useful in the design of better financial regulation and monitoring of financial systems.

The empirical exercise provides some guidance on how the methodology works in a financial environment dominated by universal banks (characteristic of the Brazilian banking system). Demirgüç-Kunt and Huizinga (2010) argue that after the crises the universal banking model has emerged as a more desirable structure for financial institutions since it is more resilient to adverse shocks due to risk diversification an increase of returns. Therefore, an empirical exercise that evaluates the behavior of universal banks, from a networks perspective, may provide some insights on their inherent risks.

The first risk-related network measurement that we propose is denominated impact susceptibility. The computation of the impact susceptibility is built upon the concept of network communicability, originally introduced by Estrada and Hatano (2008). In general terms, the impact susceptibility is defined as how likely it is for a financial institution (FI) to receive impacts originated from arbitrary points in the network. Thus, this measurement gauges the potential contagion of market participants. The impact susceptibility computation relies on a weighted combination of shortest paths and walks of several lengths that we compute using the vulnerability matrix constructed from the financial network. A directed connection exists in the vulnerability matrix when the default of an entity leads another institution into default. Hence, the vulnerability matrix captures the notion of the importance of the FIs' liabilities in relation to available capital buffers of their neighborhood.

We use that matrix to build a graph that delineates the potential contagion paths that may arise due to arbitrary defaults in the network. The impact susceptibility, thus, measures the potential that these feasible contagion paths end up impacting FIs. The motivation of using not only shortest paths, but also other longer paths is as follows. The contagion transmission may not occur only through the shortest path between different FIs. For example, it may cascade through longer paths in an isolated or additive manner, using paths that are easier to "breakthrough" because of the presence of FIs with low capital buffers. The impact susceptibility captures both types of contagion transmissions.

The impact susceptibility can also provide guidelines for the regulator by pointing out potential candidates for a close surveillance. As such, we can also consider the impact susceptibility as a financial stability-monitoring tool. In our work, we link the impact susceptibility coefficient of each bank to its potential of receiving impacts from the network using a topological approach. We obtain theoretical results that show that, when the impact susceptibility coefficient of an FI is one, the analysis of its local vulnerabilities is roughly sufficient to determine whether or not that FI may lead to amplifications of contagion routes. This is true because, in this situation, the FI communicability to other vertices in the network, except to its direct neighbors, is zero. Opposed to that, when the impact susceptibility of that FI is larger than one, we show that only assessing the financial health of its neighborhood may not provide sufficient information as to whether or not that entity will amplify contagion. This holds true because remote institutions can communicate with these kinds of entities very easily. We relate this phenomenon to the concept of remote vulnerability, which can be understood as the susceptibility of market participants of receiving indirectly hits via contagion processes.

One of our main theoretical contributions in this work is to show that the impact susceptibility of an FI is greater than one if and only if it is remotely vulnerable. Using this result, potential good candidates for a close surveillance are those entities that are remotely vulnerable, because distant defaults are likely to reach them. Note, however, that this is a measure of potential for contagion, not of contagion itself, meaning that whether or not the market participant will default depends on its current capability of absorbing losses from its direct exposures.

In the Brazilian financial market, we find that non-banks are dominant over banks in terms of impact susceptibility. We show that there are periods in which the impact susceptibility of non-banking entities, such as credit unions, is greater than one, revealing their remote vulnerability. Within these periods, we cannot simply evaluate the health of these non-banking institutions by assessing their local vulnerability. This surveillance strategy is insufficient because many potential contagion paths can ultimately affect these non-banks in an indirect or aggregative manner. Even though the health of their neighbors is also important to analyze, as they act as shields and can therefore repel the propagation and amplification of contagion paths, it is not sufficient to assess whether or not these neighbors will be able to absorb losses coming from other communicable players in the network.

Still with respect to our findings connected with the impact susceptibility, we verify that banking institutions have impact susceptibility indices smaller than one in the entire period. This fact suggests that indirect and communicable neighbors cannot affect them. In this way, the analysis of local vulnerabilities of these banking institutions is usually sufficient to determine as to whether or not they may lead to the amplification of contagion routes.

One natural extension of the impact susceptibility index is to transport its meaning from the vertex-level to a network-level measurement. Using this idea, we introduce the notion of impact fluidity. We define the network impact fluidity as how potentially easy can an impact travel throughout the network built up from the FIs' vulnerabilities. Domino-like effects due to an onset of a contagion process are more prone of happening in networks with high impact fluidity. This holds true because, in this situation, FIs tend to be very susceptible to receiving impacts from the network due to their high communicability. Therefore, networks with small fluidity of potential impacts are safer than those with large impact fluidity. We show that the network topology and the available capital buffers of the FIs are the main contributors for establishing the impact fluidity of the network.

With regard to our findings related to the network impact fluidity, we observe that the Brazilian financial market has low fluidity for the entire period. This characteristic suggests that, on average, contagion processes only happen to entities that have direct exposures to the defaulted neighbor. That is, indirect contagion chains are unlikely to materialize. As such, the contagion routes are short, in general. Note, however, that institution-wise surveillance is essential, as some institutions can be very susceptible to potential impacts, while the impact fluidity of the network remains small. Nonetheless, the network impact fluidity supplies a quick gist as to how the network members are susceptible to random defaults occurring in the market.

Another interesting finding that drives attention is that the Brazilian financial market, from 2008 to 2014, has the lowest impact fluidity index exactly in September 2008, which is the period at the heart of the crisis. We relate the low impact fluidity to the fact that FIs were less willing to provide liquidity to their counterparties in that period of turmoil. Hence, the number of exposures that could lead to a potential default substantially decreased in that period.

Another contribution of this paper is the definition of a risk-related network measure that captures the notion of how influent one institution is in terms of diffusing impacts over the network in the quantitative sense. Note that the impact diffusion influence is the opposite perspective of the impact susceptibility, which in turn measures how susceptible one institution is to potential impacts. We can understand the diffusion influence of an FI q in terms of the variation it provokes on the communicability indices of all of the participants when we remove q's power of diffusing impacts in the network built up from the vulnerability matrix. If the communicability decreases at a large extent, then that removed FI plays an important role in diffusing impact throughout the network. Conversely, if its removal slightly modifies the communicability between FIs, then it does not influence the impact diffusion process in the network.

We theoretically show how to factor out the variations of the communicability indices due to the removal mechanism into two terms that are intuitive and important from a viewpoint of impact propagation. These terms are the influence exerted by q in starting and in intermediating the impact propagation. We also provide guidelines for employing the diffusion influence indices as financial stability-monitoring tools. With this purpose, we introduce the concept of remote contagiousness, which is the possibility of an FI q to lead into default other network members that are not directly exposed to q. In addition, we show that, if q is remote contagious, then its impact diffusion influence must be greater than one. This result enables us to find the most harmful entities in the network in a systematic manner.

In financial networks, institutions with high impact diffusion influence have the potential to propagate and amplify impact or losses to the network. This measure, however, does not quantify how harmful these impact propagations can be to the entire financial system. For instance, one institution can have large influence in propagating impacts, but only to non-important institutions. *A contrario sensu*, one institution may have low influence in propagating impact or losses and still have the potential to cause a devastating impact on the financial system. Therefore, we also define a weighted version of the impact diffusion influence by modulating each potential impact to a corresponding proxy of importance of the impacted institution. Note that, while the non-weighted version provides a quantitative measure of how many entities are impacted, the weighted version also brings into play the importance of each of the impacted entities.

We find that banking institutions have larger impact diffusion influence than non-banking institutions in the entire period. Specifically, we show that non-banking institutions are not at all active agents for diffusing impact in the network, as they typically cannot communicate to anyone because they are solely investors in the financial network.

Another key finding of this work is that we unveil the presence of a portion of non-large banking institutions that is more influential than large banks in diffusing impact throughout the network. In terms of financial system stability, regulators must be aware of these most influential entities, as they can render the system more risky. One factor that contributes to this finding is that non-banking institutions prefer to create relationships with non-large banks than with large banks. These non-banking institutions establish financial relations with non-large banks probably because they offer high return rates at the cost of higher risks. In this context, the impact diffusion influence correctly captures this notion that some non-large banks are candidates of potentially diffusing more impact than large banks. Finally, we also show that the large banks' distribution of impact diffusion influence is roughly uniform, while the distribution of impact diffusion influence exerted by non-large banks is highly skewed. This suggests the presence of different classes of non-large banks, which we define in terms of their contribution to the overall systemic risk of the financial network.

1.1 Notation

In order to analyze the network, we extract some network measures from the graph $\mathbf{G} = (\mathscr{V}, \mathscr{E})$ constructed from the borrowing and lending relationships established by members of the financial market. To build up such network, let $\mathscr{V}$ denote the set of vertices (FIs) and $\mathscr{E}$, the set of edges. The cardinality of $\mathscr{V}$, $N = |\mathscr{V}|$, represents the number of vertices or FIs in the network. The matrix $\mathbf{L}$ expresses the liabilities matrix (weighted adjacency matrix), in which the (i, j)-th entry corresponds to the liabilities of the FI (vertex) i towards j. We define the set of edges $\mathscr{E}$ by the following filter over $\mathbf{L}$: $\mathscr{E} = \{\mathbf{L}_{ij} > 0 : (i, j) \in \mathscr{V}^2\}$. In our analysis, there is no netting between i and j.[3] As such, if an arbitrary pair of FIs owe to each other, then $\mathbf{L}$ will present two directed independent edges linking each other in opposite directions. An interesting property of maintaining the gross exposures in the network is that, if an FI defaults, its debtors remain liable for their debts. We also define the matrix of exposures or assets between the FIs as $\mathbf{A} = \mathbf{L}^T$, where T is the transpose operator. In this paper, when we do not mention the type of network that we are using, it is assumed to be the liabilities matrix $\mathbf{L}$.

1.2 Organization

The structure of the remainder of the paper is as follows. In Section 2, we describe the methodology employed in our analysis. In Section 3, we provide meta-information about the Brazilian financial network. In Section 4, we discuss the main results and findings. Finally, in Section 5, we draw some conclusions about the obtained results.

2 Methodology

In this paper, we move forward in the complex network and systemic risk literature by contributing with three new risk-related network measurements: impact susceptibility, network impact fluidity, and the impact diffusion influence (weighted and non-weighted versions). The proposed measures are global indicators, as they make use of all of the relationships contained in the network to derive information. In addition, we provide clear systemic-risk interpretations for each of those indicators.

We start by presenting the communicability concept, opening way to understanding our proposed measures. Afterwards, we detail the contributions of this work.

[3] We do not net out pairwise liabilities so as to maintain consistency with the Brazilian law, because financial compensation is not always legally enforceable.

2.1 Relevant background: communicability concept

In this section, we review the communicability concept in complex networks, which we use to build up our risk-related network measures in this paper. Communicability is defined for every pair of vertices $p \in \mathscr{V}$ and $q \in \mathscr{V}$. In essence, the communicability from p to q quantifies how easily vertex p can communicate with q by means of a combination of shortest paths and random walks with varying lengths. The communicability concept first appeared in the work of Estrada and Hatano (2008). Mathematically, Estrada and Hatano (2008) define the communicability of vertex p to q as:

$$\mathbf{G}_{pq}(\mathbf{M}) = \frac{1}{s!}\mathbf{P}_{pq} + \sum_{k>s} \frac{1}{k!}(\mathbf{M}^k)_{pq} = (e^{\mathbf{M}})_{pq}, \tag{1}$$

in which $\mathbf{P}_{pq}$ denotes the number of paths with the shortest length from p to q; s is the length of such paths; and $\mathbf{M}$ is the binary adjacency matrix obtained from liabilities matrix $\mathbf{L}$. The term $\mathbf{M}_{pq}^{(k)}$ is the (p,q)-th element of the k^{th} power of matrix $\mathbf{M}$, which gives the number of walks of length k from p to q along the liabilities adjacency matrix $\mathbf{M}$, where $k > s$. The communicability of $\mathbf{G}_{pq}$ and $\mathbf{G}_{qp}$ may be different for directed graphs, such as for the financial market network. A large $\mathbf{G}_{pq}$ reveals that p can reach q by several routes. Conversely, when $\mathbf{G}_{pq}$ is small, there are few possibilities for p to reach q.

For a financial system network, the communicability $\mathbf{G}_{pq}$ is a measure of the potential contagion transmitted from p to q, which takes into account the network topology. The onset of a default on p may propagate to q by several paths, either by their shortest ones or through other longer walks. For example, there could be a fragile long path of FIs with insufficient capital to absorb the default of p, in a way that the shock propagates through this route towards q. It is worth noting that a large $\mathbf{G}_{pq}$ does not imply a default of q when p defaults; rather, it simply indicates that q's assets are prone to being impacted in many different ways. On the other extreme, if $\mathbf{G}_{pq}$ is small, p's default may not impact q at all, as there is a small number of possible paths from p to q through which a potential shock can propagate.

2.2 Risk-related network analysis

In this section, we present the main contributions of this work. Note that all of our proposed measures gauge potential contagion in the network.

2.2.1 Impact susceptibility

The original liabilities matrix $\mathbf{L}$ representing the financial network is not a representative matrix for evaluating possible contagion routes among pairs of FIs, in that it only takes into account pairwise exposures. A more depictive network would be one that allowed for computing how well an FI would be able to absorb impacts coming from its exposures. The vulnerability index, which we define as the FI's exposures to its capital buffer ratio, exactly conveys that type of information. With that consideration in mind, we first define a truncated and binary version of the vulnerability matrix $\mathbf{V}$ in the liabilities sense, denoted here as $\bar{\mathbf{V}}$, as follows:

$$\bar{\mathbf{V}}_{ij} = \begin{cases} 1, \text{if } \frac{\mathbf{L}_{ij}}{E_j} \geq 1, \\ 0, \text{otherwise.} \end{cases} \tag{2}$$

Observe that $\bar{\mathbf{V}}_{ij} = 1$ only when bank i's liabilities towards j surpasses j's capital buffer. This configuration illustrates the situation in which the default of i can lead to a subsequent default of j, as j is highly exposed to its neighbor i. Conversely, $\bar{\mathbf{V}}_{ij} = 0$ whenever j can absorb the losses in case i defaults.[4] Note now that the graph topology represented by $\bar{\mathbf{V}}$ quantifies the possible contagion paths delineated by the current conditions and exposures of the market participants.

The graph $\bar{\mathbf{V}}$ not only provides information of direct contagion, but indirect contagion once we take higher powers of $\bar{\mathbf{V}}$. For instance, the entry $(\bar{\mathbf{V}}^k)_{ij} \in \mathbb{N}$ indicates the quantity of contagion paths of length k that starts from i and are transmitted to j due to high vulnerabilities of other FIs. The contagion transmission, however, may not only occur through the shortest path between different FIs. That is, it may cascade through longer paths in an isolated or additive manner, using, for instance, paths that are easier to "breakthrough" because of the presence of members having low capital buffers. The computation of the communicability index in (1) exactly provides this information, once we take as input the truncated vulnerability matrix in (2) instead of the original graph $\mathbf{M}$. We define the institution q's impact susceptibility, here symbolized as S_q, as:

$$S_q(\mathbf{G}(\bar{\mathbf{V}})) = \begin{cases} \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \mathbf{G}_{pq}(\bar{\mathbf{V}}), & \text{when } k_q^{(\text{in})}(\bar{\mathbf{V}}) > 0 \\ 0, & \text{otherwise} \end{cases}, \tag{3}$$

where $k_q^{(\text{in})}(\bar{\mathbf{V}})$ is the number of counterparties to which q is directly exposed and can lead it to default, i.e., it is the number of counterparties p such that $\mathbf{G}_{pq}(\bar{\mathbf{V}}) > 0$. S_q is a linear combination of the communicabilities of all of the other market participants $p \in \mathscr{V}$ to q, except q itself. Note that we have explicitly informed that the communicability is calculated based on the truncated vulnerability matrix $\bar{\mathbf{V}}$, using the notation $\mathbf{G}_{pq}(\bar{\mathbf{V}})$. We

[4]Note that we could also introduce stress between banks by fixing $\bar{\mathbf{V}}_{ij} = \min\left(\frac{\mathbf{L}_{ij}}{E_j}, 1\right)$.

have removed the elements in the main diagonal of the communicability matrix $\mathbf{G}(\bar{\mathbf{V}})$ because they do not convey a measure of intercommunicability between pairs of different FIs. Instead, Estrada and Rodríguez-Velázquez (2005) show that the i-th entry of the main diagonal represent the subgraph centrality or self-communicability of the FI i. In brief terms, the subgraph centrality measures the importance of a vertex by taking into account the number of closed subgraphs of which that vertex is member. Here, in contrast, we are interested in quantifying how easily pairs of FIs can communicate.

We now provide some useful concepts and mathematical derivations that allow us to better understand the properties of the impact susceptibility index.

Definition 1. ***Remote vulnerability:*** *we say that institution q is remotely vulnerable when $\exists p \in \mathscr{V}$ and $k > 1 : (\bar{\mathbf{V}}^k)_{pq} > 0$.*

Remark 1. *Besides being remotely vulnerable to its indirect neighbors, FI q can be remotely vulnerable to its direct neighbors p whenever there exists an indirect path from p to q. The left panel of Fig. 1 shows an example of remote vulnerability coming from an indirect neighbor. The right panel in Fig. 1 shows a situation in which q is remotely vulnerable to FI 2 via the possible contagion path "$2 \Rightarrow 1 \Rightarrow q$." In this case, all of the members in the path are direct neighbors of q.*

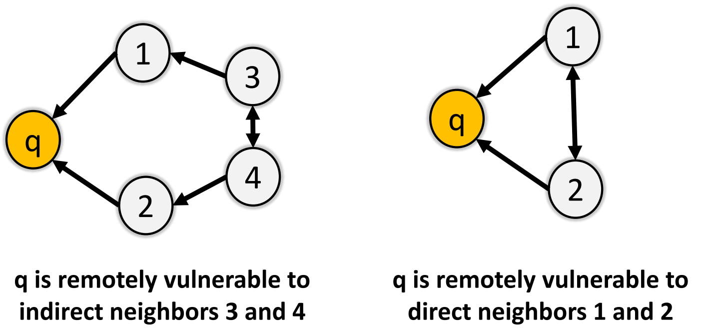


Figure 1: *Illustrations that show situations in which q is remote vulnerable.*

Definition 1 states that remote vulnerability of q can be understood in terms of the existence of paths linking another arbitrary institution p to q in an indirect manner. We move forward in this concept by relating it to our impact susceptibility index in the next demonstrations.

Lemma 1. *If institution q is a singleton, then it cannot be remotely vulnerable.*

Proof. If q is singleton, then $(\bar{\mathbf{V}}^k)_{pq} = 0, \forall p \in \mathscr{V}$ and $k \in \mathbb{N}$. Hence, q cannot be remote vulnerable. ∎

Lemma 2. *If p is a direct neighbor of q, $p \neq q$, then $\mathbf{G}_{pq}(\bar{\mathbf{V}}) \geq 1$.*

Proof. We can express the communicability definition in (1) in terms of a matrix exponential. Rewriting the matrix exponential as a power series, we get:

$$(e^{\bar{\mathbf{V}}})_{pq} = \sum_{k=0}^{\infty} \frac{(\bar{\mathbf{V}}^k)_{pq}}{k!} = (\mathbf{I})_{pq} + (\bar{\mathbf{V}})_{pq} + \frac{(\bar{\mathbf{V}}^2)_{pq}}{2!} + \ldots, \tag{4}$$

where $\mathbf{I}$ is the identity matrix. As $p \neq q$, the first term in the RHS of (4), the identity matrix element $(\mathbf{I})_{pq}$ has a zero value. The second term, $(\bar{\mathbf{V}})_{pq}$ results in 1 as q is a neighbor of p by hypothesis. The order-k term yields non-zero entries if there exists paths of length k from p to q. Putting these facts in together in (4), we get:

$$\begin{aligned}(e^{\bar{\mathbf{V}}})_{pq} &= \mathbf{I}_{pq} + \bar{\mathbf{V}}_{pq} + R_{pq} \\ &= 0 + 1 + R_{pq} \\ &= 1 + R_{pq} \geq 1 \end{aligned} \tag{5}$$

where R_{pq} represents the residual sum of the paths of length $k > 1$ from p to q, whose expression is:

$$R_{pq} = \sum_{k=2}^{\infty} \frac{(\bar{\mathbf{V}}^k)_{pq}}{k!}. \tag{6}$$

From (6), we see that $R_{pq} \geq 0$, because that expression is composed of an infinite weighted linear combination of high-order matrix powers. As the vulnerability matrix possesses only non-negative entries, if we take high-order powers of it, we always get non-negative numbers.

■

Corollary 1. *The equality of Lemma 2 ($\mathbf{G}_{pq}(\bar{\mathbf{V}}) = 1$) only holds when $R_{pq} = 0$ in (5). According to (6), this can only happen when no paths of length $k \geq 2$ exist between p and q. Conversely, it can only exist paths of unitary length linking p to q. In this case, such path is the direct exposure of q to p (or direct liability of p towards q).*

Lemma 3. *If $\mathbf{G}_{pq}(\bar{\mathbf{V}}) > 1$, then q is remotely vulnerable.*

Proof. Invoking Lemma 2, the strict inequality only holds in the case $R_{pq} > 0$ in (5). In accordance to (6), this is true when paths of length $k \geq 2$ exist between p and q. Hence, q must be remotely vulnerable. ■

Theorem 1. *$S_q > 1$ if and only if q is remotely vulnerable.*

Proof. We first note that q cannot be singleton. Otherwise, from Lemma (1), q would not be remotely vulnerable. Therefore, with no loss of generality, we assume that q is not singleton, i.e., $k_q^{(\text{in})} > 0$.

Initially, we prove that the RHS of Theorem 1 implies the LHS: if q is remotely vulnerable, then $S_q > 1$. We start the proof by manipulating algebraically the expression in (3). These steps are valid for proving both ways of the theorem. The derivation of the referred equation is given as follows:

$$
\begin{aligned}
& S_q(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \mathbf{G}_{pq}(\bar{\mathbf{V}}) \\
\Leftrightarrow \quad & S_q(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \left(\mathbf{I}_{pq} + \bar{\mathbf{V}}_{pq} + \sum_{k=2}^{\infty} \frac{(\bar{\mathbf{V}}^k)_{pq}}{k!} \right) \\
\Leftrightarrow \quad & S_q(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \left(\bar{\mathbf{V}}_{pq} + \sum_{k=2}^{\infty} \frac{(\bar{\mathbf{V}}^k)_{pq}}{k!} \right).
\end{aligned} \tag{7}
$$

Note that:

$$
k_q^{(\text{in})}(\bar{\mathbf{V}}) = \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \bar{\mathbf{V}}_{pq} = \sum_{p \in \mathscr{V}} \bar{\mathbf{V}}_{pq} \tag{8}
$$

Plugging (8) into (7) results in:

$$
S_q(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \left[k_q^{(\text{in})}(\bar{\mathbf{V}}) + \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \sum_{k=2}^{\infty} \frac{(\bar{\mathbf{V}}^k)_{pq}}{k!} \right]
$$

$$
\Leftrightarrow \quad S_q(\bar{\mathbf{V}}) = 1 + \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \sum_{k=2}^{\infty} \frac{(\bar{\mathbf{V}}^k)_{pq}}{k!} \tag{9}
$$

$$
\Leftrightarrow \quad S_q(\bar{\mathbf{V}}) = 1 + \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} R_{pq} \tag{10}
$$

We assume, by hypothesis, that q is remotely vulnerable to at least one FI p. Hence, $\exists p \in \mathscr{V}$ and $k > 1 : (\bar{\mathbf{V}}^k)_{pq} > 0$. In other words, $R_{pq} > 0$, for some p. Incorporating this fact into (10) yields $S_q(\bar{\mathbf{V}}) > 1$, and the first part of the proof is complete.

Observe that $S_q(\bar{\mathbf{V}}) \geq 1$ whenever q is not singleton. This is a lower bound for connected institutions in the graph.

Now, we prove that the LHS of Theorem 1 implies the RHS. Using the hypothesis of the LHS of Theorem 1, we get:

$$\begin{aligned}
& S_q(\bar{\mathbf{V}}) > 1 \\
\Leftrightarrow \quad & S_q(\bar{\mathbf{V}}) = 1 + \frac{1}{k_q^{(\text{in})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} R_{pq} > 1 \\
\Leftrightarrow \quad & \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} R_{pq} > 0
\end{aligned} \tag{11}$$

Equation (11) only holds when there exists at least one path of length $k > 1$ between any $p \in \mathscr{V}$, $p \neq q$, and q, i.e., when q is remotely vulnerable to p. Using Lemma 3, we conclude that q is remote vulnerable and the proof is complete. ■

Note that (3) assumes a non-continuous interval of values. If entity q is invulnerable to any other entities, then $k_q^{(\text{in})}(\bar{\mathbf{V}}) = 0$, i.e., q is singleton in the vulnerability matrix and no other FI can communicate with q. In this situation, q's susceptibility for impacts is $S_q(\bar{\mathbf{V}}) = 0$. When q is not singleton, the range of values assumed by the impact susceptibility index is $S_q(\bar{\mathbf{V}}) \in [1, G_N^{\max}]$, where $G_N^{\max}$ is the maximum communicability between any pairs of vertices in a graph with N vertices. Note that the communicability reaches its maximum value in complete graphs. According to Estrada et al. (2008), $G_N^{\max}$ is given by:

$$G_N^{\max} = \frac{1}{Ne}\left(e^N - 1\right), \tag{12}$$

that is, the maximum communicability $G_N^{\max} \to \infty$ as $N \to \infty$.

Equation (3) provides important indicative information as to whether FI q may be impacted in the event of the default of any FIs in the network. If $S_q(\bar{\mathbf{V}})$ is large, the probability that a default of a randomly chosen FI p reaches FI q is higher, whereas if $S_q(\bar{\mathbf{V}})$ is low, that probability is comparatively lower. The impact susceptibility $S_q(\bar{\mathbf{V}})$ alone does not determine contagion risk, but rather provides subsidies to systematically identify whether the presence of vulnerable FIs may render the system more or less risky.

2.2.2 Network impact fluidity

In this section, we introduce another novel proposed network measurement. The potential fluidity of an impact in the network $F(\bar{\mathbf{V}})$ is given by:

$$F(\bar{\mathbf{V}}) = \frac{1}{N} \sum_{q \in \mathscr{V}} S_q(\bar{\mathbf{V}}), \tag{13}$$

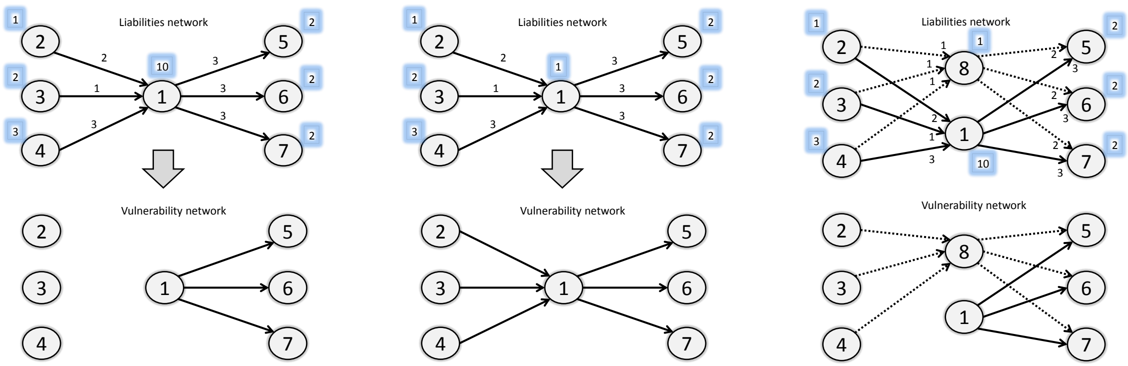
i.e., we quantify the potential fluidity in terms of average total impact susceptibility in the network. It is worth nothing that the network impact fluidity, despite being in a similar format, is not equal to the total communicability concept introduced by Benzi and Klymko (2013), in the sense that, in our measure, the impact susceptibilities are weighted adaptively according to the number of investors of the FIs in accordance with (3).

The idea behind such index is as follows. When most FIs have large impact susceptibility, it is easier that shocks suffered by a randomly chosen FI to propagate to other FIs along the network. In other terms, the vulnerability network formed by the FIs favors the fluidity of impacts in the network and, hence, $F(\bar{\mathbf{V}})$ is large. In contrast, when most of the FIs have small impact susceptibility, few contagion routes exist that ultimately lead to them. In this case, the network retains most of the impact at the vicinity of where the default or original impact happened. As such, $F(\bar{\mathbf{V}})$ is small.

The fluidity of the network can vary mainly due to two reasons: capital buffer and network topology. Figure 2a shows a network with low network impact fluidity, as the FIs in the left side are totally incommunicable with FIs located at the right side. This is because FI 1 acts as a shield in preventing the propagation of impacts from one side to another, as it has a large available capital buffer. Note, however, that if FI 1's capital buffer diminishes, the impact fluidity of the network can largely increase, as Fig. 2b reveals. Another factor that can lead to the increase of $F(\bar{\mathbf{V}})$ is the network topology. Figure 2c shows an example in which the introduction of a new FI permits the transmission of shocks from the left to the right side. In practical terms, the shocks always escape from the most vulnerable FI, in case, FI 8, due to its low capital buffer. In any case, note that the network topology plays a crucial role in both situations. For instance, we can have several FIs with very low capital buffers and still do not have high impact fluidity. This is the case when none of those FIs invests in the market, i.e., they are not exposed to any network members.

2.2.3 Impact diffusion influence

In this section, we detail another contribution of this work, which is the impact diffusion influence index. We have devised hitherto measures of how likely one vertex in a network can be susceptible to impacts occurring on other arbitrary vertices in the network. Now, we take the opposed perspective and construct a measure that shows the potential influence exercised by an FI on the diffusion or propagation of impacts in the network. Observe that, while the susceptibility conveys the concept of how exposed



***(a)** Small fluidity* ***(b)** Large fluidity (capital buffer)* ***(c)** Large fluidity (topology)*

***Figure 2:** Illustrative networks with different network impact fluidities. The numbers inside the blue glowing rectangles symbolize the FI's capital buffer. The numbers near the edges portray exposures of the FIs near the arrows.*

one member of the network is in relation to the remainder participants, the influence on the impact diffusion process suggests how harmful one member of the network is to the others.

The diffusion influence of an FI q can be understood in terms of the variation it provokes on the communicability indices between all of the participants when q's power of diffusing impacts is removed from the network that is built up from the vulnerability matrix. This can be effectively performed by deleting the out-edges emanating from q. This type of filtering transforms q in a sink vertex in the network, for every path that reaches q must end in there. Figure 3 presents an schema of the vertex removal mechanism and how the vulnerability matrix changes when we want to compute the impact diffusion influence of vertex $q \in \mathscr{V}$.

The reasoning behind that procedure is as follows. If q is responsible for diffusing a significant portion of impact throughout the network, then its removal will reduce the overall network impact fluidity, causing the impact susceptibility and communicability of all of the FIs to diminish considerably. In contrast, if q diffuses impact to the network only to a small extent, then the network impact fluidity will remain almost unaltered. In this scenario, the impact susceptibilities and communicabilities of all of the FIs in the network will suffer slight modifications in relation to their original values.

In this context, we express the potential influence that q exerts on diffusing impact to the network, here denoted as $I_q(\bar{\mathbf{V}})$, as:

$$I_q(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\mathrm{out})}(\bar{\mathbf{V}})} \sum_{p \in \mathscr{V}} \sum_{\substack{r \in \mathscr{V} \\ r \neq p}} \left[\mathbf{G}_{pr}(\bar{\mathbf{V}}) - \mathbf{G}_{pr}(\bar{\mathbf{V}}^{(q-)}) \right], \tag{14}$$

where $k_q^{(\text{out})}(\bar{\mathbf{V}})$ is the number of counterparties exposed directly to q that can be lead to default by a q's default. $\bar{\mathbf{V}}^{(q-)}$ denotes the modified truncated vulnerability matrix, in which all of the out-edges that emanates from q are removed (recall Fig. 3). The factor $\left[\mathbf{G}_{pr}(\bar{\mathbf{V}}) - \mathbf{G}_{pr}(\bar{\mathbf{V}}^{(q-)})\right]$ indicates the communicability index of walks from p to r that visit q. This term is evaluated by first computing the communicability index of p to r in the original vulnerability network. From that, we subtract the fraction of that communicability that is not due to a path that has q along the way. Consequently, Equation (14) effectively quantifies the shortfall occurred in the pairwise network communicability, when q's power of diffusing impacts is disabled. In addition, observe that we do not compute self-communicability indices as they do not convey the notion of forward propagation of impacts.

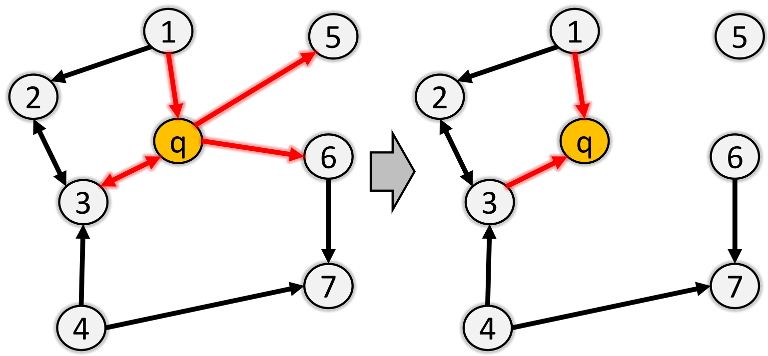


Figure 3: *Illustrative process detailing the vertex removal mechanism used to compute the impact diffusion influence. Vertex q is set to be removed. The bolded red edges are removed because q is in one their endpoints.*

With a close inspection of (14), the terms inside the summation can be conveniently factored out into two groups that are intuitive and important from a viewpoint of impact propagation, which are: the influence exerted by q in starting impact propagation and that related to q intermediating that propagation. The next Theorem provides the mathematical derivations.

Theorem 2. *$I_q(\bar{\mathbf{V}})$ can be decomposed into two orthogonal and complementary terms:*

$$I_q(\bar{\mathbf{V}}) = I_q^{(\text{start})}(\bar{\mathbf{V}}) + I_q^{(\text{inter})}(\bar{\mathbf{V}}), \tag{15}$$

where $I_q^{(\text{start})}(\bar{\mathbf{V}})$ quantifies the potential influence of q on starting impacts and $I_q^{(\text{inter})}(\bar{\mathbf{V}})$ indicates the same for impacts that do not start at q, but necessarily pass through it in the chaining effect. In this second term, q acts as a potential intermediator by transmitting the impact, in principle, to its direct neighbors.

The two terms in (15) are given by:

$$I_q^{(\text{start})}(\bar{\mathbf{V}}) = \begin{cases} \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \mathbf{G}_{qp}(\bar{\mathbf{V}}), & \textit{if } k_q^{(\text{out})}(\bar{\mathbf{V}}) > 0 \\ 0, & \textit{otherwise} \end{cases} \tag{16}$$

$$I_q^{(\text{inter})}(\bar{\mathbf{V}}) = \begin{cases} \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \sum_{\substack{r \in \mathscr{V} \\ r \notin \{q,p\}}} \left[\mathbf{G}_{pr}(\bar{\mathbf{V}}) - \mathbf{G}_{pr}(\bar{\mathbf{V}}^{(q-)}) \right], & \textit{if } k_q^{(\text{out})}(\bar{\mathbf{V}}) > 0, \\ 0, & \textit{otherwise} \end{cases} \tag{17}$$

Proof. The strategy is to start off from (14) and analyze the communicabilities matrices to reach (15). Let $\mathbf{G}(\bar{\mathbf{V}})$ and $\mathbf{G}(\bar{\mathbf{V}}^{(q-)})$ represent the original and modified communicability matrices, respectively. The modified communicability matrix is evaluated by transforming q into a sink vertex. As such, they can be represented by:

$$\mathbf{G}(\bar{\mathbf{V}}) = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{q1} & g_{q2} & \cdots & g_{qN} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N1} & g_{N2} & \cdots & g_{NN} \end{pmatrix}, \qquad \mathbf{G}(\bar{\mathbf{V}}^{(q-)}) = \begin{pmatrix} g'_{11} & g'_{12} & \cdots & g'_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g'_{N1} & g'_{N2} & \cdots & g'_{NN} \end{pmatrix}, \tag{18}$$

where g_{pr} and g'_{pr} are the communicability indices from p to r evaluated from the original and modified communicability matrices. Note that the q-th row of $\mathbf{G}(\bar{\mathbf{V}}^{(q-)})$ must be a zero row, because the diffusion power of q is effectively disabled in the modified vulnerability matrix.

The difference of both matrices in (18) is given by:

$$\mathbf{G}(\bar{\mathbf{V}}) - \mathbf{G}(\bar{\mathbf{V}}^{(q-)}) = \begin{pmatrix} \Delta g_{11} & \Delta g_{12} & \cdots & \Delta g_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{q1} & g_{q2} & \cdots & g_{qN} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta g_{N1} & \Delta g_{N2} & \cdots & \Delta g_{NN} \end{pmatrix}, \tag{19}$$

where $\Delta g_{pr} = g_{pr} - g'_{pr}$. Note that in the q-th row, we simplify $\Delta g_{qr} = g_{qr} - 0 = g_{qr}, \forall r \in \mathscr{V}$. Attempting to approach the functional form in (14), we divide (19) by $k_q^{(\text{out})}(\bar{\mathbf{V}})$ and remove the self-communicability entries $\Delta g_{pp}, \forall p \in \mathscr{V}$, as follows:

$$\frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})}\left[\mathbf{G}(\bar{\mathbf{V}}) - \mathbf{G}(\bar{\mathbf{V}}^{(q-)}) - \mathbf{G}^{(\text{self})}(\bar{\mathbf{V}})\right] = \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})}\begin{pmatrix} 0 & \Delta g_{12} & \dots & \Delta g_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{q1} & g_{q2} & \cdots & g_{qN} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta g_{N1} & \Delta g_{N2} & \dots & 0 \end{pmatrix}, \tag{20}$$

where $\mathbf{G}^{(\text{self})}(\bar{\mathbf{V}})$ is the matrix with zero-valued entries except its main diagonal, which assumes the values $\mathbf{G}_{pp}^{(\text{self})}(\bar{\mathbf{V}}) = \Delta g_{pp}, \forall p \in \mathscr{V}$. This matrix is introduced to remove the self-communicabilities entries from the term $\mathbf{G}(\bar{\mathbf{V}}) - \mathbf{G}(\bar{\mathbf{V}}^{(q-)})$.

Note that we can obtain (14) from (20) by simply summing over all of the matrix entries in (20). The summation of these terms can be performed in a formal way using the following matricial expression:

$$I_q(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})}\mathbf{1}^T\left[\mathbf{G}(\bar{\mathbf{V}}) - \mathbf{G}(\bar{\mathbf{V}}^{(q-)}) - \mathbf{G}^{(\text{self})}(\bar{\mathbf{V}})\right]\mathbf{1} \tag{21}$$

where $\mathbf{1}$ is a vector with dimension $N \times 1$, and T is the transpose operator.

With a matricial representation of $I_q(\bar{\mathbf{V}})$, we are now able to decompose its constituents terms. First, the influence of q on starting a impact diffusion process is related to its communicability to the remainder of the network. This information can be extracted from the summation of the terms in the q-th row in the expression (20):

$$I_q^{(\text{start})}(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})}\left[g_{q1} + g_{q2} + \ldots + g_{qN}\right] \tag{22}$$

$$= \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})}\sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \mathbf{G}_{qp}(\bar{\mathbf{V}}), \tag{23}$$

which retrieves (16).

The remainder of the rows in the factor $\left[\mathbf{G}(\bar{\mathbf{V}}) - \mathbf{G}(\bar{\mathbf{V}}^{(q-)}) - \mathbf{G}^{(\text{self})}(\bar{\mathbf{V}})\right]$ in (20), i.e., rows in the set $\{1, 2, \ldots, q-1, q+1, \ldots, N\}$, account for the influence of q on the role of intermediating the impact diffusions that start from other vertices in the network. That is:

$$I_q^{(\text{inter})}(\bar{\mathbf{V}}) = \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})} \left[\Delta g_{12} + \ldots + \Delta g_{1N} + \ldots + \Delta g_{(q-1)N} + \Delta g_{(q+1)1} + \ldots + \Delta g_{N(N-1)}\right] \tag{24}$$

$$= \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})} \sum_{\substack{p \in \mathscr{V} \\ p \neq q}} \sum_{\substack{r \in \mathscr{V} \\ r \notin \{q,p\}}} \left[\mathbf{G}_{pr}(\bar{\mathbf{V}}) - \mathbf{G}_{pr}(\bar{\mathbf{V}}^{(q-)})\right], \tag{25}$$

which retrieves (17) and the proof is complete. ■

The potential impact exerted by q on initiating a diffusion process is related to how ease q can reach others in the network. This feature is effectively translated by the communicability index, which captures both shortest and longer paths in the vulnerability network. In contrast, the intermediation role of q on diffusing impacts is related to how it influences the communicability indices of other pairs of FIs. If its removal causes the communicability from p to q to reduce in a significant manner, then q's influence on the paths from p to q is important. However, if the communicability is slightly or not modified at all, then q does not play the role of intermediator of impacts of p to q.

In financial networks, institutions with high impact diffusion influence have the potential to propagate impact or losses to a large number of FIs in the network. This measure, however, does not inform how harmful these impact propagations can be to the financial system. For instance, one institution can have large influence in propagating impacts, but only to non-important institutions. On the other hand, one institution may have low influence in propagating impact or losses and still have the potential to cause a devastating impact on the financial system.

Similar to what we have performed for the impact susceptibility, we derive some mathematical formulae to better understand the properties of the impact diffusion influence.

Definition 2. ***Remote contagious:*** *we say that institution q is remotely contagious when $\exists p \in \mathscr{V}$ and $k > 1 : (\bar{\mathbf{V}}^k)_{qp} > 0$.*

Lemma 4. *$I_q^{(\text{start})} > 1$ if and only if q is remotely contagious.*

Proof. Once we transpose the communicability matrix $\mathbf{G}(\bar{\mathbf{V}})$, the same steps followed in proving Theorem 1 can be straightforwardly applied to demonstrate the sufficient and necessary conditions of Lemma 4. ■

Theorem 3. *If q is remotely contagious, then $I_q > 1$.*

Proof. From (15), we know that $I_q = I_q^{(\text{start})} + I_q^{(\text{inter})}$. Employing the hypothesis that q is remote contagious into Lemma 4, we find that $I_q^{(\text{start})} > 1$. Thus, the inequality $I_q > 1$

always holds, because $I_q^{(\text{inter})} \geq 0$. Hence, q must be remote contagious and the proof is complete. ■

Remark 2. *The converse of Lemma 3 is not always true. This is because q can contribute to the diffusion process by both starting and intermediating it. In this process,* $I_q^{(\text{start})} \leq 1$ *and* $I_q^{(\text{inter})} \leq 1$ *and* $I_q > 1$ *can still hold in a way that q is not remote contagious.*

2.2.4 Weighted impact diffusion influence

The impact diffusion influence evaluated in (15) assesses how hazardous one entity can be to other in a quantitative manner: the more entities it can lead to default, direct or indirectly, the higher is its influence. The index, however, does not take into account the importance or value of each of the institutions that are potentially led to default. It is rather intuitive to give larger weights in the impact propagation process to those exposures that lead institutions that are more important. In line with that, we propose a weighted impact diffusion influence measurement as follows:

$$I_q^{(\text{w})}(\bar{\mathbf{V}}, P^{(\text{value})}) = \frac{1}{k_q^{(\text{out})}(\bar{\mathbf{V}})} \sum_{p \in \mathscr{V}} \sum_{\substack{r \in \mathscr{V} \\ r \neq p}} \left[\mathbf{G}_{pr}(\bar{\mathbf{V}}) - \mathbf{G}_{pr}(\bar{\mathbf{V}}^{(q-)}) \right] \cdot P_r^{(\text{value})}, \tag{26}$$

where $P^{(\text{value})}$ is a proxy for the value or importance of all of the financial institution in the market. Note that $I_q(\bar{\mathbf{V}}) = I_q^{(\text{w})}(\bar{\mathbf{V}}, P^{(\text{value})})$ when $P^{(\text{value})} = [1, \ldots, 1]$, i.e., the weighted influence indicator reduces to the original non-weighted influence index once the importance of all banks is set in a uniform manner.

We reinforce that the importance in (26) is attributed to the final destination reached by the walks, instead of their starting point. This allows one to compute a proxy for the extent of the damage potentially caused by a given institution.

3 Data

In this paper, we use a unique Brazilian database with supervisory data.[5] From this database, we take quarterly information on Brazilian domestic financial market exposures, supervisory variables and balance sheet statements. We use accounting information to evaluate the FIs' capital buffers from March 2008 through December 2014. This information is vital to compute some network measurements, such as the impact susceptibility and its derived measures.

[5] The collection and manipulation of the data were conducted exclusively by the staff of the Central Bank of Brazil.

Following Souza et al. (2015), we define the capital buffer of an FI as the FI's total capital (Tier 1 + Tier 2 capitals) that exceeds 8% of its risk-weighted assets (RWA). In Brazil, the capital requirement is 13% or 15% for specific types of credit unions and 11% for other FIs, including banks. Most FIs hold positive capital buffers (their regulatory capital exceeds the requirement). FIs that are not compliant with this requirement are warned by the Supervision and must present a plan to recover compliance in a given period. If the plan is not credible or not feasible, the Authority intervenes. We set 8% RWA as a reference for the computation of capital buffers as we assume that if an FI holds less than what is recommended by the Basel Committee on Banking Supervision (BCBS), i.e., 8% of its RWA, it will take longer to raise its capital to an adequate level and will likely suffer an intervention.

Although exposures among FIs may be related to operations in the credit, capital and foreign exchange markets, here we focus solely on unsecured operations in the money market. The money market comprises operations on private securities. We have information on operations with private securities that is provided by the Cetip[6]: interfinancial deposits, debentures and repurchase agreements collateralized by debentures issued by leasing companies of the same financial conglomerate[7]. In this work, we term the last financial instrument as "repo issued by the borrower financial conglomerate." Figure 4 portrays the total amount of active operations established between members of the financial network from 2008 to 2014. Noticeably, we see that the amount of interfinancial deposits prevails over operations related to debentures and repos issued by the borrower financial conglomerate. The total amount invested in this market by its participants varies from R$28.09 billion to R$61.98 billion in the period analyzed, corresponding to 1.5% of the FIs' total assets and 14% of their aggregated Tier 1 Capital.

We use exposures among financial conglomerates and individual FIs that do not belong to a conglomerate. Intra-conglomerate exposures are not considered. They can be either banking or non-banking financial institutions. Banking FIs can be commercial banks, investment banks, savings banks and development banks. Credit unions represent non-banking institutions. Banks and non-banks are classified by size according to the same methodology applied to their groups, i.e., the group of banks and the group of non-

[6]Cetip is a depositary of mainly private fixed income, state and city public securities and other securities representing National Treasury debts. As a central securities depositary, Cetip processes the issue, redemption and custody of securities, as well as, when applicable, the payment of interest and other events related to them. The institutions eligible to participate in Cetip include commercial banks, multiple banks, savings banks, investment banks, development banks, brokerage companies, securities distribution companies, goods and future contracts brokerage companies, leasing companies, institutional investors, non-financial companies (including investment funds and private pension companies) and foreign investors.

[7]Recall that repurchase agreements are technically secured operations. However, since the borrower in this type of repo guarantees the operation using collateral of a leasing company of the same financial conglomerate, the collateral bears the same credit risk of the borrower financial conglomerate. Thus, in practical terms, the financial operation turns out to be unsecured.

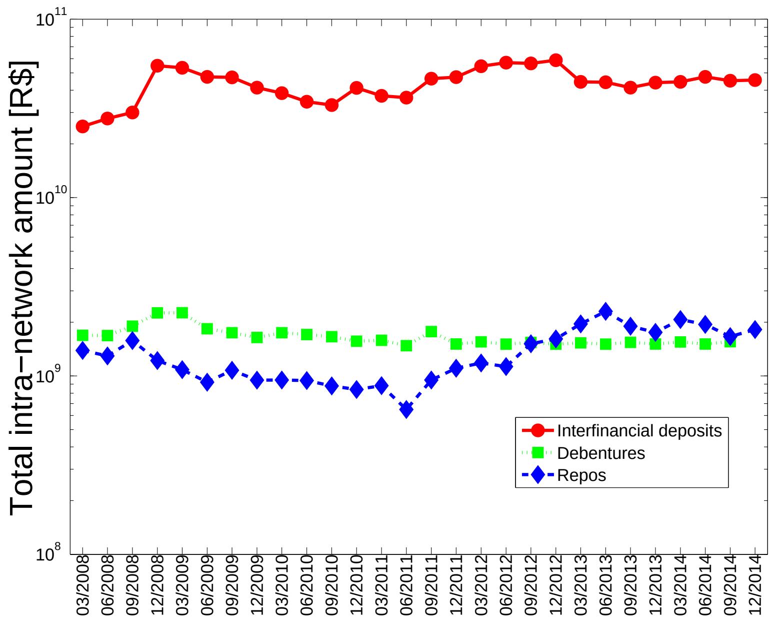


__Figure 4:__ Total amount of active operations established between members of the financial network in the studied period. The y-axis is in log-scale.

banks. We use a simplified version of the size categories defined by the Central Bank of Brazil in the Financial Stability Report published in the second semester of 2012 (see BCB (2012)), as follows[8]: 1) we group together the micro, small, and medium banks into the "non-large" category, and 2) the official large category is maintained as is in our simplified version. Therefore, instead of four segments representing the banks' sizes, we only employ two.

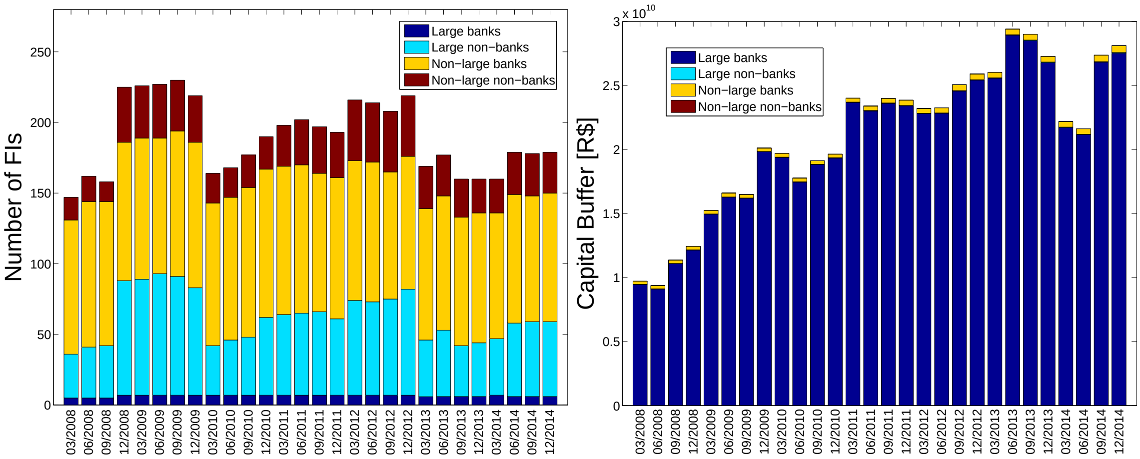
For each pairwise exposure between financial conglomerates or individual institutions, we remove the share that is guaranteed by the Brazilian Credit Guarantee Fund (FGC)[9]. Among the types of financial institutions that we are employing in our analysis, only credit unions are not registered members in the FGC. All of the financial instruments that we are using are covered by the FGC. Until May 2013, the FGC guarantees up to R\$70 thousand for each deposit holder against each registered institution. After that date, due to Resolution 4222 published by the National Monetary Council, that amount increased to R\$250 thousand. Say that the liability of p to q at time t is $\boldsymbol{L}_{pq}(t)$. If p is not a credit union, then we adjust that liability to $\max\left[0, \boldsymbol{L}_{pq}(t) - \mathrm{FGC}(t)\right]$, where:

[8]The Financial Stability Report ranks FIs according to their positions in a descending list ordered by FIs' total assets. The Report builds a cumulative distribution function (CDF) on the FIs' total assets and classifies them depending on the region that they fall in the CDF. It considers as large FIs that fall in the 0% to 75% region. Similarly, medium-sized FIs fall in the 75% to 90% category, small-sized, 90% to 99% mark, and those above are micro-sized.

[9]The Credit Guarantee Fund, whose legal establishment is authorized by the Resolution 2197 emitted by the National Monetary Council, is a private institution responsible for the protection of checking/saving account holders and investors against registered financial institutions in case of intervention, liquidation or bankruptcy.

$$\mathrm{FGC}(t) = \begin{cases} 70{,}000, & \text{if } t < \mathrm{May}/2013, \\ 250{,}000, & \text{otherwise.} \end{cases} \tag{27}$$

Figure 5a displays the evolution of the total number of participants in the financial market. Non-large banks are the majority in the entire period. Large non-banks and non-large non-banks are present in similar quantities in the sample. The number of large banks is the minority and remains roughly constant throughout the period. Although the proportions of the analyzed segments, which are divided into banks and non-banks and size groups as defined above, remain roughly the same, the total number of FIs does change. In special, the number of FIs consistently grows until March 2013, date in which it suffers a considerable drop due to a reduction in the number of large non-banks. Afterwards, we can verify again the upward trend on the number of FIs. Figure 5b presents the mean capital buffer for the same categories of FIs. We can see that large banks have much higher capital buffers than the other entities, which reflects their size differences.



(a) *Total number of FIs* ***(b)*** *Capital buffer*

Figure 5: *Evolution of several features extracted from the Brazilian financial network. We discriminate the trajectories by size (large or non-large) and type of entity (banking or non-banking).*

4 Results

In this section, the main findings of the paper are discussed using our proposed risk-related measurements on the Brazilian financial market. Recall that we build up this financial market using interfinancial deposits, debentures, and repos issued by the borrower financial conglomerate among banking and non-banking institutions.

4.1 Impact susceptibility

We investigate the contribution of the network structure to the fragility of the financial system. To do this, we compute the impact susceptibility of each bank, so that we can identify which banks are more prone to default as a consequence of defaults of other randomly chosen banks. We note that institutions with higher impact susceptibility indices (much higher than 1) should be monitored strictly, specially if they also have high impact diffusion influence.

Figures 6a and 6b display the impact susceptibility coefficient for large and non-large FIs in the financial market from 2008 to 2014. These figures make clear that non-banking institutions are dominant with respect to the average impact susceptibility throughout the studied period. We distinguish the areas denoting remote and local vulnerabilities. We observe that every remotely vulnerable entity must also be locally vulnerable.

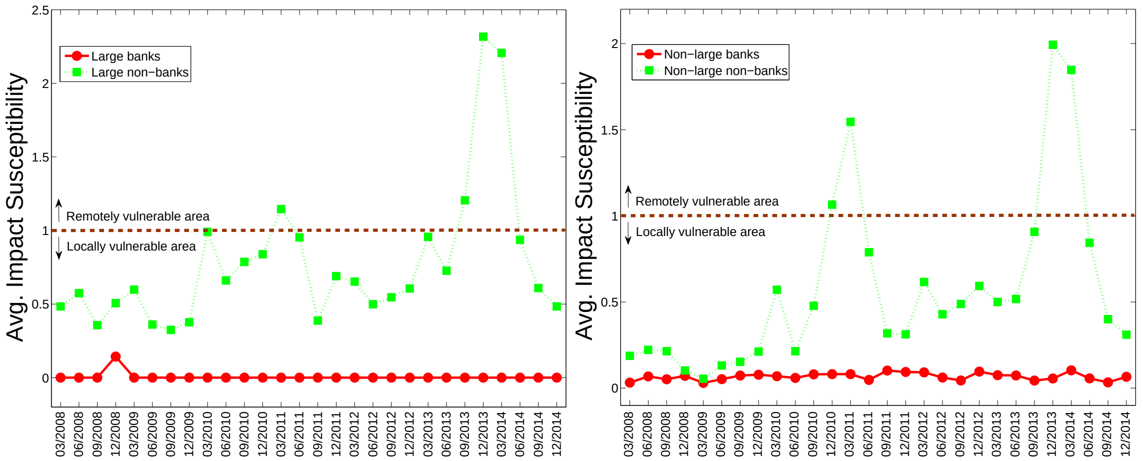
In addition, Figure 6a shows that large banks have zero impact susceptibility indices for almost the entire interval. As such, they are not susceptible to individual defaults because they have a combination of sufficiently high capital buffers (recall Fig. 5b) and relative small exposures to the financial network. In this way, their connectivity is sparse in the vulnerability matrix domain. Nevertheless, there is a single point in which the impact susceptibility of large banks is non-zero: in December 2008. Out of the six existent large banks in that period, one pair of large banking institutions has a large exposure between one another that can potentially lead the creditor side into default if the liability is not honored by the debtor. This explains the positive average impact susceptibility of $1/6 \approx 0.17$ for large banks in December 2008. Large banks, in all of the other periods, are essentially singletons[10] in the vulnerability network.

Figure 6b exhibits a similar picture for non-large banks: very low, but still positive, impact susceptibility values. In addition, for all of the banking institutions we find that the impact susceptibility is less than one, revealing that these entities, in general, are not prone of being remotely impacted from an indirect and communicable neighbor. That is, they are only locally vulnerable. In this way, the analysis of their local vulnerability is generally sufficient to determine as to whether or not they may lead to the amplification of contagion routes.

Still in Figs. 6a and 6b, we see that non-banking institutions are prevalent in terms of impact susceptibility values when we compare against banking institutions. Having in mind that non-banks are almost purely investors in the Brazilian financial market, they are likely to be more exposed than the others. Note that there are periods in which the impact susceptibility is greater than one. For large non-banks, they are: during the first

[10] A singleton vertex is defined as a vertex with no out- and in-edges.

quarter of 2011 and from the second semester of 2013 to the first quarter of 2014. For non-large non-banks: from the fourth quarter of 2010 to the first quarter of 2010 and from the fourth quarter of 2013 to the first quarter of 2014. Within these regions, the financial healthiness of these non-banking institutions cannot simply be evaluated by their local vulnerability. They are remotely vulnerable. Even though the health of their neighbors remains important for a first analysis, as they act as shields and can therefore repel the propagation and amplification of contagion paths, it is not sufficient to assess whether or not these entities with high impact susceptibility will be able to absorb losses coming from arbitrary players in the network.

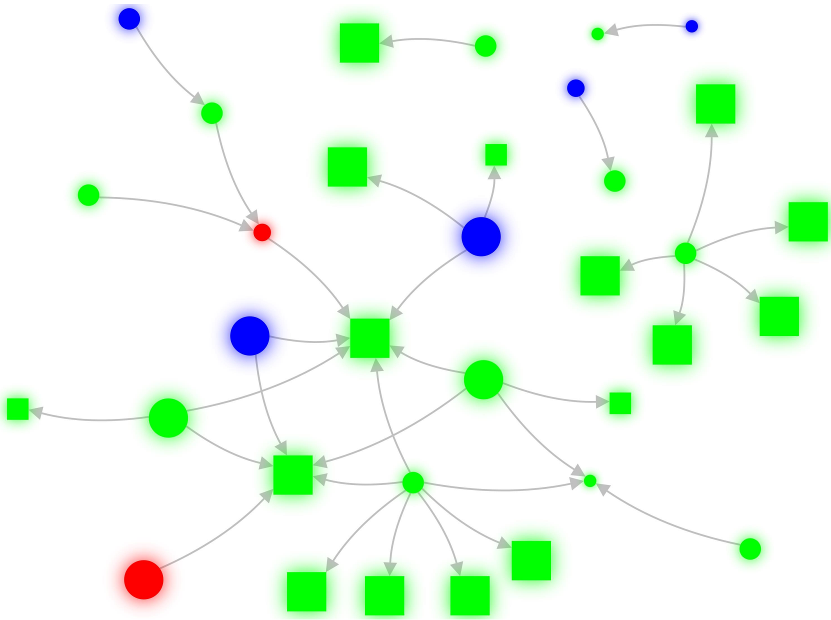


***(a)** Impact susceptibility of large FIs* ***(b)** Impact susceptibility of non-large FIs*

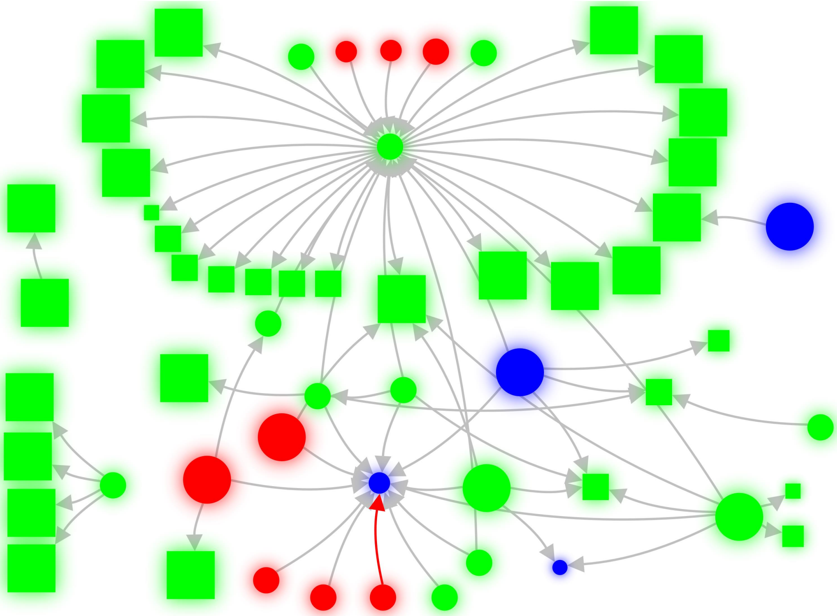
***Figure 6:** Trajectory of the average impact susceptibility of large and non-large FIs for static snapshots of the financial network in different periods.*

In order to get a better glimpse of the discussed results, we show the networks constructed from the truncated vulnerability matrices for September 2008 and December 2013 in Figs. 7a and 7b, respectively. Recall that in the vulnerability matrix an edge from p to q exists only if the default of p forces q into default. We choose the network snapshot for September 2008 because it is a date at the heart of the subprime crisis. We also display the network snapshot for December 2013 in that it is an interesting point in which the impact susceptibility of non-banking institutions is larger than one, according to Figs. 6a and 6b.

These figures pictorially clarify the reason non-banking institutions own the largest impact susceptibilities: the majority of the potential contagion paths end up in them for both periods. They are, thus, leaves or sinks in the graph. This is in line with their investors-only market profiles previously described. Furthermore, we see that it is much more usual potential vulnerabilities of non-banking institutions to non-large banks than to large banks. In September 2008, one non-large bank can potentially lead six large non-banks and another non-large bank institution to default. In contrast, this scenario is accentuated by a large extent in December 2013, in which there is a non-large bank that



(a) Vulnerability network in September 2008



(b) Vulnerability network in December 2013

Figure 7: *Vulnerability networks of the Brazilian financial market computed for two dates. An edge from i to j exists if i's default lead j to default as well. The circles denote banking institutions, while the squares, non-banking institutions. The green color portrays domestic private institutions; the red color, government-owned institutions; the blue color, private with foreign control institutions; and the black color, private with foreign participation institutions. The vertices' sizes are proportional to the corresponding institutions' sizes.*

can potentially impact more than 20 non-banking institutions in a direct hit, and even more indirectly.

In December 2013, we see a higher frequency of pairs of non-large banks leading one another to default than in September 2008. This is one of the reasons non-banking institutions have an average impact susceptibility greater than one, as these vulnerable pairs of banking institutions almost always communicate with non-banking entities. Hence, the latter can be indirectly impacted in a domino-like effect if the indirect, but communicable, banking institution defaults. As such, non-banking institutions must be aware not only of the health on their immediate vicinities, but also of indirect neighbors.

4.2 Network impact fluidity

Given the topology of the vulnerabilities network associated to the financial system, the network impact fluidity measures the network's potential contagion intensity. Put differently, it gauges how far a default of an FI can reach, on average, leading other FI into default. Thus, domino-like effects due to a contagion process are more prone of happening in networks with high impact fluidity. This holds true because, in this situation, FIs tend to be very susceptible to receiving impacts from the network due to their high communicability. Note, however, that the network impact fluidity measures the potential impact and not the realized loss that really occurs. The latter depends on how the agents behave, and how high the capital buffers are of the institutions when the incident occurs.

Figure 8 exhibits the trajectory of the network impact fluidity evaluated on the Brazilian financial market from 2008 to 2014. Observe that the fluidity remains below the mark of one in the entire period, suggesting that, on average, contagion processes only happen to entities that have direct exposures to the defaulted neighbor. That is, contagion processes in an indirect manner are unlikely. As such, the contagion routes are short in general. Note, however, that institution-wise surveillance must be performed, as some entities can be very susceptible to impacts, while the impact fluidity of the network is small. Nonetheless, the network impact fluidity supplies a quick gist as to how the network members are susceptible to random defaults occurring in the market.

Looking back at Figs. 7a and 7b, which shows the vulnerability networks in September 2008 and December 2013, respectively, one fact that drives attention is of the significantly larger quantity of edges in December 2013 than in September 2008. In fact, the densities of the vulnerability networks in September 2008 and December 2013 are 0.11% and 0.18%, respectively.[11] In principle, this observation evidences that the network impact fluidity in December 2013 is much higher than in September 2008, as we can see from Fig. 8. In fact, the network in September 2008 marks the lowest network impact fluidity in the studied period. One reason for that stems from a regulatory resolution

[11]Contrasting to that, the densities formed by the liabilities networks in September 2008 and December 2013 amount to 1.85% and 1.25%. This means that, even though there are more pairwise relationships between institutions, the lent and borrowed amounts are smaller. This fact further strengths the argument that impact fluidity in networks is not necessarily correlated to the density of the liabilities network. Instead, it both depends on the network topology and the FIs' capital buffers, for they may provide fragility or robustness to the system.

published by the National Monetary Council[12] and accessory regulations emitted by the Central Bank of Brazil that came into effect in July 2008[13]. That resolution modified the computation of the capital requirements of financial institutions. In special, the resolution largely broadened the risk coverage of supervised institutions by the Central Bank of Brazil. As a result, we observe an increase of the tier 1 capital share of the available capital buffers of FIs. This may explain in part the reason of the robustness of the Brazilian financial market in September 2008, which is captured by its low network impact fluidity in the referred period.

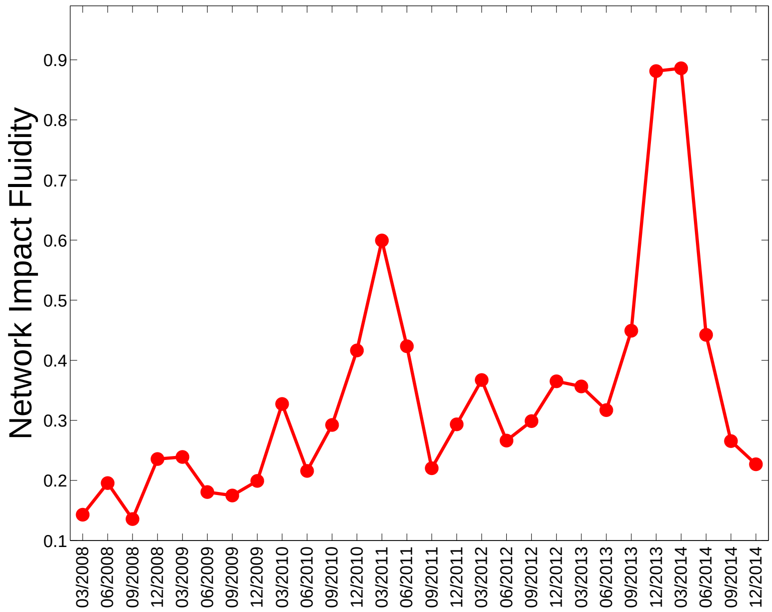


Figure 8: *Trajectory of the network impact fluidity for static snapshots of the financial network in different periods.*

4.3 Impact diffusion influence

Opposed to the impact susceptibility that measures if a bank may default as a consequence of the default of others, the impact diffusion influence measures the potential influence of an institution on the impact propagation over the network. That is, the more institutions it can potentially propagate impacts to, the larger is its exerted influence. In addition, the impact diffusion influence of an institution is composed of two orthogonal terms: the potential influence that it exerts when it starts the diffusion process, i.e., when it is the first institution in the propagation chain, and when it acts as an intermediator by being part of a contagion chain originated by another institution. Figures 9a and 9b exhibit the average impact diffusion influence of institutions when they play the role of

[12]The National Monetary Council is the major institution of the Brazilian National Financial System. It is in charge of formulating monetary and credit policies, aiming at the preservation of the Brazilian monetary stability, and the promotion of economic and social development.

[13]The reader is referred to BCB (2008) for more information. The resolution number published by the National Monetary Council is 3.490, which was first published in August, 29th, 2007.

starters in the diffusion process for large and non-large institutions, respectively. In turn, Figs. 9c and 9d depict the impact diffusion influence for large and non-large institutions, respectively, when they act as intermediators in the diffusion process. Summing up both contributions, Figs. 9e and 9f portray the total impact diffusion influence of large and non-large institutions, respectively, in the Brazilian financial market from 2008 to 2014.

An interesting finding is that impact diffusion influence exerted by large banks is solely due to its potential influence on starting a diffusion process. In this way, they do not act as potential intermediators in an ongoing contagion process. This is an interesting property that provides robustness for the Brazilian financial market. One reason is that the network presents very strong patterns of disassortative mixing and core-periphery structure,[14] in such a manner that almost all of the non-large institutions are likely to be directly connected to a large banking institutions. In this configuration, the large bank can stop the propagation process at the initial phase of the contagion process. Hence, contagion chains are expected to be small. Another advantage of this setup is that large banks often have very large capital buffers (recall Fig. 5b), in such a way that it is very unlikely for a contagion process to propagate through them.

In contrast to that, the impact diffusion influence imposed by non-large banking institutions is due to their routes as starters and intermediators in a potential diffusion process in the network. In special, we see two prominent peaks happening for both diffusion influence terms in the first half of 2011 and from the fourth quarter of 2013 to the first quarter of 2014. We can get a gist as to why there is a peak of the impact diffusion influence for non-large banks in December 2013 by looking at Fig. 7b, which shows the vulnerability network at that period. In that occasion, there is a non-large bank that can lead to default more than twenty non-banking institutions. Moreover, such non-large bank is susceptible to be impacted by several other banking institutions. This setup skyrockets the impact diffusion influence indices for institutions that are communicable to that specific non-large bank. Hence, the appearance of that contrasting peak.

We can see that non-banking institutions are not the main actors in diffusing impacts over the network, as they typically cannot communicate to anyone. In fact, inspecting Fig. 7, they correspond to leaf vertices most of the time, because they are "dead-end" vertices. In special, the impact diffusion influence is zero for non-large non-banks in the entire period. Large non-banks, notwithstanding, have zero impact diffusion influence from September 2008 to September 2012. After that point, the average influence index is small, but remains positive until December 2014, specially because of the impact diffusion influence that measures how important they are as intermediates in the diffusion process. The region ranging from the fourth quarter of 2012 to the third quarter of 2013

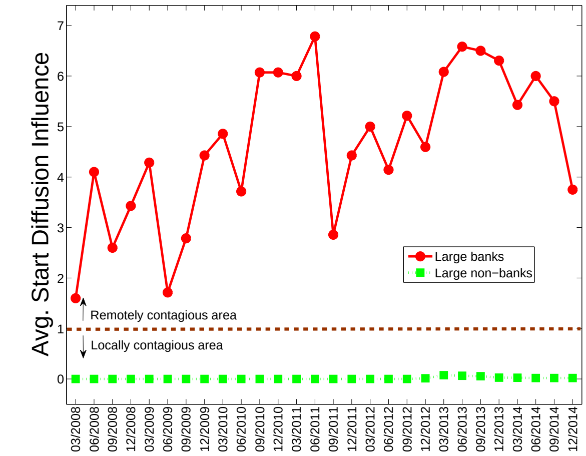
[14] See Silva et al. (2015) for a qualitative discussion on the network topology of the Brazilian financial network.

shows that a few large non-banking institutions have non-zero values for intermediating diffusion processes. (The average is low because most of those institutions have intermediating diffusion process influences equal to zero.) Examples of these features can be visually noticed in the vulnerability networks depicted in Fig. 7. For instance, inspecting the vulnerability network in September 2008 in Fig. 7a, it is clear that non-banking institutions are incommunicable to the remainder of the network. Diverging from that, looking at the vulnerability network in December 2013 exhibited in Fig. 7b, we do see few non-banking institutions that can possibly lead other FIs into default, mostly non-large banks. As such, they can potentially inflict damage to others; hence, the positive impact diffusion influence.

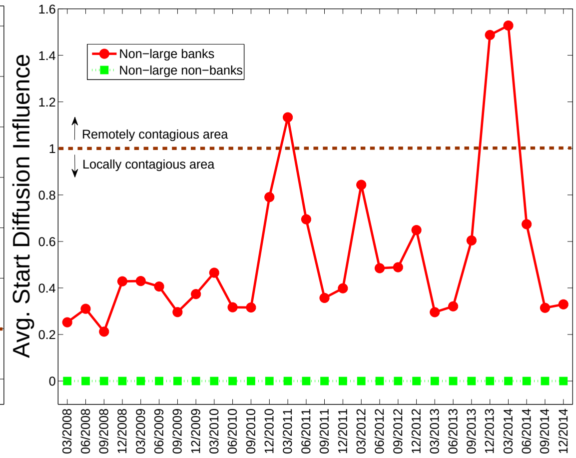
Returning to Fig. 9, we notice that banking institutions have larger impact diffusion influence than non-banking institutions in the entire period. Interestingly, the influence of large banks in potentially propagating impacts is, on average, of the same magnitude order of those of non-large banks. However, by inspecting the data, on one hand, we verify that all of the large banks have similar non-zero influence. On the other hand, the distribution of impact diffusion influence exerted by non-large banks is highly skewed. For instance, looking at Fig. 7, we can see that some non-large banks play central roles in diffusing impacts, and, in many cases, these roles are much more central than that of large banks. One argument to support that is that the vulnerability network in December 2013 has a non-large bank that can diffuse potential impact to more than twenty non-banking institutions in a direct hit. The large bank that potentially can inflict more damage, however, only affects seven other institutions.

In order to check for this skewness, we plot in Fig. 10 the average impact diffusion influence of large banks against the top $M(t)$ non-large banks, where $M(t)$ represents the number of large banks existent in the network at time t (recall Fig. 5a). Note that these top non-large banks influence much more in the process of diffusing impact over the network as the vulnerability networks in Fig. 7 clearly show. We see that non-banking institutions prefer to create relationships with non-large banks than with large banks. Non-large banks are often more demanded by them probably because they offer high return rates at the cost of higher risks. In view of that, our proposed index for measuring impact diffusion influence correctly captures this notion that top non-large banks are candidates of potentially diffusing more impact than large banks.

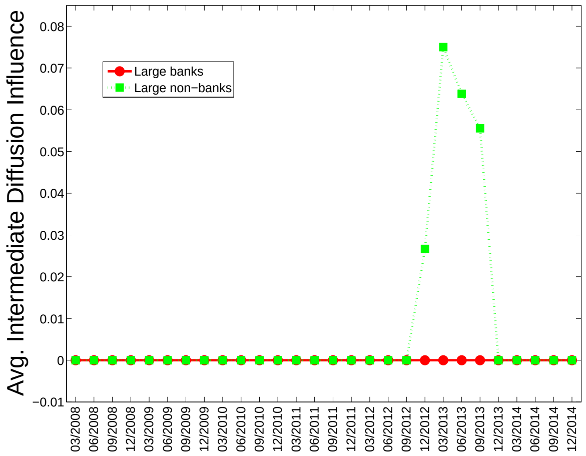
Figure 11 provides another subsidy to confirm the skewness of the distribution comprising the diffusion influences exerted by non-large banks. Numerically, a large portion of the non-large banks have zero impact diffusion influence in the studied period, which contributes to diminishing their corresponding average influence index. It is clear that, on average, more than 90.20% of non-large banks do not take part in diffusing potential impacts throughout the network. In contrast, however, there is a small quantity of non-



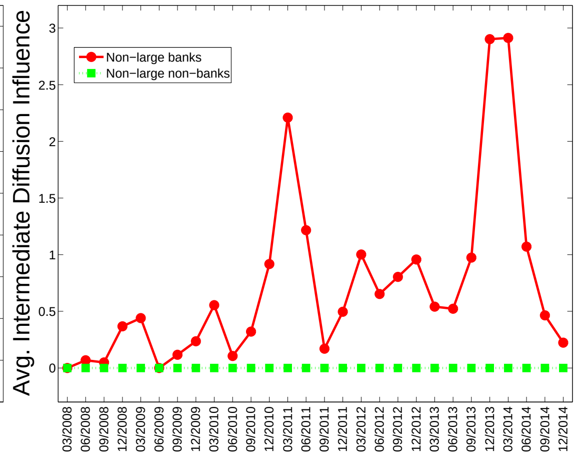
***(a)** Start impact diffusion influence of large FIs*



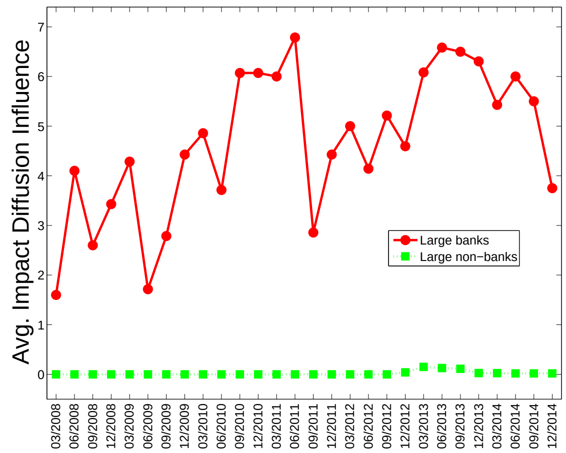
***(b)** Start impact diffusion influence of non-large FIs*



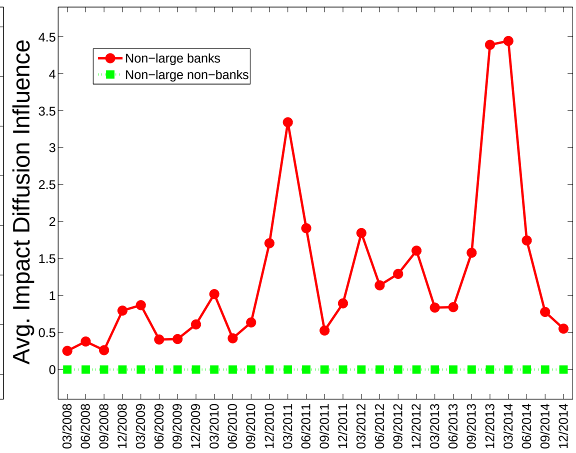
***(c)** Intermediate impact diffusion influence of large FIs*



***(d)** Intermediate impact diffusion influence of non-large FIs*



***(e)** Impact diffusion influence of large FIs*



***(f)** Impact diffusion influence of non-large FIs*

***Figure 9:** Trajectory of the average impact diffusion influence and its constituent parts (start and intermediate factors) for static snapshots of the financial network in different periods.*

large banks, which appear in the vulnerability networks depicted in Fig. 7, that is more influential than the average impact diffusion influence exerted by large banks. In the pe-

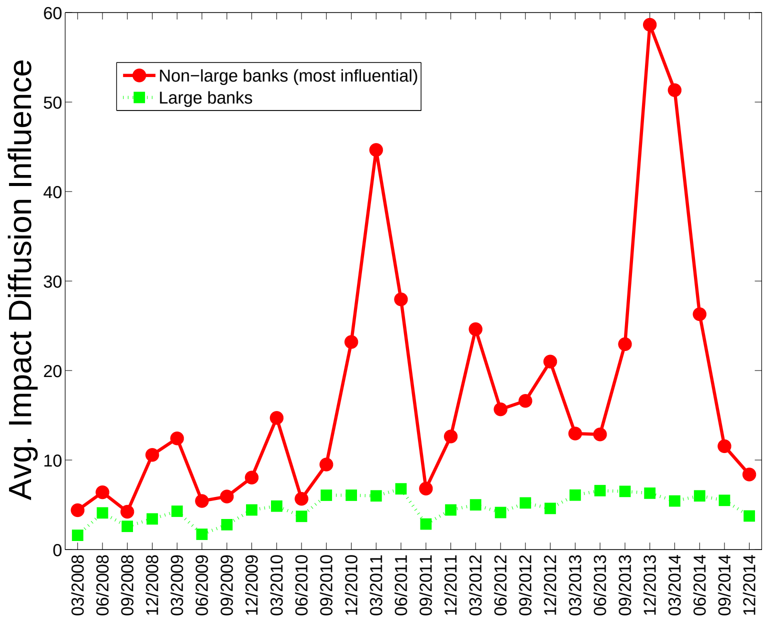


Figure 10: *Comparison of the average impact diffusion influence of large banks and the top $M(t)$ non-large banks, where $M(t)$ is the number of large banks at instant t.*

riod, this portion of most influential non-large banks is composed, on average, of 3.92% of the non-large banks. There is a reminiscent of non-large banks that have a smaller, but non-zero, impact diffusion influence than the average of large banks. They account, on average, for just 5.88% of the non-large banks. In terms of surveillance on the financial system, regulators must be aware of the these most influential entities, as they can render the system more risky. In the Brazilian financial market, they are very few, but may potentially harm the system as an entirety in the sense of the impact diffusion influence index.

4.4 Weighted impact diffusion influence

The impact diffusion influence gives an equal importance for all of the impact diffusing institutions in the network. However, in general, causing the default of a large bank is much more catastrophic than of a non-large non-bank in terms of overall assets losses. That considered, providing a measure of value or importance to each of the institutions is essential, as we are able to get quantitative instead of only qualitative information as the non-weighted influence index supplies. Thus, we use, as a proxy for importance, the liabilities that banks have on the financial market, i.e., we employ $P_p^{(\text{value})} = s^{(\text{out})}$ in (26), as we are working with the liabilities matrix. We use liabilities instead of assets because, whenever an FI defaults, it may not be able to honor all its creditors. In fact, the recovery rate is effectively zero for its creditors in the short-run.

In order to get a better grasp of the magnitude of the weighted impact diffusion influence, we opt to report it using a normalized version of (26) as:

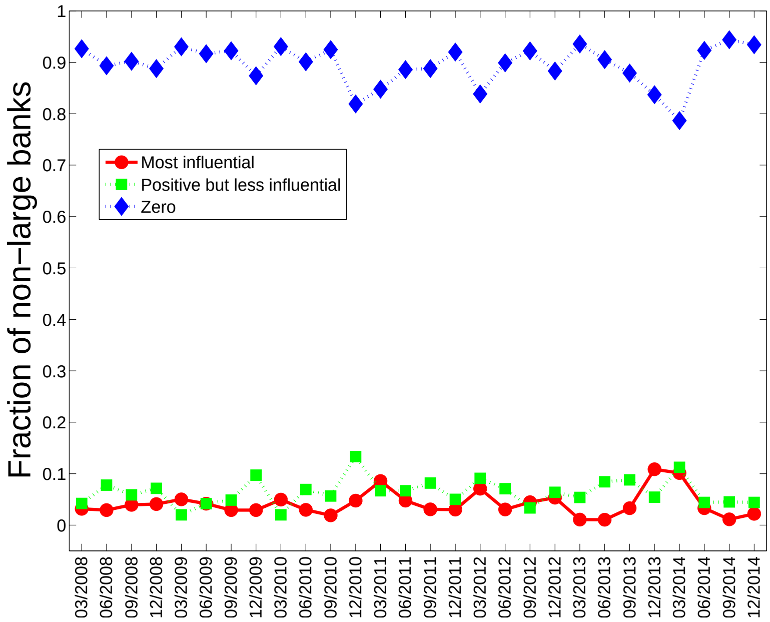


Figure 11: *Fractions of (i) "Most influential:" non-large banks that are more influential than the average influence exerted by large banks; (ii) "Zero:" non-large banks that have no influence in diffusing impact in the network; (iii) "Positive but less influential:" non-large banks that have a smaller, but non-zero, impact diffusion influence than the average influence of large banks.*

$$\bar{I}_q^{(\mathrm{w})}(\bar{\mathbf{V}}, s^{(\mathrm{out})}) = \frac{I_q^{(\mathrm{w})}(\bar{\mathbf{V}}, s^{(\mathrm{out})})}{\sum_{i \in \mathscr{V}} s_i^{(\mathrm{out})}}. \tag{28}$$

i.e., we report the potential influence in terms of the total liabilities within the financial network. Note, however, that the values in the y-axis cannot truly be seem as monetary values, because the communicability index may possibly contain attenuated cycles in its computation. It is rather an approximate potential value of the impact caused by entities and serves as a comparison measure to verify who is more prevalent against others in potentially causing harm to the financial system.

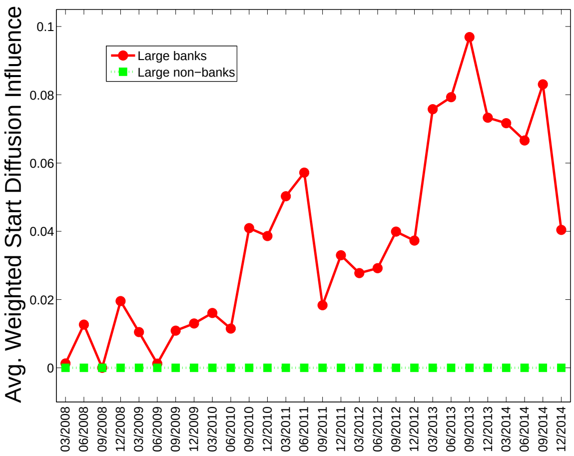
We can compute the average weighted impact diffusion influence over all the network to obtain a measure of the average potential reach of an institution's default along the network. We use as weights the total liabilities of the institutions, which gives more weight to the influence of the larger institutions. Thus, to compare the impact diffusion influence for size and type categories of institutions, we perform the average weighted impact diffusion influence computations separately for each of those groups of institutions. Figures 12a and 12b exhibit the average weighted impact diffusion influence of institutions when they play the role of starters in the diffusion process for large and non-large institutions, respectively. Similarly, Figs. 12c and 12d depict the weighted impact diffusion influence for large and non-large institutions, respectively, when they act as intermediators in the diffusion process. Summing up both contributions, Figs. 9e and 9f portray the total weighted impact diffusion influence of large and non-large institutions, respectively, in the Brazilian financial market from 2008 to 2014. We find that, on average, large banks are about five times more influent than large non-banks, and that non-banking institutions are far less influent than banking ones.

Figure 13 shows the weighted influence of non-large banks that are more influential, less influential but with positive values of diffusion influence and non-large banks with zero weighted diffusion influence. Again, we confirm the skewness of the distribution comprising the weighted diffusion influences exerted by non-large banks. We can verify how the weight in the influence can alter the overall results by comparing Figs. 13 and 10 that portray the weighted and non-weighted influences, respectively. We can see that, while the non-weighted diffusion influence exerted by the most influential non-large banking institutions remains consistently above of large banks in the non-weighted version, this behavior does not extend to the weighted version. In special, we can see that the weighted influence exerted by large banks in the second and third quarters of 2011, in the last quarter of 2013, and in the last semester of 2014 surpasses that of the most influential non-large banks. This suggests that, in these periods, non-large banks influence a larger quantity of other FIs than large-banks. These influenced FIs, however, often do not have large quantities of liabilities in the financial market. Therefore, their importance is effectively reduced by our proxy. In those periods, large banks, in contrast, influence a smaller quantity of FIs that are, on average, more important as they present expressive liabilities in the financial network.

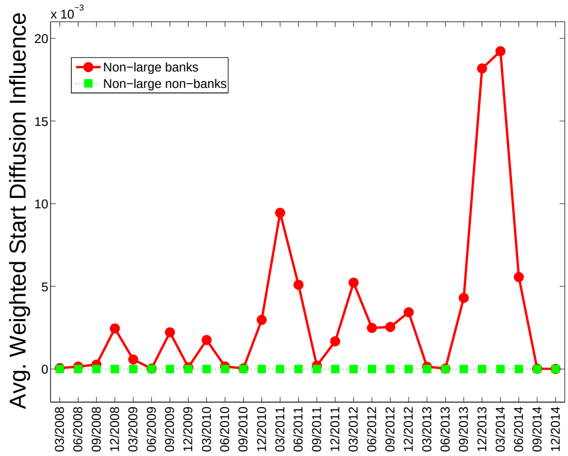
5 Conclusion

In this work, we have investigated the roles FIs play within the Brazilian financial market using an approach based on complex networks. One prominent advantage of employing network-based theory is that it is able to capture topological and structural characteristics of the players' relationships from the data representation itself.

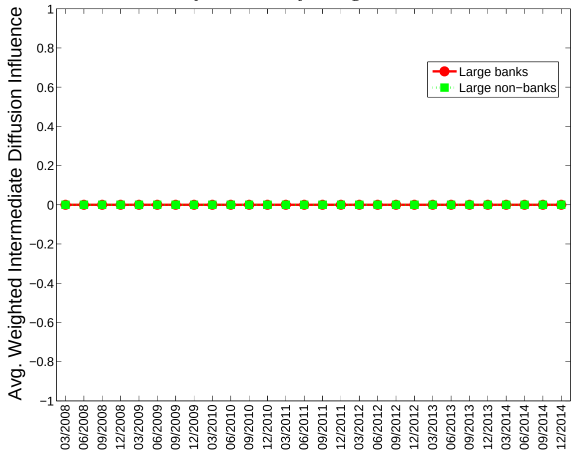
In this paper, we have proposed four new measures of potential contagion in a financial network: impact susceptibility, network impact fluidity, and weighted and non-weighted impact diffusion influence. We derive theoretical properties for the impact susceptibility and impact diffusion influence that permit us to understand how FIs are prone or susceptible to random systemic events in the network. We observe that these measurements are applied to the network "as-is," not requiring any initial external shock to their computation. This interesting feature contrasts with other developed measures in the literature, like the Eisenberg and Noe (2001)'s fictitious default algorithm and the Battiston et al. (2012b)'s DebtRank measure, that do require those initial shocks. Therefore, one complication of using these measures is how one should design these shocks.



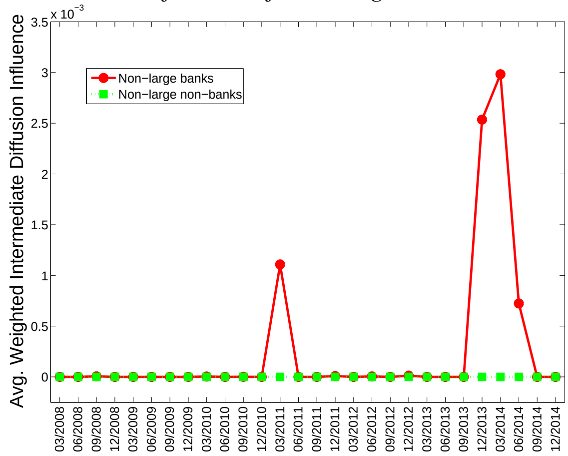
(a) Weighted start impact diffusion influence of large FIs



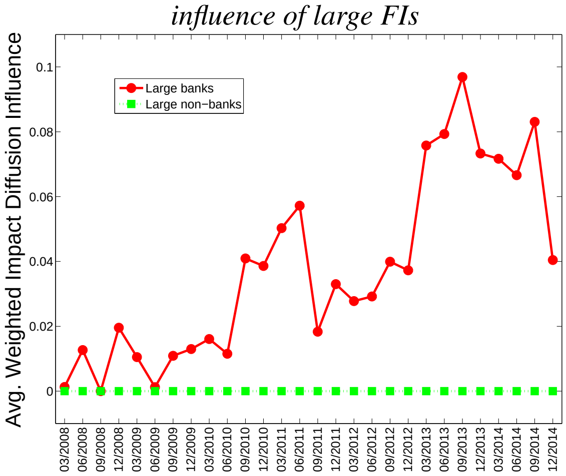
(b) Weighted start impact diffusion influence of non-large FIs



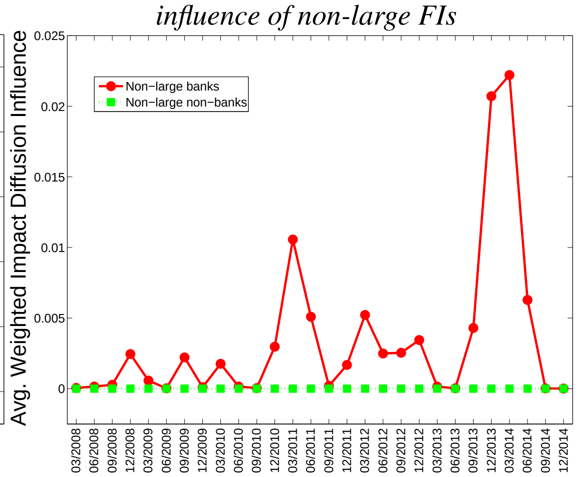
(c) Weighted intermediate impact diffusion influence of large FIs



(d) Weighted intermediate impact diffusion influence of non-large FIs



(e) Weighted impact diffusion influence of large FIs



(f) Weighted impact diffusion influence of non-large FIs

Figure 12: *Trajectory of the average weighted impact diffusion influence and its constituent parts (start and intermediate factors) for static snapshots of the financial network in different periods.*

Our measures, opposed to those, gauge potential contagion exempting us from the need of applying exogenous shocks: they extract the inherently built-in risk of the financial

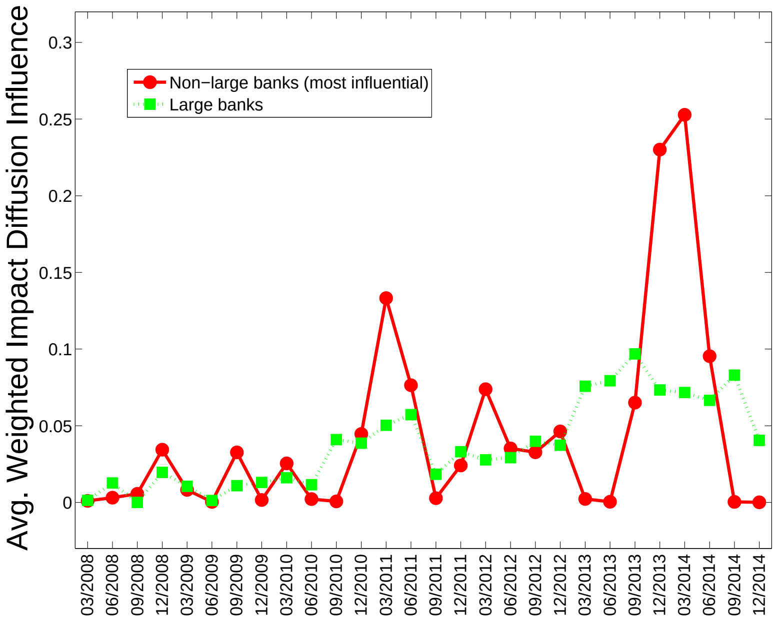


Figure 13: *Comparison of the average weighted impact diffusion influence of large banks and the top $M(t)$ non-large banks, where $M(t)$ is the number of large banks at instant t.*

relationships.

Using the impact susceptibility, for the period ranging from 2008 to 2014, we find that banking institutions are barely susceptible to indirect impacts, while non-banking ones are, on average, susceptible to them in the first quarter of 2011 and from the second half in 2013 to the first in 2014. Silva et al. (2015) find that non-banking institutions are rather exposed to banking institutions than to non-banking ones. Given that banking institutions are well capitalized and are barely susceptible to indirect impacts, we can say that the Brazilian financial system is rather stable.

We employ the network impact fluidity to assess how much exposed is the Brazilian financial market network to the onset of contagion chains, finding that the appearance of those chains are unlikely and that, if those emerge, they have short path lengths. We have also found that the lowest impact fluidity index happens in September 2008. We relate this low impact fluidity to the fact that FIs were less willing to provide liquidity to their counterparties in that period of turmoil. Hence, the number of exposures that could contribute to potential defaults substantially decreased in that period.

We use the impact diffusion influence to gauge the influence of financial institutions, as shocks sources or transmitters, and discover the presence of a portion of non-large banking institutions that is consistently more influential than large banks in potentially diffusing impacts throughout the network. That is a tool that financial system monitors and regulators can use to identify financial institutions that should receive more attention in times of crisis.

References

Allen, F., Babus, A., and Carletti, E. (2012). Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics*, 104(3):519 – 534.

Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108(1):1–33.

Battiston, S., Gatti, D. D., Gallegati, M., Greenwald, B., and Stiglitz, J. E. (2012a). Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control*, 36(8):1121 – 1141.

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012b). DebtRank: Too central to fail? Financial networks, the FED and systemic risk. *Scientific reports*, 2:541.

BCB (2008). Financial stability report. 7(2):1–174.

BCB (2012). Financial stability report. 11(2):1–63.

Benzi, M. and Klymko, C. (2013). Total communicability as a centrality measure. *Journal of Complex Networks*, 1(2):124–149.

Demirgüç-Kunt, A. and Huizinga, H. (2010). Bank activity and funding strategies: The impact on risk and returns. *Journal of Financial Economics*, 98(3):626 – 650.

Eisenberg, L. and Noe, T. H. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

Estrada, E. and Hatano, N. (2008). Communicability in complex networks. *Physical Review E*, 77:036111.

Estrada, E., Higham, D. J., and Hatano, N. (2008). Communicability and multipartite structures in complex networks at negative absolute temperatures. *Physical Review E: Statistical, Nonlinear, and Soft Matter Physics*, 78(2):026102.

Estrada, E. and Rodríguez-Velázquez, J. A. (2005). Subgraph centrality in complex networks. *Physical Review E: Statistical, Nonlinear, and Soft Matter Physics*, 71(5):056103.

Gai, P., Haldane, A., and Kapadia, S. (2011). Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453 – 470.

Gai, P. and Kapadia, S. (2010). Contagion in financial networks. *Proceedings of the Royal Society*, 4666(2120):2401–2424.

Hojman, D. A. and Szeidl, A. (2008). Core and periphery in networks. *Journal of Economic Theory*, 139(1):295 – 309.

IMF, BIS, and FSB (2009). Guidance to assess the systemic importance of financial institutions, markets and instruments: initial considerations. Report to the G20 Finance Ministers and Governors. *International Monetary Fund, Bank for International Settlements, and Financial Stability Board.*

Lux, T. (2015). Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52(0):A11 – A23.

Martínez-Jaramillo, S., Alexandrova-Kabadjova, B., Bravo-Benítez, B., and Solórzano-Margain, J. P. (2014). An empirical study of the Mexican banking system's network and its implications for systemic risk. *Journal of Economic Dynamics and Control*, 40(0):242 – 265.

Minoiu, C. and Reyes, J. A. (2013). A network analysis of global banking: 1978-2010. *Journal of Financial Stability*, 9(2):168 – 184.

Silva, T. C., de Souza, S. R. S., and Tabak, B. M. (2015). Network structure analysis of the Brazilian interbank market. Working Paper 391, Central Bank of Brazil.

Souza, S. R., Tabak, B. M., Silva, T. C., and Guerra, S. M. (2015). Insolvency and contagion in the Brazilian interbank market. *Physica A: Statistical Mechanics and its Applications*, 431:140–151.