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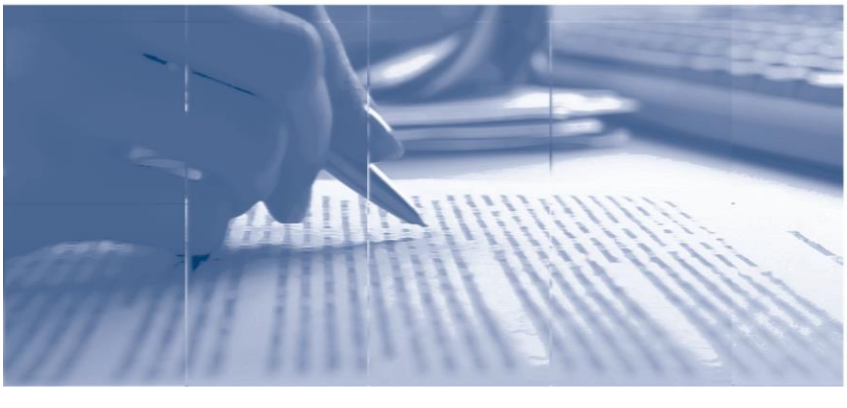

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The Ramsey Steady State under Optimal Monetary and Fiscal Policy for Small Open Economies

Angelo Marsiglia Fasolo*

Abstract

This paper describes the steady state allocations and prices for small open economies under optimal monetary and fiscal policy in a medium-scale DSGE model. The model encompasses the most common nominal and real rigidities normally found in the literature in a single framework. The Ramsey solution for the optimal monetary and fiscal policy is computed for a large space of the parameter set and for different combinations of fiscal policy instruments. Results show that, despite the large number of frictions in the model, optimal fiscal policy follows the usual results in the literature, with high taxes over labor income and low taxes (subsidies) on capital income. On the other hand, the choice of fiscal policy instruments is critical to characterize optimal monetary policy. Frictions associated with the small open economy framework do not play a critical role in characterizing the Ramsey planner's policy choices.

JEL Codes: E52, E61, E63, F41, F42, F44

Keywords: Ramsey Policy; DSGE models; Small Open Economies; Monetary and Fiscal Policy

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1 Introduction

What is the optimal combination of monetary and fiscal policy instruments for a small open economy? How does optimal policy changes when structural parameters characterize an emerging economy, instead of a developed economy? How do allocations and prices under optimal policy differ in a small open economy if the instrument set available for the benevolent planner changes? A lot of effort has been devoted to characterize optimal policy for developed small open economies, without discussing if the associated welfare ranking of policy recommendations is still the same for more volatile, less developed economies. The high volatility observed in emerging economies usually translates in medium-scale dynamic stochastic general equilibrium (DSGE) models to parameter values far from those estimated to developed economies. This paper describes the allocations under optimal policy of a conventional medium-scale model for small open economies. In this paper, the main differences between Emerging Economies (EMEs, henceforth) and developed, Small Open Economies (SOEs, henceforth) will be restricted to structural parameters characterizing each economy. The exercise here focus on the description of the Ramsey policy steady state, clarifying the trade-offs faced by a benevolent central planner.

The development of DSGE models for closed economies[1], comprising a large set of nominal and real rigidities, changed research on optimal monetary and fiscal policies, not only because of the departure from analytically solving Ramsey's (1927)[33] problem in tractable models, but also from a theoretical point of view. Models with such large number of frictions demand an equally large number of non-distortionary instruments in order to recover the first-best allocation as the equilibrium outcome of the optimal policy. In a framework constrained by the lack of such large set of instruments, the literature focused on the numerical characterization of the optimal policy's outcome in models where the steady state is distorted as a consequence of the nominal and real rigidities. In these simulations, the government chooses values for a set of instruments in order to maximize (minimize) an utility (loss) function.

Few authors in the literature provide a comprehensive discussion about the properties of the steady state under the Ramsey policy. Woodford (2003)[43] provides a complete description of the steady state policy of the basic New Keynesian model for closed economies. The author explores differences in the Ramsey policy outcome when imposing additional restrictions like those included here, as the "timeless perspective" of the optimal policy problem. Still in the closed economy framework, but now dealing with variations of models similar in structure to Christiano, Eichenbaum and Evans (CEE, henceforth) (2005)[11], Schmitt-Grohé and Uribe (SGU, henceforth) (2005, 2006, 2007)[36] [37] [38] explore the properties of the steady state under Ramsey optimal monetary and fiscal policies. These medium-scale models do not have a closed form solution, like the basic structures described in Woodford (2003)[43]. Therefore, the only way to understand and describe optimal policy is by means of numerical simulations. The results in terms of steady state of prices usually point out for price stability as the main outcome of the Ramsey planner, with small variations depending on the number of nominal and real rigidities included in the model.

Despite the adoption of medium-scale models for monetary policy analysis in some Central Banks, the research on optimal policy for SOEs in these models is still very incipient. The main focus of the literature is on the evaluation of optimal monetary policy in models with small departures from the basic sticky price framework proposed in Galí and Monacelli (2005)[20] and Monacelli (2005)[32]. Extensions try to deal with specific features of open economies: deviations from the Law of One Price (Kollmann (2002)[25], Ambler, Dib and Rebei (2004)[5]); incomplete foreign asset markets (Ambler, Dib and Rebei (2004)[5], Justiniano and Preston (2009)[24]); fiscal policy dimension of the open economy framework (Benigno and De Paoli (2009)[8]). In the literature, no distinctions are made between SOEs

[1] The models are usually some variation of the framework in Christiano, Eichenbaum and Evans (2005)[11]. See Schmitt-Grohé and Uribe (2006 and 2007)[37][38] for optimal policy computation in those models.

and EMEs, resulting in the same policy recommendations for both types of countries despite large differences documented in the literature between their structural parameters[2].

The literature on EMEs focus on the description of these economies, adding structure over a basic model in order to capture distinctive aspects of data. The higher volatility in data, when compared with SOEs[3], brings attention to topics like: foreign currency demand (Felices and Tuesta (2007)[19]); investment financed by foreign currency and "balance sheet effects" (Devereux, Lane and Xu (2006)[17] and Elekdag and Tchakarov (2007)[18], Batini, Levine and Pearlman (2009)[7]); a commodity sector, in order to highlight the importance of natural resources (Laxton and Pesenti (2003)[26], Batini, Levine and Pearlman (2009)[7]); households heterogeneity in credit market access (Batini, Levine and Pearlman (2009)[7]). Most of these papers focus on the computation of optimal monetary policy rules, with little focus on fiscal policy or the structural parameters[4], or, sometimes, assuming a steady state that might be different from the Ramsey optimal solution. Among the papers in this non-exhaustive list, Batini, Levine and Pearlman (2009)[7] is the closest reference in terms of the theoretical framework adopted here, as they compute optimal monetary and fiscal policy rules in a model with several nominal and real rigidities. However, the flexible price allocation can always be recovered as an optimal outcome of the policy due to the assumption of a lump sum taxation as one of the fiscal policy instruments.

From a theoretical perspective, the model here departs from the literature as it does not consider a set of lump sum mechanisms in order to eliminate the distortions caused by nominal rigidities: the Ramsey planner has access to distortionary consumption, capital and labor income taxes, besides the control of money supply and debt to balance the budget. As a consequence, the optimal policy allocations are not necessarily equivalent to those under flexible prices, just like the case for closed economies described in SGU (2005 and 2006)[36][37]. The computation of optimal policy is based on the solution for the Ramsey problem, where a planner tries to maximize the discounted expected utility of the representative household. This approach differs from other studies where optimal policy is derived from the minimization of an arbitrary loss function as a measure of welfare[5]. The steady state under optimal policy here is characterized under the same theoretical model, but fully exploring changes in parameter space, fiscal policy framework and nominal rigidities – in this case, rigidities located not only in domestic markets, but also in the price-setting mechanism of imported and exported goods.

One might ask about the importance of generalizing result on optimal policy under commitment for EMEs, given the high volatility and structural changes characterizing these economies. In reality, most of fiscal and monetary policy settings in EMEs are characterized by some type of commitment. The adoption of controlled exchange rate regimes in the early 90's is an explicit commitment to keep exchange rate fluctuations under constraint. In the early 2000's, several EMEs adopted a combination of high fiscal surpluses and inflation targeting regimes, while showing signs that little interference will be made in the exchange rate markets – again, a new form of commitment. Even in abnormal periods, like "sudden stops" episodes, governments usually sign "letters of intentions" to institutions like the IMF, committing to a new macroeconomic arrangement in order to guarantee emergency loans. Thus, the policy problem of EMEs can be viewed as setting the right commitment for these economies, instead of a proposition between commitment versus "discretionary policies".

In terms of results, price stability seems to be the main goal of the Ramsey planner, given parameters

[2]Silveira (2006)[15] estimated the basic Galí and Monacelli (2005)[20] model using Brazilian data. The posterior values for the elasticity of the labor supply and the elasticity of substitution between imported and domestically produced consumption goods are outside the boundaries found in the literature. Another example is Elekdag, Justiniano and Tchakarov (2005)[41], with discrepancies in the values for the intertemporal elasticity of substitution and on the elasticity of the labor supply, when compared to the calibration used for closed economies.

[3]See Aguiar and Gopinath (2007)[3].

[4]There is an effort to put these models in an estimated framework. As an example, Elekdag, Justiniano and Tchakarov (2005)[41] present an estimation of the model published later in Elekdag and Tchakarov (2007)[18].

[5]Some examples of the loss function approach are found in Svensson (2000)[40], Levin and Williams (2003)[27], Laxton and Pesenti (2003)[26] and Justiniano and Preston (2009)[24]

used in calibration. However, the number of available taxes for the government plays a key role in setting the optimal taxes and interest rates under different assumptions on nominal and real rigidities. To be more specific, the inclusion of consumption taxes as one of the taxes available in the model usually results in price stability as the optimal outcome, eliminating almost all trade-offs related to the combination of nominal and real rigidities. This result confirms, for a model designed for small open economies, the propositions in Correia, Nicolini and Teles (2008)[14] regarding the role of a tax over the final good of the economy, vis-a-vis a tax over intermediate inputs. The classical result from Judd (2002)[23], of a high subsidy to capital relative to the returns of labor, is robust for fiscal policy, irrespective to the set of instruments available to the benevolent government.

This paper is organized as follows. Chapter 2 presents the DSGE model, with focus on the equations characterizing the equilibrium and the connections between the structural frictions and the literature on DSGE models, finishing with the definitions of the competitive and the Ramsey equilibria. Chapter 3 shows the baseline calibration of the model. Chapter 4 presents a detailed analysis of the steady state of the model, assuming structural parameters normally observed in the literature. Chapter 5 concludes.

2 A Medium-Scale Model for Small Open Economies

In this section, a brief description of the model is provided with the characterization of the household and the firms' problem, the policy rules for the government in a competitive equilibrium, the foreign sector and aggregation[6]. After that, the definitions of both a competitive and Ramsey equilibria are presented. The model is an extension for a small open economy of the closed economy model for monetary policy analysis proposed in CEE (2005)[11] and Altig, Christiano, Eichenbaun and Lindé (2005)[4]. Similar models are used in Adolfson, Laseén, Lindé and Villani (2007)[2] and in Christiano, Trabandt and Walentin (2007)[12]. These models combine the sticky price framework of Galí and Monacelli (2005)[20] and Monacelli (2005)[32] to add nominal and real frictions based in CEE (2005)[11].

From the households' perspective, the model presents external habit persistence in consumption, adjustment costs for investment, portfolio and capital utilization. Households' objective is to maximize the discounted value of expected utility. They own capital, demand money to buy consumption goods and set wages after observing the demand for his specific type of labor. Households in each period buy both domestically produced and imported goods for consumption, sell labor to satisfy the demand by firms after the acceptance of the proposed wage and set the rate of capital utilization. In order to transfer wealth across periods, households trade bonds domestically and in international markets and accumulate capital built from both domestically produced and imported goods. They also face a cash-in-advance constraint, requiring domestic currency to buy a share of total consumption goods.

Firms in the tradable and non-tradable sectors of the domestic economy rent capital and labor from households to produce goods. They set prices in a Calvo style, with an exogenous probability of optimizing prices in period t. Firms from the tradable sector have to compete with imported goods retailers. These retail firms buy goods produced abroad and sell them domestically, also adjusting prices in domestic currency in a Calvo style. Also, firms from the tradable sector can sell goods for the exported goods retailers. These firms buy domestically produced goods and sell them abroad, setting price in foreign currency in a Calvo style – thus, local currency pricing in both domestic and foreign markets justifies pricing-to-market discrimination, as commonly seen in the literature[7]. A demand for foreign currency is justified by a working capital constraint for imported goods retailers, with those firms selling bonds to obtain foreign currency to finance the acquisition of foreign inputs.

[6] Appendix A lists the final set of equilibrium conditions.

[7] Some models with at least partial local currency pricing are Kollmann (2002)[25], Ambler, Dib and Rebei (2004)[5], Devereux, Lane and Xu (2006)[17], Christiano, Trabandt and Walentin (2007)[12] and Justiniano and Preston (2009)[24].

The government in a competitive equilibrium sets nominal interest rates according to a Taylor rule based on current inflation, in order to match an exogenous, time-varying inflation target. In terms of fiscal policy, the government has three instruments available to finance an exogenous stream of consumption: money, bonds sold domestically, and distortionary taxes. The government might tax in different rates consumption and the income from capital, labor and profits. In the competitive equilibrium, taxes on labor are set according to a simple policy rule based on total government liabilities. Taxes on consumption, capital and on profits, as well as government spending, are exogenous.

The foreign sector is described by a VAR including the foreign currency supply, output, inflation, interest rates and the risk premium. The model has 16 shocks, with five from the foreign sector, plus the following: one on the price of imported goods in foreign currency; two stationary sectorial productivity shocks; a non-stationary aggregate productivity shock; a non-stationary, investment-specific shock; government spending; three tax shocks; monetary policy shock and a inflation target shock.

2.1 Households

There is a continuum of infinitely-lived households i ($i \in [0,1]$) populating the domestic economy, each of them with an endowment of labor type i, $h_t(i)$. There is no population growth and labor can not be sold for firms in the rest of the world. In the intertemporal problem, households maximize discounted utility choosing consumption, capital utilization and investment for each sector, wages, hours worked and setting the money demand, next period's foreign and domestic bond holdings and physical capital stock. The general intertemporal household problem, given the non-Ponzi games constraints, is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma) \log \left(C_t(i) - \zeta C_{t-1} \right) + \gamma \log \left(1 - h_t(i) \right) \right]$$

$$s.t.: \quad P_t \left(1+\tau_t^c\right) C_t(i) + \Upsilon_t^{-1} P_t \left(I_{x,t}^d(i) + I_{n,t}^d(i) \right) + P_t M_t(i) + R_{t-1} B_{h,t}(i)$$

$$+ S_t R_{t-1}^f IB_t(i) + W_t \tfrac{\phi_w}{2} \left(\tfrac{W_t(i)}{\pi_t^{\chi w} W_{t-1}(i)} - \mu^I \right)^2 + \tfrac{\psi_1}{2} Y_t \left(\tfrac{B_{h,t+1}(i)}{Y_t} - \tfrac{B_h}{Y} \right)^2$$

$$+ \tfrac{\psi_2}{2} Y_t \left(\tfrac{S_t IB_{t+1}(i)}{P_t Y_t} - \tfrac{rer\ IB}{Y} \right)^2 = P_{t-1} M_{t-1}(i) + \left(1 - \tau_t^h\right) W_t(i) h_t(i)$$

$$+ \left(1 - \tau_t^{\phi}\right) P_t \Phi_t(i) + \left(1 - \tau_t^k\right) P_t \left[\left(R_{n,t}^k \mu_{n,t} - \Upsilon_t^{-1} a\left(\mu_{n,t}\right) \right) \overline{K}_{n,t}(i) \right.$$

$$\left. + \left(R_{x,t}^k \mu_{x,t} - \Upsilon_t^{-1} a\left(\mu_{x,t}\right) \right) \overline{K}_{x,t}(i) \right] + B_{h,t+1}(i) + S_t IB_{t+1}(i)$$

$$\overline{K}_{j,t+1}(i) = (1-\delta) \overline{K}_{j,t}(i) + I_{j,t}^d(i) \left(1 - \aleph \left(\frac{I_{j,t}^d(i)}{I_{j,t-1}^d(i)} \right) \right)$$

$$a\left(\mu_{j,t}\right) = \theta_1 \left(\mu_{j,t} - 1\right) + \frac{\theta_2}{2} \left(\mu_{j,t} - 1\right)^2$$

$$K_{j,t} = \mu_{j,t} \overline{K}_{j,t}$$

$$\aleph \left(\tfrac{I_{i,t}^d}{I_{i,t-1}^d} \right) = \tfrac{\phi_i}{2} \left(\tfrac{I_{i,t}^d}{I_{i,t-1}^d} - \mu^I \right)^2 \qquad j = \{x, n\}$$

$$\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu_{t+1}^{\Upsilon} = \left(1 - \rho_{\Upsilon}\right) \mu^{\Upsilon} + \rho_{\Upsilon} \mu_t^{\Upsilon} + \epsilon_{t+1}^{\Upsilon}; \quad \epsilon_t^{\Upsilon} \sim N\left(0, \sigma_{\Upsilon}\right)$$

$$h_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varpi} h_t$$

$$M_t(i) \geq \nu^m \left(1 + \tau_t^c\right) C_t(i)$$

In this problem, β is the intertemporal discount factor of the utility function. The utility function is log-separable in terms of consumption and labor, with consumption adjusted by external habit persistence[8]. The degree of habit persistence is defined by the parameter $\zeta \in [0,1)$.

Households accumulate physical capital, $\overline{K}_{j,t}$, for $j = \{x, n\}$ representing the sectors of the economy, buying from the firms investment goods that depreciate at a rate δ. Define Υ_t^{-1} as the non-stationary inverse of the relative price of investment in terms of consumption goods. The relative price of investment goods can also be interpreted as a technology shock affecting the linear production function available to households to transform consumption goods in investment goods[9]. Investment is subject to an adjustment cost $\aleph(.)$, in the same fashion as in CEE (2005)[11] and Altig, Christiano, Eichenbaun and Linde (2005)[4] such that $\aleph(1) = 0, \aleph'(1) = 0, \aleph''(1) > 0$.[10] The functional form follows SGU (2006)[37], with μ^I defining the steady state growth of investment. Households rent capital for firms after setting the rate of capital utilization for each sector $(\mu_{j,t})$, paying a cost given by $a(\mu_{j,t})$ to change the utilization level in each period and in each sector. The after-tax private return of capital in each sector is defined, thus, as $\left(1-\tau_t^k\right) P_t \left(R_{j,t}^k \mu_{j,t} - \Upsilon_t^{-1} a(\mu_{j,t})\right) \overline{K}_{j,t}(i)$.

The supply of labor is decided by each household taking as given the aggregate wage and demand for labor of the economy, W_t and h_t, and the adjustment cost for wages. As a monopolist of a specific type of labor, household chooses the nominal wage $W(i)$ and supplies all the demanded labor $h_t(i)$ given the acceptance of $W(i)$. The elasticity of substitution across different types of labor is given by $\varpi > 1$. The nominal wage adjustment cost function allows for partial indexation based on current inflation, determined by χ_w $(\chi_w \in [0,1])$. The presence of sticky wages results in an additional distortion, defined by mcw_t, which is the markup imposed over real wages as households supply a specific type of labor. The quadratic adjustment cost[11] is consistent with the absence of lump sum instruments to correct for wealth dispersion across households. Wage-setting processes based on Calvo model create dispersion in wage income across households, with the representative household recovered by lump sum subsidy schemes or an asset market structure capable of insuring households against wage dispersion. Both instruments would be controversial with the evaluation of optimal policy assuming that government does access lump sum schemes to support agents. Another alternative is to assume a centralized union coordinating the supply of labor, as in SGU (2006)[37]. The assumption of a labor union with such market power, however, does not seem reasonable for developed small open economies outside a few European countries[12].

Still in the budget constraint, households allocate wealth over time buying one-period, non-state contingent nominal bonds from the government, $B_{h,t+1}(i)$, or from the rest of the world, $IB_{t+1}(i)$. In the later case, bonds are priced in foreign currency, and S_t is the nominal exchange rate. In order to adjust portfolio, and to induce stationarity in the model, households incurs in costs based on the variance of the stock of bonds as a proportion of the GDP[13]. Households also receive dividends from firms $\Phi_t(i)$.

Finally, following SGU (2007)[38], households demand money, $M_t(i)$ in order to pay for a share $\nu^m \geq 0$ of consumption. The sequence of events in each period is the same as in CEE (2005)[11], with the households first deciding consumption and capital allocation, then deciding, in sequence, the financial portfolio, wages and the labor supply, and the final composition of portfolio between bonds and money.

[8]In terms of notation, the general variable $x_t(i)$ represents the choice of household i on period t about x. The variable x_t is the aggregate value of $x_t(i)$ for the economy.

[9]See Greenwood, Hercowitz and Krusell (2000)[22] and SGU (2006)[37].

[10]Adjustment costs and the structure of shocks, despite not affecting the deterministic steady state of the model, are kept in the description of the model for the sake of completeness. It is left, also, as a suggested structure for reference in order to study the dynamic properties of the Ramsey equilibrium.

[11]See, for instance, Chugh (2006)[13] and García-Cicco (2009)[21].

[12]According to data from OECD (2004), only Denmark, Sweden, Finland, Iceland and Belgium presented a steady increase in trade-union density from 1960 to 2000. In Latin American economies, the trade-union density is not only lower, compared to Scandinavian countries, but also declining since the 1990's – see Visser Martin Tergeist, 2008[42].

[13]See SGU (2003b)[35]. The use of the ratio to GDP in the functional form adopted here makes it easier to obtain the stationary form of the model.

Thus, domestic currency is expressed as an end-of-period aggregate.

Define $\widetilde{\lambda}_t/P_t$, $\widetilde{\lambda}_t\widetilde{q}_{j,t}$, $\lambda_t^m\widetilde{\lambda}_t$ and $\left(\widetilde{\lambda}_t\left(1-\tau_t^h\right)W_t\right)/\left(P_t\, mcw_t\right)$ the Lagrange multipliers on the budget constraint, on the capital accumulation equations, on the cash-in-advance constraint and on the labor demand function, respectively. Noting that, in the symmetric equilibrium, $C_t(i)=C_t$ and $W_t(i)=W_t$, the final set of equilibrium conditions of the intertemporal problem of households is given by:

$$\frac{\left(1-\tau_t^h\right)\widetilde{W}_t}{\left(1+\tau_t^c\right)\left(C_t-\zeta C_{t-1}\right)}=\frac{\gamma}{\left(1-\gamma\right)}\frac{mcw_t\left(1+\nu^m\left(\frac{\widetilde{R}_t-1}{\widetilde{R}_t}\right)\right)}{\left(1-h_t\right)} \tag{1}$$

$$\frac{\left(1-\gamma\right)}{C_t-\zeta C_{t-1}}=\left(1+\tau_t^c\right)\widetilde{\lambda}_t\left(1+\nu^m\left(\frac{R_t-1}{R_t}\right)\right) \tag{2}$$

$$\widetilde{\lambda}_t\left[1-\psi_1\left(\frac{B_{h,t+1}}{Y_t}-\frac{B_h}{Y}\right)\right]=\beta R_t E_t\left(\frac{\widetilde{\lambda}_{t+1}}{\pi_{t+1}}\right) \tag{3}$$

$$\widetilde{\lambda}_t\left[1-\psi_2\left(\frac{S_t IB_{t+1}}{P_tY_t}-\frac{rer\ IB}{Y}\right)\right]=\beta R_t^f E_t\left(\frac{S_{t+1}}{S_t}\frac{P_t}{P_{t+1}}\widetilde{\lambda}_{t+1}\right) \tag{4}$$

$$\widetilde{\lambda}_t\widetilde{q}_{x,t}=\beta E_t\left\{\widetilde{\lambda}_{t+1}\left[\left(1-\tau_{t+1}^k\right)\left(R_{x,t+1}^k\mu_{x,t+1}-\Upsilon_{t+1}^{-1}a\left(\mu_{x,t+1}\right)\right)+\widetilde{q}_{x,t+1}\left(1-\delta\right)\right]\right\} \tag{5}$$

$$\widetilde{\lambda}_t\widetilde{q}_{n,t}=\beta E_t\left\{\widetilde{\lambda}_{t+1}\left[\left(1-\tau_{t+1}^k\right)\left(R_{n,t+1}^k\mu_{n,t+1}-\Upsilon_{t+1}^{-1}a\left(\mu_{n,t+1}\right)\right)+\widetilde{q}_{n,t+1}\left(1-\delta\right)\right]\right\} \tag{6}$$

$$K_{n,t}=\mu_{n,t}\overline{K}_{n,t} \tag{7}$$

$$K_{x,t}=\mu_{x,t}\overline{K}_{x,t} \tag{8}$$

$$\theta_1+\theta_2\left(\mu_{n,t}-1\right)=\frac{R_{n,t}^k}{\Upsilon_t^{-1}} \tag{9}$$

$$\theta_1+\theta_2\left(\mu_{x,t}-1\right)=\frac{R_{x,t}^k}{\Upsilon_t^{-1}} \tag{10}$$

$$R_t=\frac{1}{r_{t,t+1}} \tag{11}$$

$$\widetilde{R}_t=R_t\left(1-\psi_1\left(\frac{B_{h,t+1}}{Y_t}-\frac{B_h}{Y}\right)\right)^{-1} \tag{12}$$

$$\begin{aligned}\widetilde{\lambda}_t\Upsilon_t^{-1} \;=\;& \widetilde{\lambda}_t\widetilde{q}_{x,t}\left[1-\Psi\left(\frac{I_{x,t}^d}{I_{x,t-1}^d}\right)-\left(\frac{I_{x,t}^d}{I_{x,t-1}^d}\right)\Psi'\left(\frac{I_{x,t}^d}{I_{x,t-1}^d}\right)\right]\\ &+\beta E_t\left[\widetilde{\lambda}_{t+1}\widetilde{q}_{x,t+1}\left(\frac{I_{x,t+1}^d}{I_{x,t}^d}\right)^2\Psi'\left(\frac{I_{x,t+1}^d}{I_{x,t}^d}\right)\right]\end{aligned} \tag{13}$$

$$\begin{aligned}\widetilde{\lambda}_t\Upsilon_t^{-1} \;=\;& \widetilde{\lambda}_t\widetilde{q}_{n,t}\left[1-\Psi\left(\frac{I_{n,t}^d}{I_{n,t-1}^d}\right)-\left(\frac{I_{n,t}^d}{I_{n,t-1}^d}\right)\Psi'\left(\frac{I_{n,t}^d}{I_{n,t-1}^d}\right)\right]\\ &+\beta E_t\left[\widetilde{\lambda}_{t+1}\widetilde{q}_{n,t+1}\left(\frac{I_{n,t+1}^d}{I_{n,t}^d}\right)^2\Psi'\left(\frac{I_{n,t+1}^d}{I_{n,t}^d}\right)\right]\end{aligned} \tag{14}$$

$$\overline{K}_{x,t+1}(i)=\left(1-\delta\right)\overline{K}_{x,t}(i)+I_{x,t}^d(i)\left(1-\aleph\left(\frac{I_{x,t}^d(i)}{I_{x,t-1}^d(i)}\right)\right) \tag{15}$$

$$\overline{K}_{n,t+1}(i)=\left(1-\delta\right)\overline{K}_{n,t}(i)+I_{n,t}^d(i)\left(1-\aleph\left(\frac{I_{n,t}^d(i)}{I_{n,t-1}^d(i)}\right)\right) \tag{16}$$

$$\left(\frac{1-\varpi}{\varpi}+\frac{1}{mcw_t}\right)\varpi h_t\left(1-\tau_t^h\right)=-\frac{\phi_w}{\pi_t^{\chi_w-1}}\left(\frac{\widetilde{W}_t}{\widetilde{W}_{t-1}}\right)\left(\frac{\widetilde{W}_t}{\pi_t^{\chi_w-1}\widetilde{W}_{t-1}}-\mu^I\right)$$

$$+\beta E_t\left[\frac{\widetilde{\lambda}_{t+1}\phi_w}{\widetilde{\lambda}_t\pi_{t+1}^{\chi_w-1}}\left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\right)^2\left(\frac{\widetilde{W}_{t+1}}{\pi_{t+1}^{\chi_w-1}\widetilde{W}_t}-\mu^I\right)\right] \tag{17}$$

From the first order conditions, notice that the uncovered interest parity (UIP) condition between domestic and foreign interest rates can be recovered after linearizing equations 3 and 4. The UIP condition holds in its strict sense only in the steady state, since the non-linear dynamics is also influenced by the presence of domestic and foreign portfolio adjustment costs. This is a departure from other studies, like Adolfson, Laseén, Lindé and Villani (2007)[2], where the only source of discrepancy between the domestic and foreign interest rates from the UIP condition is the debt-elastic foreign interest rate. As the description of the foreign block of the model will make clear, the UIP condition combines the debt-elastic foreign interest rate and the portfolio adjustment cost proposed in SGU (2003b)[35].

The household also solves a sequence of minimization problems constrained by the CES function in order to choose the composition of the consumption and investment goods. Expressing first the consumption problem, households decide between imported and domestically produced goods in the tradable goods basket, and then chooses the optimal composition of tradable and non-tradable goods. For simplicity, assume that portfolio adjustment costs are paid with a share of the consumption goods acquired by the households. As a consequence, the cost minimization problem is given by:

$$\min_{C_{n,t},C_{t,t},C_{m,t},C_{x,t}} P_{n,t}C_{n,t}+P_{t,t}C_{t,t}$$

$$C_t+PAC_{b,t}+PAC_{ib,t}=\left[(1-\omega)^{\frac{1}{\varepsilon}}C_{n,t}^{\frac{\varepsilon-1}{\varepsilon}}+\omega^{\frac{1}{\varepsilon}}C_{t,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{18}$$

$$C_{t,t}=\left[(1-\varkappa)^{\frac{1}{\varrho}}C_{x,t}^{\frac{\varrho-1}{\varrho}}+\varkappa^{\frac{1}{\varrho}}C_{m,t}^{\frac{\varrho-1}{\varrho}}\right]^{\frac{\varrho}{\varrho-1}} \tag{19}$$

$$PAC_{b,t}=\frac{\psi_1}{2}Y_t\left(\frac{B_{h,t+1}}{Y_t}-\frac{B_h}{Y}\right)^2$$

$$PAC_{ib,t}=\frac{\psi_2}{2}Y_t\left(\frac{S_tIB_{t+1}}{P_tY_t}-\frac{IB}{Y}\right)^2$$

Combine the first order conditions to obtain the demand for each type of tradable good:

$$C_{m,t}=\varkappa\left(\frac{P_{m,t}}{P_{t,t}}\right)^{-\varrho}C_{t,t} \tag{20}$$

$$C_{x,t}=(1-\varkappa)\left(\frac{P_{x,t}}{P_{t,t}}\right)^{-\varrho}C_{t,t} \tag{21}$$

By analogy, the optimal decision between tradable and non-tradable goods is given by:

$$C_{t,t}=\omega\left(\frac{P_{t,t}}{P_t}\right)^{-\varepsilon}(C_t+PAC_{b,t}+PAC_{ib,t}) \tag{22}$$

$$C_{n,t}=(1-\omega)\left(\frac{P_{n,t}}{P_t}\right)^{-\varepsilon}(C_t+PAC_{b,t}+PAC_{ib,t}) \tag{23}$$

Households solve a similar problem setting the investment good in each sector. For simplicity, assume that the weights and the elasticities of substitution among different investment goods are the same as those for consumption goods. Adjustment costs in capital utilization are paid in terms of aggregate investment. The demand for home produced and imported investment goods become:

$$\Upsilon_t^{-1} I_t = \Upsilon_t^{-1} \left(I_{n,t}^d + a\left(\mu_{n,t}\right) \overline{K}_{n,t} + I_{x,t}^d + a\left(\mu_{x,t}\right) \overline{K}_{x,t} \right) \tag{24}$$

$$I_{m,t} = \varkappa \left(\frac{P_{m,t}}{P_{t,t}} \right)^{-\varrho} I_{t,t} \tag{25}$$

$$I_{x,t} = (1-\varkappa) \left(\frac{P_{x,t}}{P_{t,t}} \right)^{-\varrho} I_{t,t} \tag{26}$$

$$I_{t,t} = \omega \left(\frac{P_{t,t}}{P_t} \right)^{-\varepsilon} \Upsilon_t^{-1} I_t \tag{27}$$

$$I_{n,t} = (1-\omega) \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon} \Upsilon_t^{-1} I_t \tag{28}$$

2.2 Firms

There are four sectors in the economy, each sector composed by a continuum of firms in a monopolistic competitive framework. Firms in non-tradable (n) and tradable (x) sectors demand labor and capital to produce. Firms in the imported (exported) goods sector, m (xp), buy the final good and sell it in the domestic economy (rest of the world). Firms chooses the amount of input and set prices based on a probability $\alpha_i, i = \{n, x, m, xp\}$, that is independent across sectors and firms. Firms not allowed to optimize prices in period t change prices according to an indexation rule based on past inflation. Imported goods' firms must finance input acquisition using foreign currency. There is no firm entry into or exit out of sector i. Equations describing price dynamics would result, in a log-linearized model around price stability, in equations like the New Keynesian Phillips curve. However, since price stability might not be optimal for Ramsey planner, the recursive formulation for the first order condition in terms of prices described in SGU (2006)[37] is adopted.

2.2.1 Domestic non-tradable goods' producers problem:

Firms in the non-tradable sector produce goods used for consumption, investment and spent by the government. Setting real profits as $\Phi_{n,t}(i_n)$, the problem of producers of type i_n product ($i_n \in [0,1]$) is to maximize the expected discounted stream of profits, subject to the demand for good i_n, the production technology and the aggregate demand for non-tradable goods. Firms choose in each period the demand for labor, capital and, with probability $1-\alpha_n$, they optimize prices. The problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{n,t} \left(\frac{P_{n,t}\left(i_n\right)}{P_{n,t}} D_{n,t}\left(i_n\right) - \frac{W_t}{P_{n,t}} h_{n,t}\left(i_n\right) - \frac{P_t}{P_{n,t}} R_{n,t}^k K_{n,t}\left(i_n\right) \right)$$

$$s.t.: \quad D_{n,t}\left(i_n\right) = \left(\tfrac{P_{n,t}\left(i_n\right)}{P_{n,t}} \right)^{-\eta_n} Y_{n,t}$$

$$Y_{n,t} = C_{n,t} + G_{n,t} + \Upsilon_t^{-1} \tfrac{P_t}{P_{n,t}} I_{n,t}$$

$$a_{n,t} K_{n,t}\left(i_n\right)^{\theta} \left(z_t h_{n,t}\left(i_n\right)\right)^{1-\theta} - z_t^* \chi_n \geq D_{n,t}\left(i_n\right)$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \tfrac{z_t^*}{z_t}$$

$$\tfrac{z_{t+1}}{z_t} = \mu_{t+1}^z = \left(1-\rho_z\right)\mu^z + \rho_z \mu_t^z + \epsilon_{t+1}^z; \quad \rho_z \in [0,1); \quad \epsilon_t^z \sim N\left(0, \sigma_z\right)$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon_{t+1}^n; \quad \rho_n \in [0,1); \quad \epsilon_t^n \sim N\left(0, \sigma_n\right)$$

In this problem, $a_{n,t}$ is a stationary, sector-specific technology shock, z_t is a labor-augmenting, non-stationary technology shock, affecting firms using labor as input. In order to guarantee zero profits in the steady state, $z_t^* \chi_n$ introduces a fixed cost proportional to the evolution of the non-stationary shock, following CEE (2005)[11] and SGU (2005, 2006)[36][37]. Parameter η_n is the elasticity of substitution across varieties of non-tradable goods.

From the first order conditions in terms of $h_{n,t}(i_n)$ and $K_{n,t}(i_n)$, it is possible to prove that the capital-labor ratio and the marginal cost are the same across firms in the non-tradable sector. Setting $mc_{n,t}$ as the Lagrange multiplier on the firm's demand constraint, the equilibrium conditions are:

$$\widetilde{W}_t \frac{P_t}{P_{n,t}} = mc_{n,t}(1-\theta)\, a_{n,t} z_t \left(\frac{K_{n,t}}{z_t h_{n,t}}\right)^{\theta} \tag{29}$$

$$R^k_{n,t} \frac{P_t}{P_{n,t}} = mc_{n,t}\theta a_{n,t} \left(\frac{K_{n,t}}{z_t h_{n,t}}\right)^{\theta-1} \tag{30}$$

Prices are formed in a Calvo style with indexation, where α_n is the probability that firm i_n is not allowed to optimally adjust its price in period t. In the case firms are not allowed to optimize prices, they follow the rule $P_{n,t}(i_n) = \pi^{\kappa_n}_{n,t-1} P_{n,t-1}(i_n)$, for $0 \leq \kappa_n \leq 1$ and $\pi_{n,t+1} = \frac{P_{n,t+1}}{P_{n,t}}$. Setting the Lagrangean of the problem, considering only the relevant terms for price determination:

$$\begin{aligned} \mathcal{L}_n = E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} P_{n,t+s} & \left(\left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}}\right)^{1-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{1-\eta_n} Y_{n,t+s} \right. \\ & \left. - mc_{n,t+s} \left(\left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}}\right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{-\eta_n} Y_{n,t+s} \right) \right) \end{aligned}$$

In this problem, $r_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and $\widetilde{P}_{n,t}(i_n)$ is the new price set by firms allowed to adjust prices in period t. The first order condition for firms is:

$$\begin{aligned} E_t \sum_{s=0}^{\infty} \alpha_n^s r_{t,t+s} Y_{n,t+s} P_{n,t+s} & \left(\frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}}\right)^{-\eta_n} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right)^{-\eta_n} \times \\ & \left(\frac{(\eta_n - 1)}{\eta_n} \frac{\widetilde{P}_{n,t}(i_n)}{P_{n,t+s}} \prod_{k=1}^{s} \left(\frac{\pi^{\kappa_n}_{n,t+k-1}}{\pi_{n,t+k}}\right) - mc_{n,t+s} \right) = 0 \end{aligned}$$

Since markup over prices is the same across firms, the symmetric equilibrium is characterized by all firms in sector n allowed to adjust prices in period t setting the same price: $\widetilde{P}_{n,t}(i_n) = \widetilde{P}_{n,t}$. Following SGU (2006)[37], split the first order condition in two parts, X_t^1 and X_t^2, and define $\widetilde{p}_{n,t} = \frac{\widetilde{P}_{n,t}}{P_{n,t}}$ in order to obtain a recursive solution for the price-setting problem:

$$X_t^1 = Y_{n,t} \widetilde{p}_{n,t}^{\,-1-\eta_n} mc_{n,t} + \alpha_n r_{t,t+1} E_t \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-1-\eta_n} \left(\frac{\pi^{\kappa_n}_{n,t}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}}\right)^{-\eta_n} X^1_{t+1} \tag{31}$$

$$X_t^2 = Y_{n,t} \widetilde{p}_{n,t}^{\,-\eta_n} \frac{(\eta_n - 1)}{\eta_n} + \alpha_n r_{t,t+1} E_t \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}}\right)^{-\eta_n} \left(\frac{\pi^{\kappa_n}_{n,t}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}}\right)^{1-\eta_n} X^2_{t+1} \tag{32}$$

$$X_t^1 = X_t^2 \tag{33}$$

2.2.2 Tradable goods' producers problem:

A tradable goods producer i_x ($i_x \in [0,1]$) solves the same problem as the non-tradable producer, using labor and capital as production inputs. The production of the tradable good is divided between domestic absorption (consumption, investment and government spending) and the demand of a continuum of i_{xp} exporting firms ($D_{xp,t}$). The tradable goods' firm problem is given by:

$$\max E_0 \sum_{t=0}^{\infty} r_{0,t} P_{x,t} \left(\frac{P_{x,t}(i_x)}{P_{x,t}} D_{x,t}(i_x) - \frac{\widetilde{W}_t}{P_{x,t}} h_{x,t}(i_x) - \frac{P_t}{P_{x,t}} R^k_{x,t} K_{x,t}(i_x) \right)$$

$$s.t.: \quad D_{x,t}(i_x) = \left(\tfrac{P_{x,t}(i_x)}{P_{x,t}}\right)^{-\eta_x} Y_{x,t}$$

$$Y_{x,t} = C_{x,t} + G_{t,t} + \Upsilon_t^{-1} \tfrac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t}$$

$$a_{x,t} K_{x,t}(i_x)^{\theta} (z_t h_{x,t}(i_x))^{1-\theta} - z_t^* \chi_x \geq D_{x,t}(i_{tr})$$

$$\Upsilon_t^{\frac{\theta}{1-\theta}} = \tfrac{z_t^*}{z_t}$$

$$\tfrac{z_{t+1}}{z_t} = \mu^z_{t+1} = (1-\rho_z)\mu^z + \rho_z \mu^z_t + \epsilon^z_{t+1}; \quad \rho_z \in [0,1); \quad \epsilon^z_t \sim N(0,\sigma_z)$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon^x_{t+1}; \quad \rho_x \in [0,1); \quad \epsilon^x_t \sim N(0,\sigma_x)$$

χ_x is a fixed cost related to the non-stationary shock ensuring zero profits in steady state. Parameter η_x is the elasticity of substitution across types of tradable goods. Setting $mc_{x,t}$ as the Lagrange multiplier, the cost minimization problem, noting that capital-labor ratio is the same across firms, implies:

$$\widetilde{W}_t \frac{P_t}{P_{x,t}} = mc_{x,t}(1-\theta) a_{x,t} z_t \left(\frac{K_{x,t}}{z_t h_{x,t}}\right)^{\theta} \tag{34}$$

$$R^k_{x,t} \frac{P_t}{P_{x,t}} = mc_{x,t} \theta a_{x,t} \left(\frac{K_{x,t}}{z_t h_{x,t}}\right)^{\theta-1} \tag{35}$$

Similar to the non-tradable sector, price adjustment is based on a Calvo mechanism with indexation, with $0 \leq \kappa_x \leq 1$ defining the degree of indexation in the tradable sector. Taking the first order conditions in terms of $\widetilde{P}_{x,t}(i_x)$, and defining $\pi_{x,t+1} = \frac{P_{x,t+1}}{P_{x,t}}$, the optimal price set by each firm solves a recursive problem that can be split in two equations for Z^1_t, Z^2_t, such that $Z^1_t = Z^2_t$, and $\widetilde{p}_{x,t} = \frac{\widetilde{P}_{x,t}}{P_{x,t}}$:

$$Z^1_t = \widetilde{p}_{x,t}^{-1-\eta_x} Y_{x,t} mc_{x,t} + \alpha_x r_{t,t+1} E_t \left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-1-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}}\right)^{-\eta_x} Z^1_{t+1} \tag{36}$$

$$Z^2_t = \widetilde{p}_{x,t}^{-\eta_x} Y_{x,t} \frac{(\eta_x - 1)}{\eta_x} + \alpha_x r_{t,t+1} E_t \left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}}\right)^{-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}}\right)^{1-\eta_x} Z^2_{t+1} \tag{37}$$

$$Z^1_t = Z^2_t \tag{38}$$

2.2.3 Imported goods' firms problem:

Following Lubik and Schorfheide (2006)[29], deviations from the Law of One price arises as a consequence of price stickiness in imported and exported goods. An imported goods' firm i_m ($i_m \in [0,1]$) buys a bundle of the international homogeneous good[14] and label it as an imported good type i_m. In order to buy input from the rest of the world, the firm needs to pay using foreign currency. The firm sells intraperiod bonds in foreign markets in order to get foreign currency, but it does not transfer financial

[14]In the model, one country buys a combination of final goods from different countries, generating a gap between the world's CPI (P^*_t) and the price of the bundle imported by a given country ($P^*_{m,t}$).

wealth over time. As a consequence, firms face only an increase in the marginal cost of production. As a timing convention, traded bonds do not reflect in the end of period balance of payments. The same framework is adopted in Christiano, Trabandt and Walentin (2007)[12] and Mendoza and Yue (2008)[31]. The budget constraint of the importing firm i_m, expressed in terms of domestic prices, is given by:

$$\begin{aligned}&\frac{S_t P_t^*}{P_t} M_{m,t}^*(i_m) + \frac{S_t}{P_t} B_{m,t+1}^*(i_m) = \\ &\quad \frac{S_t}{P_t} P_{t-1}^* M_{m,t-1}^*(i_m) + \frac{S_t}{P_t} R_{t-1}^f B_{m,t}^*(i_m) + \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}(i_m) - z_t^* \chi_m - \Phi_{m,t}(i_m)\end{aligned}$$

where χ_m is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Following assumptions that firms do not keep any financial wealth across periods and that all profits are distributed to the households, obtain the expression for real profits:

$$P_t^* M_{m,t}^*(i_m) + R_t^f B_{m,t+1}^*(i_m) = 0, \qquad \forall t$$

$$\Longrightarrow \Phi_{m,t}(i_m) = \left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}(i_m) - z_t^* \chi_m - \frac{S_t P_t^*}{P_t}\left(\frac{R_t^f - 1}{R_t^f}\right) M_{m,t}^*(i_m)$$

The imported goods' firm problem becomes:

$$\max_{\widetilde{P}_{m,t}(i_m)} E_0 \sum_{t=0}^{\infty} r_{0,t}\left[\left(\frac{P_{m,t}(i_m) - S_t P_{m,t}^*}{P_t}\right) D_{m,t}(i_m) - z_t^* \chi_m - \frac{S_t P_{m,t}^*}{P_t}\left(\frac{R_t^f - 1}{R_t^f}\right)\frac{P_t^*}{P_{m,t}^*} M_{m,t}^*(i_m)\right]$$

$$s.t.: D_{m,t}(i_m) = \left(\frac{P_{m,t}(i_m)}{P_{m,t}}\right)^{-\eta_m}\left(C_{m,t} + \Upsilon_t^{-1}\frac{P_t}{P_{m,t}} I_{m,t}\right)$$

$$M_{m,t}(i_m) \geq \frac{P_{m,t}^*}{P_t^*} D_{m,t}(i_m)$$

where $P_{m,t}^*$ is the price of the imported good bought by the domestic economy, quoted in foreign prices. Define η_m as the elasticity of substitution across varieties of imported goods, α_m as the probability that firm i_m is not allowed to adjust prices in period t, $\pi_{m,t+1} = \frac{P_{m,t+1}}{P_{m,t}}$ and $0 \leq \kappa_m \leq 1$ the degree of indexation in the imported goods' sector. Taking the first order conditions in terms of $\widetilde{P}_{m,t}(i_m)$, and as a consequence of the same mark-up over prices across firms (given by the real exchange rate based on the import price level), the symmetric equilibrium is characterized by $\widetilde{P}_{m,t}(i_m) = \widetilde{P}_{m,t}$. The recursive solution for the problem is obtained after defining Y_t^1 and Y_t^2 such that $Y_t^1 = Y_t^2$, and $\widetilde{p}_{m,t} = \frac{\widetilde{P}_{m,t}}{P_{m,t}}$:

$$\begin{aligned}Y_t^1 = \widetilde{p}_{m,t}^{-1-\eta_m}\left(C_{m,t} + \frac{\Upsilon_t^{-1} P_t}{P_{m,t}} I_{m,t}\right)\frac{S_t P_{m,t}^*}{P_{m,t}}\left(1 + \frac{R_t^f - 1}{R_t^f}\right) \\ + \alpha_m r_{t,t+1} E_t\left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}}\right)^{-1-\eta_m}\left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}}\right)^{-\eta_m} Y_{t+1}^1\end{aligned} \tag{39}$$

$$Y_t^2 = \widetilde{p}_{m,t}^{-\eta_m}\left(C_{m,t} + \frac{\Upsilon_t^{-1} P_t}{P_{m,t}} I_{m,t}\right)\frac{(\eta_m - 1)}{\eta_m} + \alpha_m r_{t,t+1} E_t\left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}}\right)^{-\eta_m}\left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{\eta_m/(\eta_m-1)}}\right)^{1-\eta_m} Y_{t+1}^2 \tag{40}$$

$$Y_t^1 = Y_t^2 \tag{41}$$

2.2.4 Exported goods' firms problem:

An exported goods' firm i_{xp} ($i_{xp} \in [0,1]$) buys a share of the final tradable good in the domestic economy and sell it to the rest of the world, setting prices in foreign currency in a Calvo style – thus, prices are sticky in foreign currency. The firm problem is given by:

$$\max_{\widetilde{P}^*_{x,t}(i_{xp})} E_0 \sum_{t=0}^{\infty} r_{0,t} \left[\left(\frac{S_t \widetilde{P}^*_{x,t}(i_{xp}) - P_{x,t}}{P_t} \right) D_{xp,t}(i_{xp}) - \left(\frac{R_t - 1}{R_t} \right) M_{xp,t}(i_{xp}) - z^*_t \chi_{xp} \right]$$

$$s.t.: D_{xp,t}(i_{xp}) = \left(\frac{P^*_{x,t}(i_{xp})}{P^*_{x,t}} \right)^{-\eta_{xp}} X_t$$

where χ_{xp} is a fixed cost associated with the non-stationary shock in order to guarantee zero profits in steady state. Define η_{xp} as the foreign elasticity of substitution across varieties of domestic exported goods, $\pi^*_{x,t+1} = \frac{P^*_{x,t+1}}{P^*_{x,t}}$, $0 \leq \kappa_{xp} \leq 1$ as the degree of indexation in the exported goods' sector, α_{xp} as the probability that firm i_x is not allowed to optimize prices in period t, and $P^*_{x,t}$ as the price of the tradable good quoted in foreign currency. Taking first order conditions in terms of $\widetilde{P}^*_{x,t}(i_{xp})$, and given that the symmetric equilibrium is characterized by $\widetilde{P}^*_{x,t}(i_x) = \widetilde{P}^*_{x,t}$, the recursive solution for the pricing problem of the exporting firms is obtained after properly defining U^1_t and U^2_t and $\widetilde{p}^*_{x,t} = \frac{\widetilde{P}^*_{x,t}}{P^*_{x,t}}$:

$$U^1_t = \left(\widetilde{p}^*_{x,t}\right)^{-1-\eta_{xp}} X_t \frac{P_{x,t}}{S_t P^*_{x,t}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\widetilde{p}^*_{x,t}}{\widetilde{p}^*_{x,t+1}} \right)^{-1-\eta_{xp}} \left(\frac{\left(\pi^*_{x,t}\right)^{\kappa_{xp}}}{\left(\pi^*_{x,t+1}\right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{-\eta_{xp}} U^1_{t+1} \tag{42}$$

$$U^2_t = \left(\widetilde{p}^*_{x,t}\right)^{-\eta_{xp}} X_t \frac{(\eta_{xp} - 1)}{\eta_{xp}} + \alpha_{xp} r_{t,t+1} E_t \left(\frac{\widetilde{p}^*_{x,t}}{\widetilde{p}^*_{x,t+1}} \right)^{-\eta_{xp}} \left(\frac{\left(\pi^*_{x,t}\right)^{\kappa_{xp}}}{\left(\pi^*_{x,t+1}\right)^{\frac{\eta_{xp}}{(\eta_{xp}-1)}}} \right)^{1-\eta_{xp}} U^2_{t+1} \tag{43}$$

$$U^1_t = U^2_t \tag{44}$$

2.3 Government

In a competitive equilibrium, the government follows simple rules to set monetary and fiscal policy. For the sake of this paper, as the steady state of the model is not affected by interest rate rules, assume that the government follows a standard Taylor rule, based on the deviations of inflation from an exogenous, autocorrelated inflation target and an autoregressive component[15]:

$$\log\left(\frac{R_{t+1}}{R}\right) = \rho_R \log\left(\frac{R_t}{R}\right) + \alpha_\pi \log\left(\frac{\pi_{t+1}}{\pi^o_{t+1}}\right) + \epsilon^R_{t+1}; \quad \epsilon^R_t \sim N(0, \sigma_R) \tag{45}$$

$$\pi^o_{t+1} = (1 - \rho_{\pi^o})\pi^o + \rho_{\pi^o}\pi^o_t + \epsilon^{\pi^o}_{t+1}; \quad \epsilon^{\pi^o}_t \sim N(0, \sigma_{\pi^o}) \tag{46}$$

The government, in order to finance its exogenous expenditures, G_t, collects distortionary taxes on consumption, labor, capital and profits income ($\tau^c_t, \tau^h_t, \tau^k_t$ and τ^ϕ_t), sells bonds domestically, $B_{g,t}$ and controls the money supply, M_t. The government budget constraint is given by:

$$P_t G_t + R_{t-1} B_{g,t} = P_t T_t + P_t M_t + B_{g,t+1} - P_{t-1} M_{t-1}$$

$$G_t = z^*_t g_t$$

$$g_t = (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon^g_t; \quad \epsilon^g_t \sim N(0, \sigma_g) \tag{47}$$

$$T_t = \tau^c_t C_t + \tau^h_t \widetilde{W}_t h_t + \tau^\phi_t \Phi_t + \tau^k_t \left[\left(R^k_{n,t}\mu_{n,t} - \Upsilon^{-1}_t a(\mu_{n,t}) \right) \overline{K}_{n,t} + \left(R^k_{x,t}\mu_{x,t} - \Upsilon^{-1}_t a(\mu_{x,t}) \right) \overline{K}_{x,t} \right] \tag{48}$$

Rewriting the government budget constraint as a function of total real government liabilities (L_t):

$$L_{t-1} \equiv M_{t-1} + \frac{R_{t-1}}{P_{t-1}} B_{g,t} \tag{49}$$

[15] To be more specific, any Taylor rule that ensures a non-zero, stationary inflation around the steady state satisfy the requirements of the competitive equilibrium.

$$\Longrightarrow L_t = \frac{R_t}{\pi_t} L_{t-1} + R_t \left(G_t - T_t\right) - \left(R_t - 1\right) M_t \tag{50}$$

To close the dynamics of fiscal policy, assume that government follows a policy rule for labor income taxation based on real liabilities as a function of GDP and the output gap. Taxes on capital and profits are exogenous. The assumption of a policy rule for labor income taxes is an arbitrary choice that does not affect the steady state of the model. For simplicity, assume also that taxation on profits is constant over time. Notice that taxes on profits are lump sum transfers from households to the government. In this sense, it does not interfere with dynamics under the competitive equilibrium, where profits are zero.

$$\tau_t^h - \tau^h = \psi_{li}\left(\frac{L_t}{Y_t} - \frac{l}{y}\right) + \psi_y \left(y_t - y\right) + \epsilon_t^{\tau h}; \quad \epsilon_t^{\tau h} \sim N\left(0, \sigma_{\tau h}\right) \tag{51}$$

$$\tau_t^k = \left(1 - \rho_{\tau k}\right)\tau^k + \rho_{\tau k}\tau_{t-1}^k + \epsilon_t^{\tau k}; \quad \epsilon_t^{\tau k} \sim N\left(0, \sigma_{\tau k}\right) \tag{52}$$

$$\tau_t^\phi = \tau^\phi \tag{53}$$

$$\tau_t^c = \left(1 - \rho_c\right)\tau^c + \rho_c \tau_{t-1}^c + \epsilon_t^{\tau c}; \quad \epsilon_t^{\tau c} \sim N\left(0, \sigma_c\right) \tag{54}$$

Additionally, the government solves the same problem as households to determine the consumption of tradable and non-tradable goods. By assumption, the government does not consume imported goods[16]:

$$G_{n,t} = \left(1 - \omega\right)\left(\frac{P_{n,t}}{P_t}\right)^{-\varepsilon} G_t \tag{55}$$

$$G_{t,t} = \omega \left(\frac{P_{t,t}}{P_t}\right)^{-\varepsilon} G_t \tag{56}$$

2.4 International Financial Markets and World's Economy

The transmission of shocks from international financial markets assumes a market capable of pricing country-specific risk on bonds traded outside domestic economy. A mechanism to induce stationarity as in SGU (2003b)[35] is used to determine the risk premium of bonds as a function of the net foreign position of the economy. The international interest rate is given by:

$$R_t^f = R_t^* \left(1 + \xi_t\right)^{\kappa_1} \left(\frac{S_t IB_{t+1}}{P_t Y_t} / \frac{IB}{Y}\right)^{\kappa_2} \tag{57}$$

In this equation, R_t^* is the nominal interest rate on a risk-free bond; ξ_t is an exogenous shock with expected value equal to the long run risk premium of the domestic economy, ξ^*; the last term is the gap of external debt as a proportion of GDP.

The world's economy is set as a stationary VAR with output, y_t^*, inflation, π_t^*, interest rates, R_t^*, growth of money supply, ΔM_t^*, and the risk premium, ξ_t, providing five shocks for the model. As the VAR does not influence the steady state, it is not necessary to be more specific with respect to additional structure. In the system, A is a 5 by 5 matrix of coefficients, $\sum$ is a 5 by 5 covariance matrix of shocks.

$$\begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R_t^*}{R^*} \\ \frac{\pi_t^*}{\pi^*} \\ \frac{y_t^*}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M_{t-1}^*}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R_{t-1}^*}{R^*} \\ \frac{\pi_{t-1}^*}{\pi^*} \\ \frac{y_{t-1}^*}{y^*} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{m*} \\ \epsilon_t^{\xi} \\ \epsilon_t^{R*} \\ \epsilon_t^{\pi*} \\ \epsilon_t^{y*} \end{bmatrix} \quad \begin{bmatrix} \epsilon_t^{m*} \\ \epsilon_t^{\xi} \\ \epsilon_t^{R*} \\ \epsilon_t^{\pi*} \\ \epsilon_t^{y*} \end{bmatrix} \overset{iid}{\sim} \left(0, \sum\right) \tag{58}$$

Two assumptions close the link between prices and quantities of goods between the domestic economy and the rest of world. First, assume that foreign households solve an expenditure minimization

[16]The same assumption is used in Lubik and Schorfheide (2006)[29].

problem to set the demand for home produced tradable goods. The solution of this problem is given by:

$$X_t = \left(\frac{P^*_{x,t}}{P^*_t}\right)^{-\eta^*} z^*_t y^*_t \tag{59}$$

Finally, the terms of trade are defined as the ratio between the exported goods and the imported goods price levels, both in foreign currency. The dynamics of the price of imported goods is given by an error-correction model that ensure the terms of trade becomes stationary, as in García-Cicco (2009)[21]:

$$tot_t = \frac{\pi^*_{x,t}}{\pi^*_{m,t}} tot_{t-1} \tag{60}$$

$$\frac{\pi^{m*}_t}{\pi^{m*}} = \upsilon_1 \frac{\pi^{m*}_{t-1}}{\pi^{m*}} + \upsilon_2 \frac{tot_{t-1}}{tot} + \S X^*_{t-1} + \epsilon^{\pi m}_t \qquad \epsilon^{\pi m}_t \sim N\left(0, \sigma_{\pi m}\right) \tag{61}$$

with $X^*_t = \left[\begin{array}{ccccc} \frac{\Delta M^*_t}{\Delta M^*} & \frac{\xi_t}{\xi^*} & \frac{R^f_t}{R^*} & \frac{\pi^*_t}{\pi^*} & \frac{y^*_t}{y^*} \end{array}\right]$.

2.5 Aggregation and Relative Prices

In order to find an expression for the aggregate constraint of the economy, start from the demand faced by a non-tradable producer firm and integrate it over all i_n firms, noting that $h_{n,t} = \int_0^1 h_{n,t}\left(i_n\right) di_n$, and that capital-labor ratio is constant across all the firms. Define $s_{n,t} = \int_0^1 \left(\frac{P_{n,t}(i_n)}{P_{n,t}}\right)^{-\eta_{n,t}} di_n$ to obtain:

$$a_{n,t} K^\theta_{n,t} \left(z_t h_{n,t}\right)^{1-\theta} - z^*_t \chi_n = s_{n,t} \left(C_{n,t} + G_{n,t} + \Upsilon^{-1}_t \frac{P_t}{P_{n,t}} I_{n,t}\right) \tag{62}$$

Obtain the recursive form of $s_{n,t}$:

$$s_{n,t} = \int_0^1 \left(\frac{P_{n,t}\left(i_n\right)}{P_{n,t}}\right)^{-\eta_n} di \Longrightarrow s_{n,t} = \left(1-\alpha_n\right) \widetilde{p}^{-\eta_n}_{n,t} + \alpha_n \left(\frac{\pi_{n,t}}{\pi^{\kappa_n}_{n,t-1}}\right)^{\eta_n} s_{n,t-1} \tag{63}$$

Also, from the definition of the non-tradable goods price index:

$$P_{n,t} = \left[\int_0^1 P_{n,t}\left(i_n\right)^{1-\eta_n} di\right]^{\frac{1}{1-\eta_n}} \Longrightarrow 1 = \left(1-\alpha_n\right) \widetilde{p}^{1-\eta_n}_{n,t} + \alpha_n \left(\frac{\pi^{\kappa_n}_{n,t-1}}{\pi_{n,t}}\right)^{1-\eta_n} \tag{64}$$

Similar expressions can be written for resource constraint, price dispersion and price index of imported and domestically produced tradable goods and the prices of exported goods in foreign currency:

$$D_{m,t} - z^*_t \chi_m = s_{m,t} \left(C_{m,t} + \Upsilon^{-1}_t \frac{P_t}{P_{m,t}} I_{m,t}\right) \tag{65}$$

$$s_{m,t} = \left(1-\alpha_m\right) \widetilde{p}^{-\eta_m}_{m,t} + \alpha_m \left(\frac{\pi_{m,t}}{\pi^{\kappa_m}_{m,t-1}}\right)^{\eta_m} s_{m,t-1} \tag{66}$$

$$1 = \left(1-\alpha_m\right) \widetilde{p}^{1-\eta_m}_{m,t} + \alpha_m \left(\frac{\pi^{\kappa_m}_{m,t-1}}{\pi_{m,t}}\right)^{1-\eta_m} \tag{67}$$

$$a_{x,t} K^\theta_{x,t} \left(z_t h_{x,t}\right)^{1-\theta} - z^*_t \chi_x = s_{x,t} \left(C_{x,t} + G_{t,t} + \Upsilon^{-1}_t \frac{P_t}{P_{x,t}} I_{x,t} + D_{xp,t}\right) \tag{68}$$

$$s_{x,t} = \left(1-\alpha_x\right) \widetilde{p}^{-\eta_x}_{x,t} + \alpha_x \left(\frac{\pi_{x,t}}{\pi^{\kappa_x}_{x,t-1}}\right)^{\eta_x} s_{x,t-1} \tag{69}$$

$$1 = \left(1-\alpha_x\right) \widetilde{p}^{1-\eta_x}_{x,t} + \alpha_x \left(\frac{\pi^{\kappa_x}_{x,t-1}}{\pi_{x,t}}\right)^{1-\eta_x} \tag{70}$$

$$D_{xp,t} - z_t^* \chi_{xp} = s_{xp,t} X_t \tag{71}$$

$$s_{xp,t} = (1-\alpha_{xp}) \left(\widetilde{p}^*_{x,t}\right)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi^*_{xp,t}}{\left(\pi^*_{xp,t-1}\right)^{\kappa_{xp}}} \right)^{\eta_{xp}} s_{xp,t-1} \tag{72}$$

$$1 = (1-\alpha_{xp}) \widetilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{\left(\pi^*_{x,t-1}\right)^{\kappa_{xp}}}{\pi^*_{x,t}} \right)^{1-\eta_{xp}} \tag{73}$$

The total amount of work hours supplied by the domestic households is given by:

$$h_{x,t} + h_{n,t} = h_t \tag{74}$$

The external equilibrium assumes that net foreign position of domestic households is proportional to trade balance in steady state. Again, notice that the external equilibrium in bond markets does not include bonds issued by imported goods' firms, as they are negotiated and liquidated inside each period:

$$P_{x,t} X_t - P_{m,t} D_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] = S_t R_{t-1}^f P_t^* IB_t - S_t P_{t+1}^* IB_{t+1} \tag{75}$$

In order to set the market clearing conditions for domestic bonds and money market, assume that foreign households and domestic firms do not demand home government bonds. As a consequence:

$$B_{g,t} + B_{h,t} = 0 \tag{76}$$

Finally, the gross domestic product and aggregate profits are given by:

$$\begin{aligned} Y_t = C_t + \frac{\psi_1}{2} Y_t \left(\frac{B_{t+1}}{Y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} Y_t \left(\frac{S_t IB_{t+1}}{P_t Y_t} - \frac{rer\ IB}{Y} \right)^2 \\ + \Upsilon_t^{-1} I_t + G_t + \frac{P_{x,t}}{P_t} X_t - \frac{P_{m,t}}{P_t} D_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f} \right) \right] \end{aligned} \tag{77}$$

$$\Phi_t = Y_t - \widetilde{W}_t h_t - R^k_{n,t} \mu_{n,t} \overline{K}_{n,t} - R^k_{x,t} \mu_{x,t} \overline{K}_{x,t} \tag{78}$$

2.6 Relative prices

The complete set of relative prices in the model is given by:

$$pt_t = \frac{P_{t,t}}{P_t} = \frac{\pi_{t,t}}{\pi_t} \frac{P_{t,t-1}}{P_{t-1}} \tag{79}$$

$$pn_t = \frac{P_{n,t}}{P_t} = \frac{\pi_{n,t}}{\pi_t} \frac{P_{n,t-1}}{P_{t-1}} \tag{80}$$

$$px_t = \frac{P_{x,t}}{P_{t,t}} = \frac{\pi_{x,t}}{\pi_{t,t}} \frac{P_{x,t-1}}{P_{t,t-1}} \tag{81}$$

$$pm_t = \frac{P_{m,t}}{P_{t,t}} = \frac{\pi_{m,t}}{\pi_{t,t}} \frac{P_{m,t-1}}{P_{t,t-1}} \tag{82}$$

$$pm^*_t = \frac{P^*_{m,t}}{P^*_t} = \frac{\pi^*_{m,t}}{\pi^*_t} \frac{P^*_{m,t-1}}{P^*_{t-1}} \tag{83}$$

$$rer_t = \frac{S_t P^*_t}{P_t} \tag{84}$$

2.7 Stationary Form and Equilibrium

The objective of this section is to characterize the stationary competitive and Ramsey Equilibria. Define stationary allocations such that, for a generic variable X_t and the appropriate trend $\check{Z}_t$, the stationary variable is given by $x_t \equiv X_t/\check{Z}_t$. The model in stationary form is described by the following variables:

- prices: $\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r^k_{x,t}, r^k_{n,t}, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi^*_t, \pi^*_{x,t}, \pi^{m*}_t, \widetilde{p}_{n,t}, \widetilde{p}_{x,t}, \widetilde{p}_{m,t}, \widetilde{p}^*_{x,t}, pt_t, pn_t, px_t, pm_t, pm^*_t, tot_t$;
- interest rates: $R_t, \widetilde{R}_t, R^*_t, R^f_t$;
- allocations: $c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i^d_{x,t}, i^d_{n,t}, y_t, \overline{k}_{x,t}, \overline{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x^1_t, x^2_t, z^1_t, z^2_t, y^1_t, y^2_t, u^1_t, u^2_t, ib_t, b_{h,t}, \xi_t, \Delta M^*_t, y^*_t, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, m_t, \phi_t$;
- government policies: $\tau^h_t, l_t, t_t, b_{g,t}$;
- domestic shocks: $g_t, \tau^k_t, \tau^c_t, \tau^\phi_t, a_{x,t}, a_{n,t}, \mu^z_t, \mu^\Upsilon_t, \pi^o_t$.

The equations describing the law of motion of the variables are given by a set of equilibrium conditions for the household (equations 1-28), firms responsible for domestic production (equations 29-38), exporting and importing firms (equations 39-44), government (equations 45-56), foreign sector (equations 57-61), aggregation and price indexes (equations 62-78) and relative prices (equations 79-84). Additionally, there are 4 exogenous processes for sectorial productivity and aggregate productivity growth $(a_{x,t}, a_{n,t}, \mu^z_t, \mu^\Upsilon_t)$. Alternatively, there are 83 equations for endogenous variables[17] and 9 domestic exogenous stochastic processes for a total of 92 variables in the model.

Prices and shocks are stationary, but allocations must be normalized in order to ensure stationarity. The set of variables given by $\{\overline{K}_{n,t+1}, K_{n,t+1}, \overline{K}_{x,t+1}, K_{x,t+1}, I_t, I_{t,t}, I_{n,t}, I_{m,t}, I_{x,t}, I^d_{x,t}, I^d_{n,t}\}$ must be normalized by $z^*_t\Upsilon_t$, while the set given by $\{Y_t, C_t, C_{t,t}, C_{n,t}, C_{m,t}, C_{x,t}, W_t, X_t, D_{m,t}, D_{xp,t}, B_{h,t+1}, B_{g,t+1}, IB_{t+1}, M_t, X^1_t, X^2_t, Z^1_t, Z^2_t, Y^1_t, Y^2_t, U^1_t, U^2_t, G_t, G_{t,t}, G_{n,t}, L_t, T_t\}$ must be adjusted by z^*_t. Finally, rental rate of capital $\{R^k_{x,t}, R^k_{n,t}\}$ and the shadow prices of investment $\{\widetilde{q}_{x,t}, \widetilde{q}_{n,t}\}$ are divided by Υ^{-1}_t, while the Lagrange multiplier of consumption, $\widetilde{\lambda}_t$, is normalized by $(z^*_t)^{-1}$ to obtain λ_t.

2.7.1 Competitive Equilibrium

Definition 1: *Given exogenous paths for shocks* $\left\{g_t, \tau^k_t, \tau^\phi_t, \tau^c_t, a_{x,t}, a_{n,t}, \mu^z_t, \mu^\Upsilon_t, \pi^o_t\right\}$, *foreign sector variables* $\{\Delta M^*_t, \xi_t, \ R^*_t, \pi^*_t, y^*_t, \pi^*_{m,t}\}$, *policy processes for interest rates* $\left\{R_t, \widetilde{R}_t, R^f_t\right\}$ *and taxes* τ^h_t, *and initial values for prices* $\{\pi_{-1}, \pi_{n,-1}, \pi_{x,-1}, \pi_{t,-1}, \pi_{m,-1}, w_{-1}, pt_{-1}, pn_{-1}, \ px_{-1}, pm_{-1}, pm^*_{-1}, tot_{-1}\}$ *and allocations* $\{c_{-1}, \ i^d_{x,-1}, i^d_{n,-1}, \overline{k}_{x,0}, \overline{k}_{n,0}, b_{h,-1}, b_{g,-1}, ib_{-1}, s_{n,-1}, s_{m,-1}, s_{x,-1}, s_{xp,-1}, l_{-1}\}$, *a stationary competitive equilibrium is a set of processes for prices* $\{\pi_t, \pi_{n,t}, \pi_{x,t}, \pi_{t,t}, \pi_{m,t}, w_t, r^k_{x,t}, r^k_{n,t}, r_{t,t+1}, mcw_t, mc_{n,t}, mc_{x,t}, rer_t, \pi^*_{x,t}, \widetilde{p}_{n,t}, \widetilde{p}_{x,t}, \widetilde{p}_{m,t}, \widetilde{p}^*_{x,t}, pt_t, pn_t, px_t, pm_t, pm^*_t, tot_t\}$ *and allocations* $\{c_t, c_{t,t}, c_{n,t}, c_{m,t}, c_{x,t}, i_t, i_{t,t}, i_{n,t}, i_{m,t}, i_{x,t}, x_t, d_{m,t}, d_{xp,t}, \mu_{x,t}, \mu_{n,t}, i^d_{x,t}, i^d_{n,t}, y_t, \overline{k}_{x,t}, \overline{k}_{n,t}, k_{x,t}, k_{n,t}, h_t, h_{n,t}, h_{x,t}, x^1_t, x^2_t, z^1_t, z^2_t, y^1_t, y^2_t, u^1_t, u^2_t, ib_t, b_{h,t}, b_{g,t}, s_{n,t}, s_{m,t}, s_{x,t}, s_{xp,t}, \lambda_t, m_t, q_{x,t}, q_{n,t}, g_{t,t}, g_{n,t}, t_t, l_t, \phi_t\}$ *such that, after stationary transformations of the respective equations: a) households maximize utility; b) firms maximize profits; c) government balances its budget; d) markets clear.*

2.7.2 Ramsey Equilibrium

The Ramsey equilibrium is evaluated from the "timeless perspective" described in Woodford (2003)[43], where the government can not change its policy from the time when the Ramsey policy is implemented

[17] Note that equation 58 is a 5-variable VAR.

to the next periods. Given that capital is a predetermined variable in the model, the Ramsey planner, without this constraint, could maximize its revenues at $t = 0$ by setting a very high value for τ_t^k and run an alternative policy for $t = 1, 2, 3...$ In this sense, this constraint eliminates changes in policy resulting from the initial state of the economy. As a consequence, the economy fluctuates around its optimal policy steady state in every period from $t = 0$.

Definition 2: *Given exogenous paths for shocks* $\left\{g_t, \tau_t^k, \tau_t^\phi, \tau_t^c, a_{x,t}, a_{n,t}, \mu_t^z, \mu_t^\Upsilon, \pi_t^o\right\}$ *and foreign sector variables* $\left\{\Delta M_t^*, \ \ \xi_t, R_t^*, \pi_t^*, y_t^*, \pi_{m,t}^*\right\}$, *previously defined, and a set of initial values for Lagrange multipliers, a Ramsey equilibrium in a "timeless perspective" is a set of processes for prices and allocations maximizing*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma) \log\left(C_t(i) - \zeta C_{t-1}\right) + \gamma \log\left(1 - h_t(i)\right)\right]$$

subject to the equilibrium conditions of the competitive stationary equilibrium and $R_t \geq 1$.

The Ramsey equilibrium for a small open economy must explicitly include an extra non-Ponzi game condition for the evolution of government liabilities. As explained in SGU (2003)[34], the absence of an explicit non-Ponzi game condition for liabilities allows the government to run explosive schemes against the rest of the world, using its own stock of liabilities to absorb all the shocks. Even with the government not trading international bonds, the optimal fiscal policy still could result in non-stationary behavior, as the government sets domestic interest rates low enough to induce households to allocate resources across time using foreign bonds. The presence of portfolio adjustment costs, both in the domestic and in the international financial markets, combined with the risk premium function on foreign interest rates, ensures that the Ramsey problem is stationary. To be more specific, the presence of portfolio adjustment costs in domestic financial markets imposes a discipline for the domestic household when setting its portfolio, forcing the optimal debt policy to have the same restriction.

3 Calibration

The steady state of the model is analytically calculated using a sequential computation of allocations and prices, starting from parameters and "big ratios" listed in table 3.1[18]. These parameters and ratios are drawn from the literature on medium-scale DSGE models, assuming time is set to quarterly frequency. In the baseline calibration under the competitive equilibrium, assume that, in steady state, the growth rate of productivity is set at 2% per year, while the growth in investment-specific technological shock is set at zero. Households spend 20% of their time endowment at work ($h = 0.2$), following the calibration proposed in SGU (2006)[37]. Other values are standard in the literature, like a depreciation rate of capital at 10% per year ($\delta = 0.025$), capital share representing 30% of output and the discount factor β targeting an annualized real interest rate of 4% in the balanced growth path.

In terms of prices, the domestic economy and the rest of the world stabilize the price level in all sectors in the baseline calibration. This assumption implies that there is no welfare loss due to price dispersion across firms in steady state. Assume also that the price elasticity of demand for each sector in the economy, η_i, $i = \{n, x, m, xp\}$, is the same, despite empirical evidence claiming that firms trading in foreign markets, especially exporting firms, present higher markups when compared to firms trading only in the domestic goods market – see De Loecker and Warzynski, 2009[28]. Parameters η_i are calibrated such that the firm markup is set at 25% [19]. The same reasoning is used to set, in the standard calibration, the Calvo parameter assigning the probability of a firm not optimizing its prices. Following the estimation of CEE (2005)[11], α_i, $i = \{n, x, m, xp\}$, is set to 0.6.

[18] The sequence presented in appendix B defines the outcome of the model under the competitive equilibrium.

[19] See Basu and Fernald (1997)[6], and SGU (2007)[38].

Table 3.1: Calibration for Steady State.

Parameter	Description	Value	Source
δ	Depreciation rate	0.025	
θ	Capital share	0.3	
β	Discount factor	0.9952	
ω	Share of tradable goods	0.55	
$\varkappa$	Share of imports in tradable	0.36364	
$\eta_n = \eta_x$	Price elasticity demand domestic goods	5.0	SGU (2007)[38]
η_m	Price elasticity demand imported goods	5.0	
η_{xp}	Price elasticity demand exported goods	5.0	
$\alpha_n = \alpha_x$	Calvo parameter domestic goods	0.6	CEE (2005)[11]
α_m	Calvo parameter imported goods	0.6	
α_{xp}	Calvo parameter exported goods	0.6	
ζ	Habit persistence	0.55	SW (2003)[39]
ϖ	Elast. subst. across labor types	21	CEE (2005)[11]
κ_1	Elast. R_t^f to exogenous risk premium	1.0	
κ_2	Elast. R_t^f to net foreign position	1.0	
η^*	Elast. subst. domestic exports to ROW	1.0	
θ_2/θ_1	Adjustment of capacity utilization	2.02	SGU (2006)[37]
	"Big Ratios", Allocations and Taxes in Competitive Equilibrium		
G/Y	Government spending-output ratio	0.17	
M/Y	Money supply-output ratio	0.1695	
B_g/Y	Net public debt-output ratio	0.42	
τ^k	Taxes on capital	0.395	Carey and Rabesona (2003)[9]
τ^n	Taxes on labor	0.234	Carey and Rabesona (2003)[9]
τ^c	Taxes on consumption	0.064	Carey and Rabesona (2003)[9]
h	Hours worked by households	0.20	SGU (2006)[37]
μ_z	Productivity growth	1.005	
$\pi = \pi^*$	Domestic and foreign inflation	1.0	

The elasticity of substitution across different labor types, ϖ, is usually calibrated in the literature. For emerging economies, García-Cicco (2009)[21] sets a value where the markup of wages over the marginal rate of substitution between labor and leisure equals 100% ($\varpi = 2$). This is the same value calibrated in Smets and Wouters (2003)[39]. The estimation of CEE (2005)[11] is the baseline for the calibration used in Altig, Christiano, Eichenbaun and Lindé (2005)[4], SGU (2005, 2006)[36][37] and Adolfson, Laseén, Lindé and Villani (2007)[2]. The estimation implies a markup of only 5% ($\varpi = 21$). The baseline scenario assumes a small markup, leaving the analysis on the effects of different degrees of wage stickiness for the next sections.[20]

Estimates of the habit persistence parameter in consumption are very unstable, with severe implication for the other parameters of the model. Justiniano and Preston (2009)[24] report, for different datasets and assumptions on the structural model, values in the range of $[0.05, 0.82]$. A similar range is found for Mexican data in García-Cicco (2009)[21]. Given the wide range in the literature, assume a baseline scenario where the habit persistence parameter, ζ, is set at 0.55, following Smets and Wouters (2003)[39] in a model for the Euro Area.

Given the functional form describing the cost of adjusting the capital utilization, and the hypothesis that, in the steady state of the competitive equilibrium, the economy operates at full capacity ($\mu_n = \mu_n = 1$), the parameter set demands a value for the ratio θ_2/θ_1. The value used for the simulations follows SGU (2006)[37], based on Altig, Christiano, Eichenbaun and Lindé (2005)[4]. Despite the fact that this

[20]Notice, however, that there is an equivalence in terms of results in the steady state depending on the combination of values between habit persistence and wage stickiness. This result is better explained in section 4.2.

ratio is irrelevant for the steady state under the competitive equilibrium, the possibility of different levels of capacity utilization under the Ramsey policy forces an assumption regarding this value.

Describing the open economy framework, notice that, due to the absence of empirical estimates for an average share of tradable goods in the GDP, these goods represent around 55% of the consumption basket. The share of imported goods in the aggregate consumption is set at 20%. The price elasticity of demand from the rest of the world for the domestic good, η^*, is set at unity, just like the elasticity of the domestic bonds traded in foreign markets to the world's risk premium and the domestic economy's net foreign asset position (κ_1 and κ_2). These last parameters are used only to determine the level of foreign variables, without any influence over the domestic economy's steady state. Closing the foreign sector, assume that the risk premium is set to zero and the trade balance is in equilibrium.

When describing the government operations, the calibration relies on standard numbers for the United States, with the ratios with respect to GDP of government spending, money supply and net public debt set at 17%, 16.95% and 42%, respectively, following SGU (2005b)[37]. The competitive equilibrium value for taxes in the US between 1990 and 2000 are taken from Carey and Rabesona (2003)[9], following an updated methodology derived from Mendoza, Razin and Tesar (1994)[30]. Taxes on capital (τ^k), labor (τ^h) and consumption (τ^c) are set at 39.5%, 23.4% and 6.4%.

4 Optimal Policy: The Ramsey Steady State

The main objective of this section is to evaluate the Ramsey steady state based on the model calibration described in the previous section. The simulations do not target matching specific moments of data. Instead, the main goal is to understand the trade-offs presented in the planner's problem and the optimal responses given the restrictions imposed by parametric assumptions and by the number of instruments available for the planner. In other words, this section tries to clearly state, for a wide range of structural parameters, the long run priorities of the Ramsey planner when defining the optimal policy. The sequence presented in appendix C defines the outcome of optimal policy under the Ramsey equilibrium.

In order to understand the policy trade-offs faced by the central planner, this section starts describing the choices of taxes and interest rates for the case where the Ramsey planner have all policy instruments available. Next, the analysis proceeds with two special cases: first, the classical Ramsey's (1927)[33] problem of choosing the optimal taxation between capital and labor is approached in a model where the government has no access to consumption taxes; next, the case where the government can not discriminate between production inputs using taxes, in a model where the government sets taxation using only income and consumption taxes. For a limiting case, this section finishes with a description of the optimal policy when the government has access only to an income tax.

4.1 An overview: The case of a complete set of instruments

Consider the case where the government has access to all the fiscal policy instruments described in the model: taxes on consumption (τ^c), labor income (τ^h), capital income (τ^k), profits (τ^ϕ), the control of money supply (m) and debt (b_g). Despite providing several degrees of freedom for the Ramsey planner to optimize the objective function, this exercise provides a benchmark for results presented ahead. The combination of income and consumption taxes has been explored in simple monetary models usually without capital. Examples in a flexible price environment for closed economies can be found in Chari, Christiano and Kehoe (1996)[10] and De Fiore and Teles (2003)[16], while models with sticky prices are presented in Correia, Nicolini and Teles (2008)[14]. The main focus of these papers is establishing conditions under which the Friedman rule can be sustained as the optimal monetary policy framework. In the open economy framework, Adao, Correia and Teles (2009)[1] show, in a model without capital,

Table 4.1: Optimal inflation and taxes.

	α_n	α_m	π	R	$\tau^k(\%)$	$\tau^h(\%)$	$\tau^c(\%)$	Obs.:
1	0.6	0.6	0.00	4.00	-15.35	100	-100	Baseline scenario
2	0	0	-3.85	0.00	-15.35	100	-100	
3	0.6	0	0.00	4.00	-15.35	100	-100	
4	0	0.6	0.00	4.00	-15.35	100	-100	
				Indexation:				
5	0.6	0.6	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	0.00	4.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
7	0	0	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.85	0.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
9	0	0.6	0.00	4.00	-15.35	100	-100	$\kappa_x = \kappa_n = 1$
				Open Economy:				
10	0.6	0.6	0.00	4.00	-15.35	100	-100	$\alpha_{xp} = 0$
11	0.6	0.6	-3.85	0.00	-15.35	100	-100	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	0.00	4.00	-15.35	100	-100	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	0.00	4.00	-13.13	100	-100	$\varkappa = 0.01$
14	0.6	0.6	0.00	4.00	-13.11	100	-100	$\omega = 0.01$
15	0.6	0.6	0.10	4.10	-15.27	100	-100	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	0.00	4.00	-15.35	100	-100	$\pi = 1.0074$ (3%p.y.)
				Profit Taxation:				
17	0.6	0.6	0.00	4.00	-15.35	100	-100	$\tau^\phi = 0$
18	0.6	0.6	0.00	4.00	-15.35	100	-100	$\tau^\phi = 1$

that if each country in a single currency area can tax domestic consumption and labor income, the real exchange rate is completely irrelevant to characterize the optimal allocations and welfare.

Table 4.1 describes the optimal choices of nominal interest rates and taxes on capital, labor and consumption under different assumptions regarding nominal rigidities, indexation, parameters characterizing the open economy and taxation on profits. Results in the first line are based on the standard calibration described in the previous section. The baseline scenario assumes that taxes on profits are set at the same rate as the tax on capital ($\tau^\phi = \tau^k$) and that there is no price indexation.

There are two striking results in the first panel of table 4.1. First, the degree of nominal rigidity does not affect the optimal policy in terms of nominal interest rates, as there are only two possible outcomes regarding monetary policy: the Friedman rule or price stability. The Friedman rule is the optimal monetary policy under price flexibility or under conditions where the output loss due to sticky prices is removed, like some cases of indexation described below. For every other combination of parameters associated to price rigidities, price stability is the optimal policy outcome. Second, also irrespective of the main parameters of the model, taxation on labor is set at 100%, while the tax on consumption is, actually, a subsidy of 100%. As a matter of fact, the two results are connected. De Fiore and Teles (2003)[16] and Correia, Nicolini and Teles (2008)[14] show that, if the conditions for uniform taxation on consumption goods are satisfied[21] and the Friedman rule is the optimal policy, than consumption must be fully subsidized and labor income fully taxed. One of the reasons for this result is that the number of policy instruments is enough to eliminate distortions generated from frictions affecting the steady state of households and firms allocations in the competitive equilibrium. If this is the case, money becomes nonessential, in the sense that any level of money holdings satisfies the households' equilibrium conditions, and the Ramsey planner eliminates the cost of shopping by fully subsidizing consumption.

There are three reasons to believe that results described in De Fiore and Teles (2003)[16] and

[21] These conditions are separability between labor and consumption and homotheticity in consumption in utility function.

Correia, Nicolini and Teles (2008)[14] also applies in this framework for small open economies. First, the log-separable utility function in consumption and labor satisfies the implementation conditions for uniform taxation in consumption. Second, the robustness of results in terms of the taxes in consumption and labor even under different parameterization of the model. Third, as presented in lines 17 and 18, is the fact that the Ramsey policy remained exactly the same as in the baseline calibration under different assumptions for taxation on profits. As discussed in SGU (2006)[37], profits are a lump sum transfer from firms to the households. If allowed to set it optimally, the Ramsey planner chooses to confiscate all income from profits to finance its spending with minimum distortion of the households and firms allocations, setting $\tau^{\phi} = 1$. In the model here, given the large number of policy instruments, the Ramsey planner is indifferent to the inclusion of a lump sum instrument.

The second panel of table 4.1 shows alternative scenarios regarding indexation. Three possible combinations of scenarios can generate the Friedman rule as an outcome for monetary policy. In line 5, with the economy under full indexation, the output loss due to price dispersion across firms is eliminated, generating, in steady state, a similar framework to complete price flexibility. However, as line 6 shows, full indexation is necessary for both production and retail firms, since the policy with indexation present only on production firms is very similar to the baseline scenario in line 1. Lines 7, 8 and 9 show that the Friedman rule might return as a policy outcome with indexation in domestic production firms if prices for imported goods firms are flexible. Thus, the Friedman rule under the current tax system emerges as a solution under a restrict set of conditions: price flexibility; full indexation in prices of domestic firms and retailers; full indexation in prices of domestic firms with price flexibility of imported goods' retailers.

When structural parameters are changed, the tax instrument affected is capital taxation, which is, under the baseline calibration, a subsidy. The intuition for the subsidy in capital was developed in Judd (2002)[23], where the presence of imperfect competition in product markets creates a distortion proportional to the price markup resulting from imperfect competition on the household's intertemporal substitution of consumption. SGU (2006)[37] explore the properties of the subsidy for the case with depreciation and time-varying capital utilization in a closed economy. Lines 13 and 14 in the table show that parameters related to the open economy framework affect the size of the subsidy, as it declines with a reduction for the demand of imported goods. In order to understand the result, note that the steady state of capital taxes and the return on capital are given by:

$$\tau^k = 1 - \left(r_i^k \mu_i - a\left(\mu_i\right)\right)^{-1} \left(\frac{\mu^z \left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}{\beta} - 1 + \delta \right)$$

$$r_i^k = mc_i \theta \left(\frac{k_i}{\mu^z \left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}} h_i} \right)^{\theta - 1}$$

As $\varkappa$ and ω approach zero, the share of total investment based on domestic production increases, as the total demand of imported goods (c_m and i_m) decreases[22]. Without the imported good, the demand for domestic goods increases, increasing the marginal return on capital (r_i^k) and reducing the subsidy necessary to reduce the distortions from the price markup. Figure 1 shows the subsidy on capital, the capital-labor ratio, the rate of capital utilization and the marginal return of capital net of adjustment costs ($r_i^k \mu_i - a\left(\mu_i\right)$) as a function of the share of imported goods in the tradable goods basket $\varkappa$.

Finally, the only difference in allocations and policies in this setup was found when the inflation in steady state for the foreign economy was larger than zero. However, as the result in line 15 shows, the increase in domestic inflation is smaller than the change in foreign inflation: an inflation of 3% per year in the rest of the world results in an increase of less than 0.1% in inflation under the Ramsey policy.

[22] Remember, from the optimal choice of households, that $\varkappa$ is the ratio of imported goods in the basket of tradable goods and ω is the share of tradable goods in the basket of total demand.

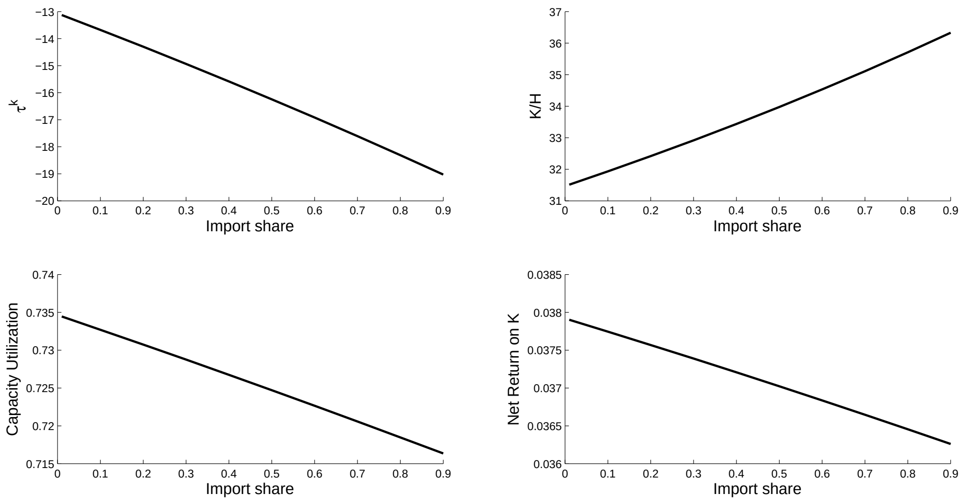

Figure 1: Optimal taxation on capital and openness

Also notice that this small deviation is a consequence only of foreign inflation, as a positive inflation for domestic prices in the competitive equilibrium does not alter results from the baseline scenario.

Given results on taxes and inflation, the next three exercises gain importance, as the constraint in the number of policy instruments will describe the policy choices of the Ramsey planner. The constraint in the number of instruments makes explicit a ranking of preferences in terms of which distortions in the model must be addressed with higher priority. It also provides a robustness test for the hypothesis of price stability as the major goal for monetary policy, as inflation, seen as a tax on money holdings, might become an alternative policy instrument. So far, given a large set of instruments, price rigidities restrict optimal choices to price stability or the Friedman rule. The presence of markups over prices also implies, just like in Judd (2002)[23], that the optimal tax on capital is actually a subsidy.

4.2 The case without consumption taxes

In this exercise, consider the case where government can not tax consumption ($\tau^c = 0, \forall t$). Despite not being able to tax consumption, the Ramsey planner is still capable of perfectly discriminate households' sources of income, as it is allowed to tax labor, capital and profits income at different rates. In order to understand the importance of the tax discrimination between labor and capital income, consider the intratemporal Euler equation of households when consumption can not be taxed:

$$\left(1-\tau^h\right) w\left(1-h\right) = \frac{\gamma}{\left(1-\gamma\right)} mcw \left(1 - \frac{\zeta}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right) c \left[1 + \nu^m \left(\frac{R-1}{R}\right)\right]$$

The expression in the left-hand side is the (after-tax) value of leisure. On the right-hand side, the second term, $mcw = \frac{\varpi}{\varpi-1}$, is the wedge between wages and the marginal disutility of labor, generated by wage stickiness, while the third term, $\left(1 - \frac{\zeta}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right) c$, is the consumption adjusted for habit persistence. The last term, $\left[1 + \nu^m \left(\frac{R-1}{R}\right)\right]$, is a consequence of the money demand by the households. Thus, there are two policy instruments in this equation (nominal interest rates and labor income taxes) set by the Ramsey planner and three distortions: habit persistence, labor wedge and the financial friction due to the cash-in-advance constraint. Note also that distortions generated by habit persistence and labor

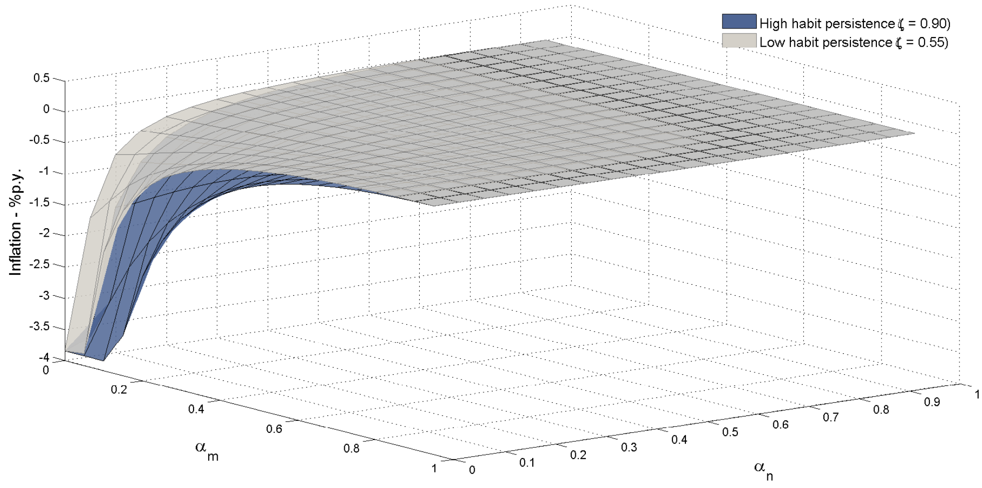

Figure 2: Real rigidities and optimal inflation: no consumption taxes

wedge operate with opposite signs: on the one hand, high elasticity of substitution across labor types, given by ϖ in the equation describing the demand for labor of household i, $h_t(i)$, implies low distortions from wage stickiness, with mcw having a lower bound at one; on the other hand, high values of ζ imply high degree of habit persistence on consumption, with the after-tax value of leisure approaching zero[23]. Thus, for appropriate combinations of ζ and ϖ, the wage gap and the habit persistence offset each other, making the cash-in-advance constraint the only relevant distortion in the labor-leisure allocation.

The perfect discrimination between capital and labor income through taxes still allows the Ramsey planner to formulate fiscal and monetary policies resulting in prices and allocations close to the Friedman rule. In order to understand this result, note that with the taxation on capital designed to eliminate the wedge from markup over prices, labor taxation can be used to minimize the gap between the efficient and the distorted intratemporal labor allocation. As seen above, for special cases of the parameters characterizing the rigidities from wage stickiness and habit persistence, the cost of holding money might emerge as the most relevant distortion affecting the labor allocation. Thus, in the limiting case of this special combination of parameters, the Friedman rule surges as the optimal policy outcome.

The role of real rigidities and the possibility of monetary policy outcomes close to the Friedman rule can be visualized in figure 2. In the figure, the vertical axe shows the Ramsey inflation as a function of the Calvo probability of a firm in the domestic production sector (α_n) and the imported goods sector (α_m) changing prices. The figure has two surfaces, with the surface computed for the baseline scenario overlapping the surface of the scenario assuming a high degree of habit persistence on consumption ($\zeta = 0.90$). Note that inflation is decreasing as prices in both sectors become more flexible, with a policy close to the Friedman rule achieved when prices in both sectors are close to full flexibility. Also note that the economy with high habit persistence presents policies closer to the Friedman rule even in the presence of a larger degree of price rigidity. In the intratemporal Euler equation, the increase in ζ reduces the distortion generated by wage stickiness, allowing the Ramsey planner to reduce the cost of holding money even in the presence of price and wage rigidities.

Table 4.2 shows the optimal combination of taxes and interest rates for different levels of nominal rigidities. Line 1 presents the baseline scenario, as described before, with no indexation, taxation of

[23]If productivity growth is equal to zero, the expression $\left(1 - \frac{\zeta}{\mu^z (\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)$ is bounded between zero and one.

Table 4.2: Optimal inflation and taxes – no consumption taxes.

	α_n	α_m	π	R	$\tau^k(\%)$	$\tau^h(\%)$	Obs.:
1	0.6	0.6	-0.11	3.88	-16.12	30.40	Baseline scenario
2	0	0	-3.85	0.00	-15.05	39.07	
3	0.6	0	-0.15	3.84	-16.12	30.41	
4	0	0.6	-0.44	3.54	-16.12	30.48	
				Indexation:			
5	0.6	0.6	-3.85	0.00	-15.03	39.18	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	-0.44	3.54	-16.12	30.48	$\kappa_x = \kappa_n = 1$
7	0	0	-3.85	0.00	-15.07	38.91	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.85	0.00	-15.07	38.91	$\kappa_x = \kappa_n = 1$
9	0	0.6	-0.44	3.54	-16.12	30.48	$\kappa_x = \kappa_n = 1$
				Open Economy:			
10	0.6	0.6	-0.11	3.88	-16.12	30.40	$\alpha_{xp} = 0$
11	0.6	0.6	-3.85	0.00	-15.05	39.07	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	-0.44	3.54	-16.12	30.48	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	-0.19	3.80	-14.47	29.25	$\varkappa = 0.01$
14	0.6	0.6	-0.19	3.80	-14.46	29.24	$\omega = 0.01$
15	0.6	0.6	-0.01	3.99	-16.05	30.28	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	-0.11	3.88	-16.12	30.40	$\pi = 1.0074$ (3%p.y.)
				Profit Taxation:			
17	0.6	0.6	-0.02	3.98	-15.35	36.49	$\tau^\phi = 0$
18	0.6	0.6	-0.02	3.98	-15.34	36.49	$\tau^\phi = 1$

profits in the same level as the taxation on capital and price stability in the competitive equilibrium both domestically and in the rest of the world. The outcome is characterized by a small deflation, and, as expected, subsidies on capital and high taxes on labor income. The second line confirms the result that, under flexible prices, the Friedman rule is the optimal outcome of the Ramsey problem, as monetary policy tries to eliminate the cost of carrying money. However, as lines 3 and 4 show, the mild deflation returns as a result if there is heterogeneity in price rigidity across sectors. The second panel of table 4.2 confirms the specific cases where the Friedman rule is optimal when changing price indexation.

The third panel of table 4.2 details the optimal policy for alternative parameters that are specific to the small open economy framework. First, in lines 10-12, as in the previous exercise, note that price flexibility for firms in the exported goods' sector do not alter results in terms of indexation and price flexibility described in the first two panels. In lines 13 and 14, reducing the share of tradable goods, ω, to 1% of the total domestic absorption, and the share of imported goods, $\varkappa$, to 1% of the domestic absorption of tradable goods, respectively, the "closed" economy features negative inflation in steady state, in a result similar to SGU (2006)[37][24]. Note that the reduction of the capital tax subsidy as ω and $\varkappa$ converges to zero also shows up in this framework. Lines 15 and 16 also confirm the results of a small effect of foreign inflation in the determination of domestic optimal level of inflation.

Finally, changes in profit taxation indeed affect allocations and the optimal policy in this framework for taxes. Setting a fixed rate for taxes on profits, instead of a choice based on capital taxation, allows the Ramsey planner to approximate the optimal policy to price stability. The adjustment to changes in taxes on profits is reflected in the level of debt supported by households.

In this exercise, the central planner was partially constrained in terms of the number of instruments, when compared to the fiscal policy setup in the previous section. Two results were robust: the Friedman rule as the optimal outcome under flexible prices or indexation structure simulating the flexible price

[24]Results in SGU (2006)[37] of positive inflation are generated by fixed lump sum transfers in steady state from the government to households. When transfers are set to zero, a small deflation is the optimal policy outcome.

economy; and the subsidy to capital compared to the high taxes on labor income, as discussed in Judd (2002)[23]. Despite low, inflation under different levels of price rigidity was not zero, like in the previous section. The next section shows that the consumption tax plays a critical role in this regard.

4.3 The case of income and consumption taxes

Assume, for this exercise, that the government is constrained on taxing all income at the same tax rate – thus, $\tau^k = \tau^h = \tau^\phi = \tau^y, \forall t$. However, differently from the previous section, the government can set the taxation on consumption. Despite both taxes affect the intratemporal decision between labor and consumption of the household, the optimal combination between taxes on income and on consumption is relevant because of the additional effects of each tax in the model. Taxes on consumption affects the transaction technology of the economy, since, for every unit of domestic or foreign good consumed, households must pay the tax. On the other hand, taxation on income distorts the intertemporal decisions of capital accumulation, based on the net expected return of capital in the next period, and the use of the current stock of capital, due to the allowance for changing capacity utilization.

Table 4.3: Optimal inflation and taxes – consumption and income taxes.

	α_n	α_m	π	R	$\tau^y(\%)$	$\tau^c(\%)$	Obs.:
1	0.6	0.6	0.00	4.00	-15.16	79.69	Baseline scenario
2	0.0	0.0	-3.71	0.15	-15.16	82.30	
3	0.6	0.0	0.00	4.00	-15.16	79.69	
4	0.0	0.6	0.00	4.00	-15.16	79.69	
Indexation:							
5	0.6	0.6	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
6	0.6	0.6	0.00	4.00	-15.16	79.69	$\kappa_x = \kappa_n = 1$
7	0	0	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = 1$
8	0.6	0	-3.79	0.06	-15.16	82.36	$\kappa_x = \kappa_n = 1$
9	0	0.6	0.00	4.00	-15.16	79.69	$\kappa_x = \kappa_n = 1$
Open Economy:							
10	0.6	0.6	0.00	4.00	-15.16	79.69	$\alpha_{xp} = 0$
11	0.6	0.6	-3.79	0.06	-15.16	82.36	$\alpha_{xp} = 0, \kappa_x = \kappa_n = \kappa_m = \kappa_{xp} = 1$
12	0.6	0.6	0.00	4.00	-15.16	79.69	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
13	0.6	0.6	0.00	4.00	-13.12	83.84	$\varkappa = 0$
14	0.6	0.6	0.00	4.00	-13.12	83.84	$\omega = 0$
15	0.6	0.6	0.10	4.10	-15.10	79.85	$\pi = \pi^* = 1.0074$ (3%p.y.)
16	0.6	0.6	0.00	4.00	-15.14	79.60	$\pi = 1.0074$ (3%p.y.)

Table 4.3 shows the optimal combination of taxes and interest rates for different levels of nominal rigidities, with the first line again describing the baseline scenario. As in the previous section, the Ramsey planner tries to tax labor more than capital, relying, in this case, in high consumption taxes to pay for a subsidy on capital – implemented, in this case, through negative income taxation. On the optimal choice of interest rates, the model replicates results presented in section 4.1, where inflation departs from zero only under full price flexibility or in the case of indexation where the distortions from price stickiness are eliminated. In these cases, the optimal policy approaches the Friedman rule. In Correia, Nicolini and Teles (2008)[14], the authors prove that allocations in a closed economy without capital are the same under flexible and sticky prices as the outcome of the Ramsey problem. The constraint necessary to

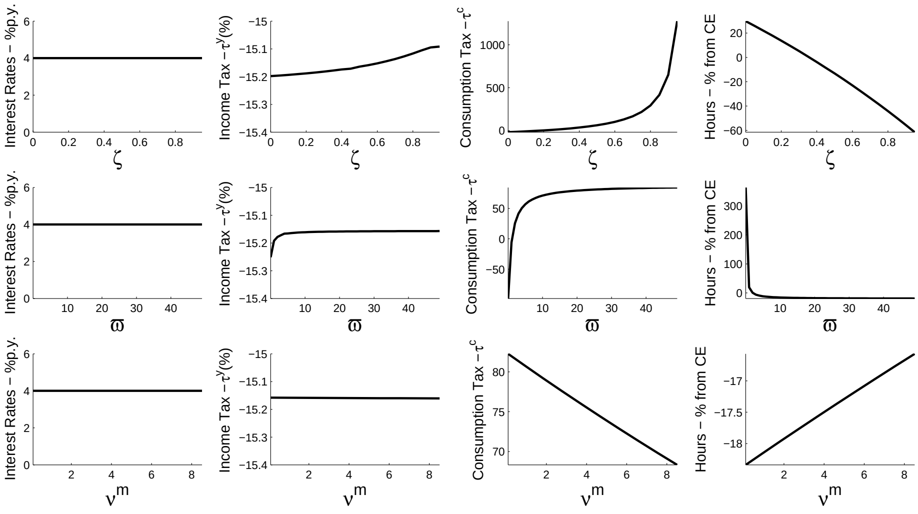

Figure 3: Optimal taxes and inflation: consumption and income taxes

prove the result is the minimum set of instruments (taxes) to operate fiscal policy, where consumption taxes is one of those taxes. Here, not only the model is designed to describe an open economy, but also the presence of capital increases the complexity of the model, both in terms of distortions in allocations and fiscal policy instruments. In this sense, the results of Correia, Nicolini and Teles (2008)[14] seem to be robust to such expansion of the model.

The intuition for the result in Correia, Nicolini and Teles (2008)[14] is that, under the appropriate instrument set, consumption taxes work as a state-contingent asset used by the government to replicate Pareto allocations even under sticky prices. Note that only consumption taxes are capable to operate this way, as the same results can not be implemented without this instrument. Thus, not only the number of instruments is relevant, but also that one of this instruments is the consumption tax. The optimal policy for prices, on the other hand, depends on the degree of price rigidity. Under flexible prices, taxes do not need to account for markups in production, and the optimal policy is the Friedman rule. Under sticky prices, price stability over all periods and states is the optimal outcome, as the elimination of price markups dominates the objective function of the planner, as in section 4.1. In this sense, consumption taxes assume the role of debt in models with complete markets, insuring households against all possible states of the economy.

In order to check the robustness of results, it is critical to evaluate prices and taxes under different parameterizations of the nominal and real rigidities affecting the consumption-leisure trade-offs. The intratemporal Euler equation when the government has access to consumption taxes is given by:

$$\frac{(1-\tau^y)}{(1+\tau^c)} w\,(1-h) = \frac{\gamma}{(1-\gamma)} mcw \left(1 - \frac{\zeta}{\mu^z\,(\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right) c \left[1 + \nu^m \left(\frac{R-1}{R}\right)\right]$$

Again, as described in the previous section, there are three main rigidities affecting the intratemporal choice: habit persistence in consumption (given by parameter ζ), wage stickiness (setting a wedge given by mcw) and the cash-in-advance constraint (restricted by parameter ν^m). Figure 3 describe taxes, nominal interest rates and labor supply choices under all possible parameter combinations for these wedges. Each line of the figure shows how interest rates, taxes and the labor supply responds when changing one of the structural parameters of the model. Notice that the Friedman rule is not a possible outcome, even eliminating most of the domestic rigidities present in the model. Optimal nominal interest rates do not diverge from 4% per year, implying that price stability is the optimal outcome. Income taxation also does not significantly change, staying always close to values found in the previous section as a subsidy for capital. The main adjustment comes from the consumption tax and the labor supply.

This exercise characterized an extension of results in Correia, Nicolini and Teles (2008)[14], showing the role of consumption taxes when setting optimal monetary policy, even when the set of fiscal policy instruments is not as complete as in section 4.1. The presence of consumption taxes is the main reason for obtaining price stability as the optimal policy outcome. As a consequence, not only the number of instruments, but also the composition of the instrument set plays a critical role, as seen by the comparison of results with previous sections. One more time, despite an alternative set of instruments where labor and capital can not be perfectly discriminated, the Ramsey planner tries to subsidize capital, when compared to taxes on labor, irrespective of parameters describing the economy.

4.4 The case of an income tax

In this section, consider the extreme case where the government is restricted to operate fiscal policy setting only a distortionary tax on total income, without access to consumption taxes ($\tau^h = \tau^k = \tau^\phi = \tau^y$, $\tau^c = 0$). The case of a single tax on income is extensively explored in the literature. For closed economies, SGU (2006)[37] detail the steady state and the dynamics in a medium scale model with several real and nominal rigidities. For an open economy, Benigno and De Paoli (2009)[8] explore the dynamics of a simple model for small open economies distorting the household intratemporal condition with a tax on total income. Ambler, Dib and Rebei (2004)[5], despite the focus on optimal monetary policy, also evaluates the dynamics of a small open economy with income taxes.

Table 4.4: Optimal inflation and taxes – income taxes.

	α_n	α_m	π	R	$\tau^y(\%)$	Obs.:
1	0.6	0.6	1.89	5.97	8.63	Baseline scenario
2	0.01	0.01	174.41	185.38	-0.29	
3	0.6	0.01	2.45	6.55	8.60	
4	0.01	0.6	6.11	10.35	8.11	
				Open Economy:		
5	0.6	0.6	1.89	5.97	8.63	$\alpha_{xp} = 0.001$
6	0.6	0.6	6.14	10.38	8.11	$\alpha_{xp} = 0, \kappa_x = \kappa_n = 1$
7	0.6	0.6	2.44	6.54	11.37	$\varkappa = 0.05$
8	0.6	0.6	2.47	6.57	11.49	$\omega = 0.05$
9	0.6	0.6	1.97	6.05	8.69	$\pi = \pi^* = 1.0074$ (3%p.y.)
10	0.6	0.6	1.89	5.97	8.65	$\pi = 1.0074$ (3%p.y.)

When the government has access to only one tax, it becomes impossible to discriminate between production inputs or between the demand of intermediate and final goods. With a single tax instrument and the money demand driven by the cash-in-advance constraint, the income tax is set in such a way that the government budget constraint is balanced. As a consequence of the lack of instruments, inflation is now considered as an additional tax from the Ramsey planner's perspective. This is the main result from the first panel of table 4.4, since, for low levels of price rigidity, inflation is significantly larger than zero. Note that this is the opposite relation between price rigidity and optimal monetary policy as the one presented, for instance, in section 4.2, where inflation was increasing as a function of price rigidity. For another comparison with the previous case, figure 4 shows the same simulation presented in figure 2,

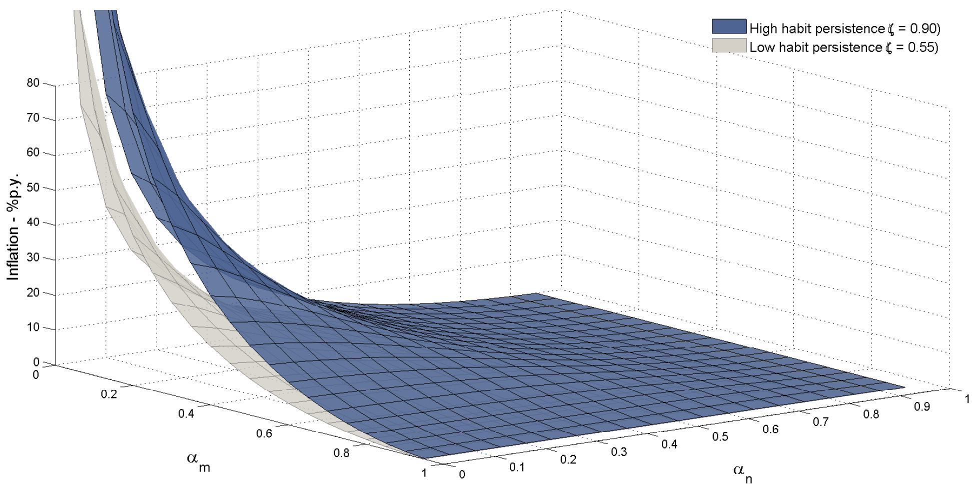

Figure 4: Real rigidities and optimal inflation: single income tax

with two surfaces relating the Calvo parameter for domestic and imported goods' firms and the optimal of inflation for two different degrees of habit persistence in consumption. The figure shows that not only the relation between price rigidity and the optimal level of inflation is negative, but also the relation between inflation and habit persistence is reversed, when compared to figure 2. Thus, the lack of fiscal policy instruments generates a negative relation between habit persistence and inflation.

There are two reasons for inflation to work as an additional instrument for the Ramsey planner. First, from the intratemporal Euler condition of the households setting the consumption-labor choice, a tax on labor income is observationally equivalent to an increase in nominal interest rates generated from the cash-in-advance constraint. Second, according to another main result from the previous sections, there is the objective of the Ramsey planner in discriminating returns from capital to the returns from labor, subsidizing the first at the expense of the later. Given that the nominal interest rate does not directly affect the capital allocation, raising steady state inflation and, as a consequence, the nominal interest rate is equivalent to imposing a large tax on labor. In order to confirm this result, simulations of the model without the cash-in-advance constraint result to price stability as the main outcome of the Ramsey policy with this configuration of taxes.[25]

Results in this exercise confirm, even under this extreme assumption regarding the number of taxes, the main priorities of the Ramsey planner: the discrimination of labor and capital income, favoring the latter; and the elimination of wage and price wedges. Under the restricted set of instruments adopted in this section, the government still implements a relative subsidy on capital, even at the expense of an output loss due to the increase in inflation. From this perspective, the objective of price stability is abandoned in order to obtain a proper discrimination between capital and labor returns to households. The amount of subsidy to capital returns, from this perspective, is limited by the size of the wedges in real wages and prices.

5 Conclusions

This paper explored in details the main trade-offs faced by the Ramsey planner when setting optimal monetary and fiscal policy in a model designed for a small open economy. The main results confirm

[25]Results available upon request.

the propositions in Correia, Nicolini and Teles (2008)[14] about the relevance of consumption tax in setting the optimal policy in terms of inflation. For every scenario not associated with flexible prices, price stability is the optimal outcome of the Ramsey problem. Also, the subsidy for capital is robust to every formulation in the model, confirming that the price markups distortions are the main target of the benevolent government when setting its policy. The small open economy framework does not have a large influence in the steady state, as the relevant distortions are still associated with the intertemporal and the intratemporal Euler equations of the household – structures that are irrelevant to the setup of an open economy.

Results also highlight the importance of underlying assumptions regarding the fiscal policy framework when dealing with the optimal policy problem, as the outcome of the problem generates significant changes in the steady state of allocations and prices. Optimal monetary policy might be characterized from the Friedman rule, where the nominal interest rates must be set at zero, to high levels of inflation, depending not only on the role of nominal and real rigidities of the model, but also on the number of instruments available to the Ramsey planner. It is obvious that the exercises performed here are not exhaustive in terms of the characterization of optimal policy for open economies in general. Several features regarding the dynamics of these economies must still be explored, both confronting the theoretical framework with the data and also computing the optimal policy. This is even more important when dealing with the nonlinearities and alternative propagation mechanisms studied in SOEs and EMEs, especially after the 2008 financial crisis.

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Appendix A Stationary Equilibrium Conditions

In order to transform the model for the stationary form, first note that:

$$z_t^* = z_t \Upsilon_t^{\frac{\theta}{1-\theta}} \Longrightarrow \frac{z_t^*}{z_{t-1}^*} = \frac{z_t}{z_{t-1}} \left(\frac{\Upsilon_t}{\Upsilon_{t-1}} \right)^{\frac{\theta}{1-\theta}} = \mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{\theta}{1-\theta}}$$

Also:

$$\frac{z_t^* \Upsilon_t}{z_{t-1}^* \Upsilon_{t-1}} = \left(\frac{z_t^*}{z_{t-1}^*} \right) \mu_t^\Upsilon = \left(\frac{z_t \Upsilon_t^{\frac{\theta}{1-\theta}}}{z_{t-1} \Upsilon_{t-1}^{\frac{\theta}{1-\theta}}} \right) \mu_t^\Upsilon = \mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{1}{1-\theta}}$$

The stationary equilibrium conditions of the model are:

$$\frac{\left(1-\tau_t^h\right)}{\left(1+\tau_t^c\right)} w_t = \frac{\gamma}{(1-\gamma)} \left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right) \frac{mcw_t \left(1 + \nu^m \left(\frac{\widetilde{R}_t - 1}{\widetilde{R}_t}\right)\right)}{(1-h_t)}$$

$$\left(c_t - \zeta \frac{c_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right)^{-1} (1-\gamma) = \left(1+\tau_t^c\right) \lambda_t \left(1 + \nu^m \frac{R_t - 1}{R_t}\right)$$

$$\lambda_t \left[1 - \psi_1 \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right)\right] = \beta R_t E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1} \left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} \right)} \right)$$

$$\lambda_t \left[1 - \psi_2 \left(\frac{rer_t \; ib_{t+1}}{y_t} - \frac{rer \; ib}{y} \right)\right] = \beta R_t^f E_t \left(\frac{rer_{t+1}}{rer_t} \frac{\lambda_{t+1}}{\pi_{t+1}^* \left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} \right)} \right)$$

$$k_{x,t} = \mu_{x,t} \overline{k}_{x,t}$$

$$k_{n,t} = \mu_{n,t} \overline{k}_{n,t}$$

$$\theta_1 + \theta_2 \left(\mu_{x,t} - 1\right) = r_{x,t}^k$$

$$\theta_1 + \theta_2 \left(\mu_{n,t} - 1\right) = r_{n,t}^k$$

$$R_t = \frac{1}{r_{t,t+1}}$$

$$\widetilde{R}_t = R_t \left(1 - \psi_1 \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right) \right)^{-1}$$

$$\log a_{n,t+1} = \rho_n \log a_{n,t} + \epsilon_{t+1}^n$$

$$\log a_{x,t+1} = \rho_x \log a_{x,t} + \epsilon_{t+1}^x$$

$$\lambda_t q_{x,t} = \beta E_t \left\{ \frac{\left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} \right)^{-1}}{\mu_{t+1}^\Upsilon} \lambda_{t+1} \left[\left(1-\tau_{t+1}^k\right) \left(r_{x,t+1}^k \mu_{x,t+1} - a\left(\mu_{x,t+1}\right) \right) + q_{x,t+1} (1-\delta) \right] \right\}$$

$$\lambda_t q_{n,t} = \beta E_t \left\{ \frac{\left(\mu_{t+1}^z \left(\mu_{t+1}^\Upsilon\right)^{\frac{\theta}{1-\theta}} \right)^{-1}}{\mu_{t+1}^\Upsilon} \lambda_{t+1} \left[\left(1-\tau_{t+1}^k\right) \left(r_{n,t+1}^k \mu_{n,t+1} - a\left(\mu_{n,t+1}\right) \right) + q_{n,t+1} (1-\delta) \right] \right\}$$

$$\overline{k}_{x,t+1} = (1-\delta) \frac{\overline{k}_{x,t}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}} + i_{x,t}^d \left(1 - \frac{\phi_i}{2} \left(\frac{i_{x,t}^d}{i_{x,t-1}^d} \mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right)$$

$$\overline{k}_{n,t+1} = (1-\delta) \frac{\overline{k}_{n,t}}{\mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}}} + i_{n,t}^d \left(1 - \frac{\phi_i}{2} \left(\frac{i_{n,t}^d}{i_{n,t-1}^d} \mu_t^z \left(\mu_t^\Upsilon\right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \right)$$

$$
\begin{aligned}
\lambda_t = \lambda_t q_{x,t} \Bigg[1 - \frac{\phi_i}{2} \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \\
- \phi_i \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}} \right) \left(\frac{i^d_{x,t}}{i^d_{x,t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}} - \mu^V \right) \Bigg] \\
+ \beta E_t \Bigg[\lambda_{t+1} \left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}} \right)^{-1} \frac{q_{x,t+1}}{\mu^\Upsilon_{t+1}} \times \\
\phi_i \left(\frac{i^d_{x,t+1}}{i^d_{x,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}} \right)^2 \left(\frac{i^d_{x,t+1}}{i^d_{x,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}} - \mu^V \right) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\lambda_t = \lambda_t q_{n,t} \Bigg[1 - \frac{\phi_i}{2} \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}} - \mu^V \right)^2 \\
- \phi_i \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}} \right) \left(\frac{i^d_{n,t}}{i^d_{n,t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{1}{1-\theta}} - \mu^V \right) \Bigg] \\
+ \beta E_t \Bigg[\lambda_{t+1} \left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}} \right)^{-1} \frac{q_{n,t+1}}{\mu^\Upsilon_{t+1}} \times \\
\phi_i \left(\frac{i^d_{n,t+1}}{i^d_{n,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}} \right)^2 \left(\frac{i^d_{n,t+1}}{i^d_{n,t}} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{1}{1-\theta}} - \mu^V \right) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{\varpi - 1}{\varpi} + \frac{1}{mcw_t} \right) \varpi h_t \left(1 - \tau^h_t\right) = \\
- \frac{\phi_w}{\pi_t^{\chi_w - 1}} \left(\frac{w_t}{w_{t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{\theta}{1-\theta}} \right) \left(\frac{w_t}{\pi_t^{\chi_w - 1} w_{t-1}} \mu^z_t \left(\mu^\Upsilon_t\right)^{\frac{\theta}{1-\theta}} - \mu^I \right) \\
+ \beta E_t \Bigg[\frac{\lambda_{t+1} \phi_w}{\lambda_t \pi_{t+1}^{\chi_w - 1}} \left(\mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}} \right)^{-1} \times \\
\left(\frac{w_{t+1}}{w_t} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}} \right)^2 \left(\frac{w_{t+1}}{\pi_{t+1}^{\chi_w - 1} w_t} \mu^z_{t+1} \left(\mu^\Upsilon_{t+1}\right)^{\frac{\theta}{1-\theta}} - \mu^I \right) \Bigg]
\end{aligned}
$$

$$
c_t + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t \; ib_t}{y_t} - \frac{rer \; ib}{y} \right)^2 = \left[(1-\omega)^{\frac{1}{\varepsilon}} c_{n,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega^{\frac{1}{\varepsilon}} c_{t,t}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}
$$

$$
c_{t,t} = \left[(1-\varkappa)^{\frac{1}{\varrho}} c_{x,t}^{\frac{\varrho-1}{\varrho}} + \varkappa^{\frac{1}{\varrho}} c_{m,t}^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}
$$

$$
c_{m,t} = \varkappa \left(pm_t \; pt_t \right)^{-\varrho} c_{t,t}
$$

$$
c_{x,t} = (1-\varkappa) \left(px_t \; pt_t \right)^{-\varrho} c_{t,t}
$$

$$
c_{t,t} = \omega \left(pt_t\right)^{-\varepsilon} \left(c_t + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t \; ib_t}{y_t} - \frac{rer \; ib}{y} \right)^2 \right)
$$

$$
c_{n,t} = (1-\omega) \left(pn_t\right)^{-\varepsilon} \left(c_t + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{B}{Y} \right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t \; ib_t}{y_t} - \frac{rer \; ib}{y} \right)^2 \right)
$$

$$i_t = i_{n,t}^d + a(\mu_{n,t}) \frac{\overline{k}_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}} + i_{x,t}^d + a(\mu_{x,t}) \frac{\overline{k}_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}}}$$

$$i_{m,t} = \varkappa (pm_t \, pt_t)^{-\varrho} i_{t,t}$$

$$i_{x,t} = (1-\varkappa)(px_t \, pt_t)^{-\varrho} i_{t,t}$$

$$i_{t,t} = \omega (pt_t)^{-\varepsilon} i_t$$

$$i_{n,t} = (1-\omega)(pn_t)^{-\varepsilon} i_t$$

$$w_t = pn_t mc_{n,t} (1-\theta) a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{n,t}} \right)^\theta$$

$$r_{n,t}^k = pn_t mc_{n,t} \theta a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{n,t}} \right)^{\theta-1}$$

$$w_t = px_t mc_{x,t} (1-\theta) a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{x,t}} \right)^\theta$$

$$r_{x,t}^k = px_t mc_{x,t} \theta a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z (\mu_t^\Upsilon)^{\frac{1}{1-\theta}} h_{x,t}} \right)^{\theta-1}$$

$$x_t^1 = \widetilde{p}_{n,t}^{-1-\eta_n} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t} \right) mc_{n,t} + E_t \alpha_n r_{t,t+1} \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}} \right)^{-1-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{(1+\eta_n)/\eta_n}} \right)^{-\eta_n} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} x_{t+1}^1$$

$$x_t^2 = \widetilde{p}_{n,t}^{-\eta_n} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t} \right) \frac{(\eta_n - 1)}{\eta_n} + E_t \alpha_n r_{t,t+1} \left(\frac{\widetilde{p}_{n,t}}{\widetilde{p}_{n,t+1}} \right)^{-\eta_n} \left(\frac{\pi_{n,t}^{\kappa_n}}{\pi_{n,t+1}^{\eta_n/(\eta_n-1)}} \right)^{1-\eta_n} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} x_{t+1}^2$$

$$z_t^1 = \widetilde{p}_{x,t}^{-1-\eta_x} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right) mc_{x,t} + E_t \alpha_x r_{t,t+1} \left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}} \right)^{-1-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{(1+\eta_x)/\eta_x}} \right)^{-\eta_x} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} z_{t+1}^1$$

$$z_t^2 = \widetilde{p}_{x,t}^{-\eta_x} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t} \right) \frac{(\eta_x - 1)}{\eta_x} + E_t \alpha_x r_{t,t+1} \left(\frac{\widetilde{p}_{x,t}}{\widetilde{p}_{x,t+1}} \right)^{-1-\eta_x} \left(\frac{\pi_{x,t}^{\kappa_x}}{\pi_{x,t+1}^{\eta_x/(\eta_x-1)}} \right)^{1-\eta_x} \mu_{t+1}^z (\mu_{t+1}^\Upsilon)^{\frac{\theta}{1-\theta}} z_{t+1}^2$$

$$y_t^1 = \widetilde{p}_{m,t}^{-1-\eta_m} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right) rer_t \frac{pt_t \; pm_t^*}{pm_t} \left(1 + \frac{R_t^f - 1}{R_t^f} \right)$$

$$+ E_t \alpha_m r_{t,t+1} \left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}} \right)^{-1-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}} \right)^{-\eta_m} \mu_{t+1}^z \left(\mu_{t+1}^\Upsilon \right)^{\frac{\theta}{1-\theta}} y_{t+1}^1$$

$$y_t^2 = \widetilde{p}_{m,t}^{-\eta_m} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t} \right) \frac{(\eta_m - 1)}{\eta_m}$$

$$+ E_t \alpha_m r_{t,t+1} \left(\frac{\widetilde{p}_{m,t}}{\widetilde{p}_{m,t+1}} \right)^{-\eta_m} \left(\frac{\pi_{m,t}^{\kappa_m}}{\pi_{m,t+1}^{(1+\eta_m)/\eta_m}} \right)^{1-\eta_m} \mu_{t+1}^z \left(\mu_{t+1}^\Upsilon \right)^{\frac{\theta}{1-\theta}} y_{t+1}^2$$

$$u_t^1 = \left(\widetilde{p}_{x,t}^* \right)^{-1-\eta_{xp}} x_t \frac{px_t \; pt_t}{rer_t \; pm_t^* \; tot_t}$$

$$+ E_t \alpha_{xp} r_{t,t+1} \left(\frac{\widetilde{p}_{x,t}^*}{\widetilde{p}_{x,t+1}^*} \right)^{-1-\eta_{xp}} \left(\frac{\left(\pi_{x,t}^* \right)^{\kappa_{xp}}}{\left(\pi_{x,t+1}^* \right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{-\eta_{xp}} \mu_{t+1}^z \left(\mu_{t+1}^\Upsilon \right)^{\frac{\theta}{1-\theta}} u_{t+1}^1$$

$$u_t^2 = \left(\widetilde{p}_{x,t}^* \right)^{-\eta_{xp}} x_t \frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$+ E_t \alpha_{xp} r_{t,t+1} \left(\frac{\widetilde{p}_{x,t}^*}{\widetilde{p}_{x,t+1}^*} \right)^{-\eta_{xp}} \left(\frac{\left(\pi_{x,t}^* \right)^{\kappa_{xp}}}{\left(\pi_{x,t+1}^* \right)^{\frac{(1+\eta_{xp})}{\eta_{xp}}}} \right)^{1-\eta_{xp}} \mu_{t+1}^z \left(\mu_{t+1}^\Upsilon \right)^{\frac{\theta}{1-\theta}} u_{t+1}^2$$

$$x_t^1 = x_t^2$$

$$z_t^1 = z_t^2$$

$$y_t^1 = y_t^2$$

$$u_t^1 = u_t^2$$

$$\log \left(\frac{R_{t+1}}{R} \right) = \rho_R \log \left(\frac{R_t}{R} \right) + \alpha_\pi \log \left(\frac{\pi_{t+1}}{\pi_{t+1}^o} \right) + \epsilon_{t+1}^R$$

$$\pi_{t+1}^o = (1 - \rho_{\pi^o}) \pi^o + \rho_{\pi^o} \pi_t^o + \epsilon_{t+1}^{\pi^o}$$

$$t_t = \tau_t^c c_t + \tau_t^h w_t h_t + \tau_t^k \left[\left(r_{n,t}^k \mu_{n,t} - a\left(\mu_{n,t} \right) \right) \overline{k}_{n,t} + \left(r_{x,t}^k \mu_{x,t} - a\left(\mu_{x,t} \right) \right) \overline{k}_{x,t} \right] + \tau_t^\phi \phi_t$$

$$g_t = (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon_t^g$$

$$l_t = m_t + R_t b_{g,t+1}$$

$$l_t = \frac{R_t}{\pi_t} \frac{l_{t-1}}{\mu_t^z \left(\mu_t^\Upsilon \right)^{\frac{\theta}{1-\theta}}} + R_t \left(g_t - t_t \right) - \left(R_t - 1 \right) m_t$$

$$\tau_t^h - \tau^h = \psi_{li} \left(\frac{l_t}{y_t} - \frac{l}{y} \right) + \psi_y \left(y_t - y \right) + \epsilon_t^\tau$$

$$\tau_t^k = (1 - \rho_{\tau k}) \tau^k + \rho_{\tau k} \tau_{t-1}^k + \epsilon_t^{\tau k}$$

$$\tau_t^\phi = \tau^\phi$$

$$\tau_t^c = (1 - \rho_{\tau c}) \tau^c + \rho_{\tau c} \tau_{t-1}^c + \epsilon_t^{\tau c}$$

$$g_{n,t} = (1 - \omega) \left(pn_t \right)^{-\varepsilon} g_t$$

$$g_{t,t} = \omega \left(pt_t\right)^{-\varepsilon} g_t$$

$$R_t^f = R_t^* \left(1+\xi_t\right)^{\kappa_1} \left(\frac{rer_t ib_t}{y_t} / \frac{ib}{y}\right)^{\kappa_2}$$

$$\begin{bmatrix} \frac{\Delta M_t^*}{\Delta M^*} \\ \frac{\xi_t}{\xi^*} \\ \frac{R_t^*}{R^*} \\ \frac{\pi_t^*}{\pi^*} \\ \frac{y_t^*}{y^*} \end{bmatrix} = A \begin{bmatrix} \frac{\Delta M_{t-1}^*}{\Delta M^*} \\ \frac{\xi_{t-1}}{\xi^*} \\ \frac{R_{t-1}^*}{R^*} \\ \frac{\pi_{t-1}^*}{\pi^*} \\ \frac{y_{t-1}^*}{y^*} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{m*} \\ \varepsilon_t^{\xi} \\ \varepsilon_t^{R*} \\ \varepsilon_t^{\pi*} \\ \varepsilon_t^{y*} \end{bmatrix}$$

$$x_t = \left(pm_t^*\ tot_t\right)^{-\eta*} y_t^*$$

$$tot_t = \frac{\pi_{x,t}^*}{\pi_t^*} tot_{t-1}$$

$$\frac{\pi_t^{m*}}{\pi^{m*}} = \upsilon_1 \frac{\pi_{t-1}^{m*}}{\pi^{m*}} + \upsilon_2 \frac{tot_{t-1}}{tot} + \S X_{t-1} + \varepsilon_t^{\pi m}$$

$$a_{n,t} \left(\frac{k_{n,t}}{\mu_t^z \left(\mu_t^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\right)^{\theta} h_{n,t}^{1-\theta} - \chi_n = s_{n,t} \left(c_{n,t} + g_{n,t} + \frac{i_{n,t}}{pn_t}\right)$$

$$s_{n,t} = \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{-\eta_n} + \alpha_n \left(\frac{\pi_{n,t}}{\pi_{n,t-1}^{\kappa_n}}\right)^{\eta_n} s_{n,t-1}$$

$$1 = \left(1-\alpha_n\right) \widetilde{p}_{n,t}^{1-\eta_n} + \alpha_n \left(\frac{\pi_{n,t-1}^{\kappa_n}}{\pi_{n,t}}\right)^{1-\eta_n}$$

$$d_{m,t} - \chi_m = s_{m,t} \left(c_{m,t} + i_{m,t} \frac{pt_t}{pm_t}\right)$$

$$s_{m,t} = \left(1-\alpha_m\right) \widetilde{p}_{m,t}^{-\eta_m} + \alpha_m \left(\frac{\pi_{m,t}}{\pi_{m,t-1}^{\kappa_m}}\right)^{\eta_m} s_{m,t-1}$$

$$1 = \left(1-\alpha_m\right) \widetilde{p}_{m,t}^{1-\eta_m} + \alpha_m \left(\frac{\pi_{m,t-1}^{\kappa_m}}{\pi_{m,t}}\right)^{1-\eta_m}$$

$$a_{x,t} \left(\frac{k_{x,t}}{\mu_t^z \left(\mu_t^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\right)^{\theta} h_{x,t}^{1-\theta} - \chi_x = s_{x,t} \left(c_{x,t} + g_{t,t} + \frac{pt_t}{px_t} i_{x,t} + d_{xp,t}\right)$$

$$s_{x,t} = \left(1-\alpha_x\right) \widetilde{p}_{x,t}^{-\eta_x} + \alpha_x \left(\frac{\pi_{x,t}}{\pi_{x,t-1}^{\kappa_x}}\right)^{\eta_x} s_{x,t-1}$$

$$1 = \left(1-\alpha_x\right) \widetilde{p}_{x,t}^{1-\eta_x} + \alpha_x \left(\frac{\pi_{x,t-1}^{\kappa_x}}{\pi_{x,t}}\right)^{1-\eta_x}$$

$$d_{xp,t} - \chi_{xp} = s_{xp,t} x_t$$

$$s_{xp,t} = \left(1-\alpha_{xp}\right) \left(\widetilde{p}_{x,t}^*\right)^{-\eta_{xp}} + \alpha_{xp} \left(\frac{\pi_{xp,t}^*}{\left(\pi_{xp,t-1}^*\right)^{\kappa_{xp}}}\right)^{\eta_{xp}} s_{xp,t-1}$$

$$1 = \left(1-\alpha_{xp}\right) \widetilde{p}_{xp,t}^{1-\eta_{xp}} + \alpha_{xp} \left(\frac{\left(\pi_{x,t-1}^*\right)^{\kappa_{xp}}}{\pi_{x,t}^*}\right)^{1-\eta_{xp}}$$

$$h_{x,t} + h_{n,t} = h_t$$

$$b_{g,t} + b_{h,t} = 0$$

$$px_t\ pt_t\ x_t - pm_t\ pt_t\ d_{m,t} \left[1 + \left(\frac{R_t^f - 1}{R_t^f}\right)\right] = rer_t R_{t-1}^f \frac{ib_t}{\mu_t^z \left(\mu_t^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} - rer_t \pi_{t+1}^* ib_{t+1}$$

$$\phi_t = y_t - w_t h_t - r_{n,t}^k \mu_{n,t} \overline{k}_{n,t} - r_{x,t}^k \mu_{x,t} \overline{k}_{x,t}$$

$$m_t = \nu^m \left(1+\tau_t^c\right) c_t$$

$$
\begin{aligned}
y_t = c_t + g_t + i_t + px_t\ pt_t\ x_t - pm_t\ pt_t\ d_{m,t} & \left[1 + \left(\frac{R^f_t - 1}{R^f_t}\right)\right] \\
& + \frac{\psi_1}{2} y_t \left(\frac{b_{h,t+1}}{y_t} - \frac{b_h}{y}\right)^2 + \frac{\psi_2}{2} y_t \left(\frac{rer_t\ ib_t}{y_t} - \frac{rer\ ib}{y}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
pt_t &= \frac{\pi_{t,t}}{\pi_t} pt_{t-1} \\
pn_t &= \frac{\pi_{n,t}}{\pi_t} pn_{t-1} \\
px_t &= \frac{\pi_{x,t}}{\pi_{t,t}} px_{t-1} \\
pm_t &= \frac{\pi_{m,t}}{\pi_{t,t}} pm_{t-1} \\
pm^*_t &= \frac{\pi^*_{m,t}}{\pi^*_t} pm^*_{t-1} \\
\frac{\Upsilon_{t+1}}{\Upsilon_t} = \mu^{\Upsilon}_{t+1} &= (1 - \rho_{\Upsilon})\, \mu^{\Upsilon} + \rho_{\Upsilon} \mu^{\Upsilon}_t + \epsilon^{\Upsilon}_{t+1} \\
\frac{z_{t+1}}{z_t} = \mu^z_{t+1} &= (1 - \rho_z)\, \mu^z + \rho_z \mu^z_t + \epsilon^z_{t+1}
\end{aligned}
$$

Appendix B Steady State Conditions: Competitive Equilibrium

This section describes the sequence of equations necessary to compute the steady state of the competitive equilibrium of the assuming that the values related to income taxation are known. The taxation on consumption is obtained using the government budget constraint, assuming that the steady state level of debt-output ratio is known. Given steady state values for taxes $\tau^h, \tau^k, \tau^\phi$, parameter values for $\beta, \theta, \delta, \omega, \varkappa, \mu^z, \mu^\Upsilon, \eta_x, \eta_n, \eta_m, \eta_{xp}, \varpi, \kappa_1, \alpha_x, \alpha_m, \alpha_{xp}, \alpha_n$, and steady state values for $h, R^*/R^f, tb/y, \tau/y, \pi^o, \pi^*, g/y, b/y, m/y, imp/y$ and the share of non-tradable goods in the output, there are 86 variables and 9 parameters to be computed in the steady state of the competitive equilibrium. The set of variables is given by: $\Big\{\pi, \pi_n, \pi_m, \pi_t, \pi_x, \pi_m^*, \pi_x^*, a_n, a_x, q_n, q_x, \mu_x, \mu_n, mcw_t, \Delta M^*, pt, pn, px, pm, pm^*, R, r, \widetilde{R}, R^f, \xi, R^*, \widetilde{p}_x, s_x, \widetilde{p}_m, s_m, \widetilde{p}_x^*, s_{xp}, \widetilde{p}_n, s_n, mc_n, mc_x, rer, r_n^k, r_x^k, h_x, h_n, \overline{k}_x, \overline{k}_n, k_x, k_n, i_x^d, i_n^d, i, g, g_t, g_n, c, c_t, c_n, c_x, c_m, i_n, i_t, i_x, i_m, ib, x, b_g, m, l, d_m, w, d_{xp}, \tau^c, tot, y^*, x^1, x^2, y^1, y^2, z^1, z^2, u^1, u^2, \lambda, y, \phi\Big\}$. The set of parameters is given by: $\{\theta_1, \theta_2, \nu^m, \chi_n, \chi_x, \chi_m, \chi_{xp}, \gamma\}$.

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \pi^o$$

$$\pi_m^* = \pi_x^* = \Delta M^* = \pi^*$$

$$a_n = a_x = 1$$

$$q_n = q_x = 1$$

$$\mu_x = \mu_n = 1$$

$$mcw_t = \frac{\varpi}{\varpi - 1}$$

$$pt = 1 \qquad pn = 1 \qquad px = 1 \qquad pm = 1 \qquad pm^* = 1$$

$$R = \tfrac{\pi}{\beta}\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}} \qquad r = \tfrac{1}{R} \qquad \widetilde{R} = R^f = R \qquad \xi = \left(\tfrac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \qquad R^* = \left(\tfrac{R^*}{R^f}\right) R^f$$

$$\widetilde{p}_x = \left(\tfrac{1-\alpha_x \pi_x^{(\kappa_x - 1)(1-\eta_x)}}{1-\alpha_x}\right)^{\frac{1}{1-\eta_x}} \qquad s_x = \tfrac{(1-\alpha_x)\widetilde{p}_x^{-\eta_x}}{1-\alpha_x \pi_x^{\eta_x(1-\kappa_x)}}$$

$$\widetilde{p}_m = \left(\tfrac{1-\alpha_m \pi_m^{(\kappa_m - 1)(1-\eta_m)}}{1-\alpha_m}\right)^{\frac{1}{1-\eta_m}} \qquad s_m = \tfrac{(1-\alpha_m)\widetilde{p}_m^{-\eta_m}}{1-\alpha_m \pi_m^{\eta_m(1-\kappa_m)}}$$

$$\widetilde{p}_x^* = \left(\tfrac{1-\alpha_{xp}(\pi_x^*)^{(\kappa_{xp} - 1)(1-\eta_{xp})}}{1-\alpha_{xp}}\right)^{\frac{1}{1-\eta_{xp}}} \qquad s_{xp} = \tfrac{(1-\alpha_{xp})(\widetilde{p}_x^*)^{-\eta_{xp}}}{1-\alpha_{xp}(\pi_x^*)^{\eta_{xp}(1-\kappa_{xp})}}$$

$$\widetilde{p}_n = \left(\tfrac{1-\alpha_n \pi_n^{(\kappa_n - 1)(1-\eta_n)}}{1-\alpha_n}\right)^{\frac{1}{1-\eta_n}} \qquad s_n = \tfrac{(1-\alpha_n)\widetilde{p}_n^{-\eta_n}}{1-\alpha_n \pi_n^{\eta_n(1-\kappa_n)}}$$

$$mc_n = \widetilde{p}_n \frac{1 - \alpha_n \; r \; \pi_n^{-\eta_n\left(\kappa_n - \frac{(1+\eta_n)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_n \; r \; \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$mc_x = \widetilde{p}_x \frac{1 - \alpha_x \; r \; \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_x \; r \; \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$rer = \widetilde{p}_m \left(1 + \frac{R^f - 1}{R^f}\right)^{-1} \frac{pm}{pt \; pm^*} \frac{1 - \alpha_m \; r \; \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_m \; r \; \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$r_n^k = \left(1 - \tau^k\right)^{-1} \left[\beta^{-1} \mu^\Upsilon \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right) - 1 + \delta\right]$$

$$r_x^k = \left(1 - \tau^k\right)^{-1} \left[\beta^{-1} \mu^\Upsilon \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}\right) - 1 + \delta\right]$$

$$\frac{k_x}{h_x} = \mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}} \left(\frac{r_x^k}{mc_x \theta}\right)^{\frac{1}{\theta - 1}}$$

$$\tfrac{h_x}{h_n} = \tfrac{mc_x Y_x}{mc_n Y_n} \qquad h = 0.2 \Longrightarrow h_n = h \left(1 + \tfrac{mc_x Y_x}{mc_n Y_n}\right)^{-1} \qquad h_x = \tfrac{Y_x}{Y_n} \tfrac{mc_x}{mc_n} h_n$$

$$k_x = \overline{k}_x = \tfrac{k_x}{h_x} h_x \qquad k_n = \overline{k}_n = h_n \tfrac{k_x}{h_x} \left(\tfrac{mc_x}{mc_n}\right)^{\frac{1}{\theta}}$$

$$i_x^d = \left(1 - \tfrac{(1-\delta)}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) \overline{k}_x \qquad i_n^d = \left(1 - \tfrac{(1-\delta)}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) \overline{k}_n$$

$$i = i_x^d + i_n^d \qquad \theta_1 = r_x^k \qquad \theta_2 = \theta_1 \tfrac{\theta_2}{\theta_1}$$

$$w = mc_n (1 - \theta) \left(\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{\theta-1}} \frac{k_n \mu_n}{h_n}\right)^\theta$$

$$g = \frac{g}{y} \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$g_n = (1 - \omega) g \qquad g_t = \omega g$$

$$c = \left(1 - \frac{tb}{y}\right) \left(wh + r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) - g - i$$

$$c_n = (1 - \omega) c \qquad c_t = \omega c \qquad c_x = (1 - \varkappa) c_t \qquad c_m = \varkappa c_t$$

$$i_n = (1-\omega)\, i \qquad i_t = \omega i \qquad i_x = (1-\varkappa)\, i_t \qquad i_m = \varkappa i_t$$

$$\frac{ib}{y} = \frac{tb}{y}\left[rer\left(\frac{R^f}{\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} - \pi^*\right)\right]^{-1}$$

$$ib = \frac{ib}{y}\left(wh + r_x^k \frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$x = \frac{tb}{y}\left(wh + r_x^k \frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right) + d_m\left(1 + \frac{R^f - 1}{R^f}\right)$$

$$m = \frac{m}{y}\left(wh + r_x^k \frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$b_g = \frac{b_g}{y}\left(wh + r_x^k \frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$\frac{l}{y} = \frac{m}{y} + R\frac{b_g}{y}$$

$$l = \frac{l}{y}\left(wh + r_x^k \frac{k_x}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} + r_n^k \frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)$$

$$d_m = s_m\left(c_m + i_m \tfrac{pt}{pm}\right) \Longrightarrow \chi_m = 0 \qquad d_{xp} = s_{xp} x \Longrightarrow \chi_{xp} = 0$$

$$d_{xp} = s_{xp} x$$

$$\tau^c = \left\{R\left[g - \tau^h wh - \tau^k\left(r_n^k k_n + r_x^k k_x\right)\right] - (R-1)\, m - l\left(1 - \frac{R}{\pi\mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)\right\}(cR)^{-1}$$

$$\nu^m = \frac{m}{(1+\tau^c)\, c}$$

$$tot = \frac{\eta_{xp}}{\left(\widetilde{p}_x^*\right)(\eta_{xp}-1)} \frac{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{(1-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_{xp}\ r\ \left(\pi_x^*\right)^{(-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z\left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\left(\frac{px\ pt}{rer\ pm^*}\right)$$

$$y^* = x\, tot^{\eta*}$$

$$\chi_n = \left(\frac{k_n}{\mu^z\left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_n^{1-\theta} - s_n\left(c_n + g_n + \frac{P}{P_n} i_n\right)$$

$$\chi_x = \left(\frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} \right)^\theta h_x^{1-\theta} - s_x \left(c_x + g_t + \frac{P_t}{P_{x,t}} i_x + d_{xp} \right)$$

$$x^1 = \frac{\widetilde{p}_n^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn} \right) mc_n}{1 - \alpha_n \; r \; \pi_n^{-\eta_n \left(\kappa_n - \frac{(1+\eta_n)}{\eta_n} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\widetilde{p}_n^{-\eta_n} \left(c_n + g_n + \frac{i_n}{pn} \right)}{1 - \alpha_n \; r \; \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$y^1 = \frac{\widetilde{p}_m^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm} \right) rer \frac{pt \; pm^*}{pm} \left(1 + \frac{R^f - 1}{R^f} \right)}{1 - \alpha_m \; r \; \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$y^2 = \frac{\widetilde{p}_m^{-\eta_m} \left(c_m + i_m \frac{pt}{pm} \right)}{1 - \alpha_m \; r \; \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\widetilde{p}_x^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right) mc_x}{1 - \alpha_x \; r \; \pi_x^{-\eta_x \left(\kappa_x - \frac{(1+\eta_x)}{\eta_x} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\widetilde{p}_x^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp} \right)}{1 - \alpha_x \; r \; \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x \; \left(\widetilde{p}_x^*\right)^{-1-\eta_{xp}}}{1 - \alpha_{xp} \; r \; \left(\pi_x^*\right)^{(-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px \; pt}{rer \; pm^* \; tot} \right)$$

$$u^2 = \frac{x \; \left(\widetilde{p}_x^*\right)^{-\eta_{xp}}}{1 - \alpha_{xp} \; r \; \left(\pi_x^*\right)^{(1-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}} \right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$\frac{\gamma}{(1-\gamma)} = \frac{\left(1 - \tau^h\right) w \left(1 - h\right)}{mcw \left(1 + \tau^c\right) \left(1 + \nu^m \left(\frac{R-1}{R}\right)\right) c \left(1 - \frac{\zeta}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right)}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \right)^{-1} \frac{(1-\gamma)}{\left(1 + \tau^c\right) \left(1 + \nu^m \frac{R-1}{R}\right)}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f} \right)$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}$$

Appendix C Ramsey Steady State

The Ramsey solution assumes the same parameters from the competitive equilibrium to compute allocations and prices, including those derived implicitly in the steady state computation. The Ramsey equilibrium is characterized by no inflation dispersion across sectors (thus, relative prices remain set at unity) and the Ramsey planner has the domestic nominal interest rates (R) and taxes (τ^h, τ^k, τ^c) as instruments to maximize the objective function, taking as given the values for domestic government expenditure, g, the taxation over profits, τ^ϕ, and the steady state values for the rest of the world.

$$\tau^h = \tau^h \qquad \tau^k = \tau^k \qquad \tau^c = \tau^c \qquad \tau^\phi = \tau^\phi \qquad R = R$$

$$R^* = R^* \qquad g = g$$

$$\pi^* = \pi_x^* = \pi_m^* = \Delta M^*$$

$$\pi = \pi_n = \pi_m = \pi_t = \pi_x = \frac{\beta R}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$a_n = a_x = pm = px = pt = pn = pm^* = 1$$

$$mcw = \frac{\varpi}{\varpi - 1} \qquad r = \frac{1}{R} \qquad R^f = \frac{\pi^*}{\pi} R \qquad \xi = \left(\frac{R^*}{R^f}\right)^{\frac{1}{\kappa_1}} - 1 \qquad \widetilde{R} = R$$

$$\widetilde{p}_x^* = \left(\frac{1 - \alpha_x (\pi_x^*)^{(\kappa_{xp}-1)(1-\eta_{xp})}}{1-\alpha_{xp}}\right)^{\frac{1}{1-\eta_{xp}}} \qquad s_{xp} = \frac{(1-\alpha_{xp})(\widetilde{p}_x^*)^{-\eta_{xp}}}{1-\alpha_{xp}(\pi_x^*)^{\eta_{xp}(1-\kappa_{xp})}}$$

$$\widetilde{p}_n = \left(\frac{1-\alpha_n \pi_n^{(\kappa_n - 1)(1-\eta_n)}}{1-\alpha_n}\right)^{\frac{1}{1-\eta_n}} \qquad s_n = \frac{(1-\alpha_n)\widetilde{p}_n^{-\eta_n}}{1-\alpha_n \pi_n^{\eta_n(1-\kappa_n)}}$$

$$\widetilde{p}_x = \left(\frac{1-\alpha_x \pi_x^{(\kappa_x - 1)(1-\eta_x)}}{1-\alpha_x}\right)^{\frac{1}{1-\eta_x}} \qquad s_x = \frac{(1-\alpha_x)\widetilde{p}_x^{-\eta_x}}{1-\alpha_x \pi_x^{\eta_x(1-\kappa_x)}}$$

$$\widetilde{p}_m = \left(\frac{1-\alpha_m \pi_m^{(\kappa_m - 1)(1-\eta_m)}}{1-\alpha_m}\right)^{\frac{1}{1-\eta_m}} \qquad s_m = \frac{(1-\alpha_m)\widetilde{p}_m^{-\eta_m}}{1-\alpha_m \pi_m^{\eta_m(1-\kappa_m)}}$$

$$rer = \widetilde{p}_m \left(1 + \frac{R^f - 1}{R^f}\right)^{-1} \frac{pm}{pt\ pm^*} \frac{1 - \alpha_m\ r\ \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_m\ r\ \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$mc_x = \widetilde{p}_x \frac{1 - \alpha_x\ r\ \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_x\ r\ \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$mc_n = \widetilde{p}_n \frac{1 - \alpha_n\ r\ \pi_n^{-\eta_n\left(\kappa_n - \frac{(1+\eta_n)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}{1 - \alpha_n\ r\ \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n - 1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$tot = \frac{\eta_{xp}}{\left(\widetilde{p}_x^*\right)(\eta_{xp}-1)} \frac{1-\alpha_{xp}\ r\ \left(\pi_x^*\right)^{(1-\eta_{xp})\left(\kappa_{xp}-\frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}}{1-\alpha_{xp}\ r\ \left(\pi_x^*\right)^{(-\eta_{xp})\left(\kappa_{xp}-\frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px\ pt}{rer\ pm^*}\right)$$

$$q_x = 1 \qquad q_n = 1$$

$$g_n = (1-\omega)\, g \qquad g_t = \omega g$$

$$\mu_n = \sqrt{\frac{2}{\theta_2}\left[\left(1-\tau^k\right)^{-1}\left(\frac{\mu^{\Upsilon}\left(\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}\right)}{\beta}-1+\delta\right)-\theta_1+\frac{\theta_2}{2}\right]}$$

$$\mu_x = \sqrt{\frac{2}{\theta_2}\left[\left(1-\tau^k\right)^{-1}\left(\frac{\mu^{\Upsilon}\left(\mu^z\left(\mu^{\Upsilon}\right)^{\frac{\theta}{1-\theta}}\right)}{\beta}-1+\delta\right)-\theta_1+\frac{\theta_2}{2}\right]}$$

$$r_x^k = \theta_2\left(\mu_x-1\right)+\theta_1 \qquad r_n^k = \theta_2\left(\mu_n-1\right)+\theta_1$$

$$\tfrac{k_x}{h_x} = \mu_x \mu^z \left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}} \left(\tfrac{r_x^k}{mc_x \theta}\right)^{\frac{1}{\theta-1}} \qquad \tfrac{k_n}{h_n} = \mu_n \mu^z \left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}} \left(\tfrac{r_n^k}{mc_n \theta}\right)^{\frac{1}{\theta-1}}$$

$$w = mc_n\left(1-\theta\right)\left(\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{\theta-1}} \tfrac{k_n \mu_n}{h_n}\right)^{\theta}\left(1+\nu_f^m \tfrac{(R-1)}{R}\right)^{-1} \qquad \tfrac{h_x}{h_n} = \tfrac{mc_x Y_x}{mc_n Y_n}$$

In order to calculate the amount of labor used in domestic production, use the non-tradable sector equilibrium condition:

$$s_n\left(c_n+g_n+i_n\right)+\chi_n = \left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta} h_n$$

$$s_n\left(1-\omega\right)\left(c+g+i_x^d+i_n^d+a\left(\mu_n\right)k_n+a\left(\mu_x\right)k_x\right)+\chi_n = \left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta} h_n$$

$$s_n\left(1-\omega\right)\left(c+g+\left(1-\frac{1-\delta}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}+a\left(\mu_n\right)+a\left(\mu_x\right)\right)\left(\frac{k_n}{h_n}h_n+\frac{k_x}{h_x}h_x\right)\right)+\chi_n = \left(\frac{1}{\mu^z\left(\mu^{\Upsilon}\right)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^{\theta} h_n$$

$$s_n(1-\omega)(c+g)+\chi_n = \left(\frac{1}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n$$
$$-\, s_n(1-\omega)\left(1-\frac{1-\delta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}+a(\mu_n)+a(\mu_x)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n$$

$$s_n(1-\omega)\left(\frac{w(1-\tau^h)R(1-h)}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n =$$
$$\left(\frac{1}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n - s_n(1-\omega)\left(1-\frac{1-\delta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}+a(\mu_n)+a(\mu_x)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n$$

$$s_n(1-\omega)\left(\frac{w(1-\tau^h)R}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n =$$
$$\left(\frac{1}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta h_n +$$
$$s_n(1-\omega)\left[\left(\frac{w(1-\tau^h)R\left(1+\frac{h_x}{h_n}\right)h_n}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)}\right)\right.$$
$$\left.-\left(1-\frac{1-\delta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}+a(\mu_n)+a(\mu_x)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)h_n\right]$$

Set:

$$HN_1 = s_n(1-\omega)\left(\frac{w(1-\tau^h)R}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)}+g\right)+\chi_n$$

$$HN_2 = \left(\frac{1}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}\frac{k_n}{h_n}\right)^\theta$$

$$HN_3 = s_n(1-\omega)\left(\frac{w(1-\tau^h)R\left(1+\frac{h_x}{h_n}\right)}{mcw(1+\tau^c)(R+\nu^m(R-1))\gamma\left(1-\frac{\zeta}{\mu^z(\mu^\Upsilon)^{\frac{\theta}{1-\theta}}}\right)}\right)$$

$$HN_4 = s_n(1-\omega)\left(1-\frac{1-\delta}{\mu^z(\mu^\Upsilon)^{\frac{1}{1-\theta}}}+a(\mu_n)+a(\mu_x)\right)\left(\frac{k_n}{h_n}+\frac{k_x}{h_x}\frac{h_x}{h_n}\right)$$

Then:

$$h_n = \frac{HN_1}{HN_2 + HN_3 - HN_4}$$

$$h = \left(1 + \frac{h_x}{h_n}\right) h_n$$

$$h_x = h - h_n$$

Continuing with the steady state calculation:

$$k_x = \tfrac{k_x}{h_x} h_x \qquad k_n = \tfrac{k_n}{h_n} h_n \qquad \overline{k}_x = k_x/\mu_x \qquad \overline{k}_n = k_n/\mu_n$$

$$i_x^d = \left(1 - \tfrac{1-\delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \tfrac{k_x}{h_x} h_x \qquad i_n^d = \left(1 - \tfrac{1-\delta}{\mu^z (\mu^\Upsilon)^{\frac{1}{1-\theta}}}\right) \tfrac{k_n}{h_n} h_n$$

$$i = i_x^d + i_n^d + +a\left(\mu_n\right) \overline{k}_n + a\left(\mu_x\right) \overline{k}_x$$

$$i_n = (1-\omega)\, i \qquad i_r = \omega i \qquad i_x = (1-\varkappa)\, i_r \qquad i_m = \varkappa i_r$$

$$c_n = \left(\left(\frac{1}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} \frac{k_n}{h_n}\right)^\theta h_n - \chi_n\right) \frac{1}{s_n} - g_n - i_n$$

$$c = \tfrac{c_n}{(1-\omega)} \qquad c_t = \omega c \qquad c_x = (1-\varkappa)\, c_t \qquad c_m = \varkappa c_t$$

$$d_{xp} = \left(\left(\frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}\right)^\theta h_x^{1-\theta} - \chi_x\right) \frac{1}{s_x} - c_x - g_t - i_x$$

$$x = \left(\chi_{xp} - d_{xp}\right)/s_{xp}$$

$$y^* = x\, tot^{\eta *} \qquad d_m = \chi_m - s_m \left(c_m + \tfrac{pt}{pm} i_m\right)$$

$$ib = \frac{x - d_m \left(1 + \frac{\left(R^f - 1\right)}{R^f}\right)}{rer} \left(\frac{R^f}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} - \pi^*\right)^{-1}$$

$$y = c + i + g + x - d_m \left(1 + \frac{R^f - 1}{R^f}\right)$$

$$m = \nu^m \left(1 + \tau^c\right) c$$

$$\phi = y - wh - r_x^k \frac{k_x}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}} - r_n^k \frac{k_n}{\mu^z \left(\mu^\Upsilon\right)^{\frac{1}{1-\theta}}}$$

$$b_g = (l - m) R^{-1}$$

$$\lambda = \left(c - \zeta \frac{c}{\mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)^{-1} \frac{(1-\gamma)}{(1+\tau^c)\left(1+\nu^m \frac{R-1}{R}\right)}$$

$$x^1 = \frac{\widetilde{p}_n^{-1-\eta_n} \left(c_n + g_n + \frac{i_n}{pn}\right) mc_n}{1 - \alpha_n \; r \; \pi_n^{-\eta_n\left(\kappa_n - \frac{(1+\eta_n)}{\eta_n}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$x^2 = \frac{\widetilde{p}_n^{-\eta_n} \left(c_n + g_n + \frac{i_n}{pn}\right)}{1 - \alpha_n \; r \; \pi_n^{(1-\eta_n)\left(\kappa_n - \frac{\eta_n}{(\eta_n-1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_n - 1)}{\eta_n}$$

$$y^1 = \frac{\widetilde{p}_m^{-1-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right) rer \frac{pt \; pm^*}{pm} \left(1 + \frac{R^f - 1}{R^f}\right)}{1 - \alpha_m \; r \; \pi_m^{(-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$y_t^2 = \frac{\widetilde{p}_m^{-\eta_m} \left(c_m + i_m \frac{pt}{pm}\right)}{1 - \alpha_m \; r \; \pi_m^{(1-\eta_m)\left(\kappa_m - \frac{(1+\eta_m)}{\eta_m}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_m - 1)}{\eta_m}$$

$$z^1 = \frac{\widetilde{p}_x^{-1-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right) mc_x}{1 - \alpha_x \; r \; \pi_x^{-\eta_x\left(\kappa_x - \frac{(1+\eta_x)}{\eta_x}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}$$

$$z^2 = \frac{\widetilde{p}_x^{-\eta_x} \left(c_x + g_t + \frac{pt}{px} i_x + d_{xp}\right)}{1 - \alpha_x \; r \; \pi_x^{(1-\eta_x)\left(\kappa_x - \frac{\eta_x}{(\eta_x-1)}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_x - 1)}{\eta_x}$$

$$u^1 = \frac{x \; \left(\widetilde{p}_x^*\right)^{-1-\eta_{xp}}}{1 - \alpha_{xp} \; r \; \left(\pi_x^*\right)^{(-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \left(\frac{px \; pt}{rer \; pm^* \; tot}\right)$$

$$u^2 = \frac{x \; \left(\widetilde{p}_x^*\right)^{-\eta_{xp}}}{1 - \alpha_{xp} \; r \; \left(\pi_x^*\right)^{(1-\eta_{xp})\left(\kappa_{xp} - \frac{(1+\eta_{xp})}{\eta_{xp}}\right)} \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}} \frac{(\eta_{xp} - 1)}{\eta_{xp}}$$

$$l = \left\{ R\left[g - \tau^h wh - \tau^k \left(\left(r_n^k - a(\mu_n)\right) k_n + \left(r_x^k - a(\mu_x)\right) k_x\right) - \tau^c c - \tau^\phi \phi\right] \right.$$
$$\left. - (R-1)\, m \right\} \left(1 - \frac{R}{\pi \mu^z \left(\mu^\Upsilon\right)^{\frac{\theta}{1-\theta}}}\right)^{-1}$$