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Macro Stress Testing of Credit Risk Focused on the Tails

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Macro Stress Testing of Credit Risk Focused on the Tails[*]

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Abstract

This paper investigates macro stress testing of system-wide credit risk with special focus on the tails of the credit risk distributions conditional on bad macroeconomic scenarios. These tails determine the ex-post solvency probabilities derived from the scenarios. This paper estimates the macro-credit risk link by the traditional Wilson (1997) model as well as by an alternative proposed quantile regression (QR) method (Koenker and Xiao, 2002), in which the relative importance of the macro variables can vary along the credit risk distribution, conceptually incorporating uncertainty in default correlations. Stress-testing exercises on the Brazilian household sector at the one-quarter horizon indicate that unemployment rate distress produces the most harmful effect, whereas distressed inflation and distressed interest rate show higher impacts at longer periods. Determining which of the two stress-testing approaches perceives the scenarios more severely depends on the type of comparison employed. The QR approach is revealed more conservative based on a suggested comparison of vertical distances between the tails of the conditional and unconditional credit risk cumulative distributions.

Keywords: macro stress test; credit risk; financial system; quantile regression
JEL Classification: C14; C15; E32; G28

1. Introduction

Macro stress testing of the credit risk of banking book exposures has attracted an increasing interest from market participants in the last years due to three main reasons. First, the Basel II capital accord (Basel Committee on Banking Supervision, 2006), more specifically the internal models approach contained therein, has led commercial banks and supervisors to focus attention on credit risk stress testing exercises as a way to further test the reliability of internal models derived capital measures. Furthermore, commercial banks are likely to use stress testing of their banking exposures for a variety of other purposes, including economic capital management, planning of contingent measures and risk transfer transactions. Second, the increasing role of financial stability as a policy goal of central banks has promoted increasing interest in system-wide exercises of credit risk macro stress testing, often using data aggregated at a higher level than the analysis performed in commercial banks. Cihák (2007) and Foglia (2009) discuss and review general methodologies for implementing stress tests in financial systems. Such tests may help central banks evaluate existing capital adequacy of commercial banks and foresee the consequences of unexpected macro shocks to the stability of the banking system. This paper focuses on this system-wide version of credit risk stress testing. Third and finally, the outbreak of the recent financial turmoil, coupled with lack of more warning signals raised before the crisis, has, if anything, reinforced the two previous points and stimulated further research on the theme and its limitations (e.g. Alfaro and Drehman, 2009). As a response to the crisis, regulation has once more devoted new attention to the area of stress-testing (e.g. Basel Committee on Banking Supervision, 2009).

While the relation between the macroeconomy and the volume of credit is relatively well studied, e.g. the credit channels of monetary policy (Bernanke and Gertler, 1995), the economic theory is still incipient to explain the link between macro variables and credit risk. In the absence of well-established theoretical models to explain the macro-credit risk link, the majority of macro stress-testing approaches currently in use by central banks or supervisory agencies are non-structural. One reduced-form approach widely employed in the applied literature is Wilson (1997a, 1997b). This paper discusses and estimates Wilson model and uses it to perform macro stress testing of the credit risk of the Brazilian household sector. Wilson model,

originally conceived basically as a credit risk portfolio model, has the interesting built-in feature that macroeconomic surprises affect the macro-credit risk relationship, which is maybe a reason for its popularity in stress-testing applications. On the other hand, Sorge and Virolainen (2006) perform a critical review of stress testing methodologies, including approaches of Wilson type, pointing to the potential instability of reduced-form parameter estimates, due to the break-down in historical patterns derived from extreme shocks (e.g. in default correlations). That motivates us to also consider an alternative model for the macro-credit risk link that incorporates stochastic macro sensitivity of the credit risk indicator. In estimating and applying these models on a system level, this paper situates itself amid a recent but fast growing literature on credit risk stress-testing applications by central banks and supervisory agencies (e.g. Kalirai and Scheicher (2002), Boss (2003), Lehman (2006), van den End *et al.* (2006), Jiménez and Mencía (2007), Breuer *et al.* (2009), Simons and Rolwes (2009)).

The basic idea behind macro stress testing of credit risk is to relate a macro scenario or shock to measures of financial loss or risk indicators. In a probabilistic stress-testing exercise, an entire distribution conditional on the macro scenario is generated. This paper proposes examining the right tail of the conditional distribution to gauge the impact of the macro scenario. In light of the recent financial turmoil, many authors have reminded that, once crises emerge, we should expect the unexpected (Alfaro and Drehmann, 2009). Besides the warning embedded, such notice, if literally interpreted, could also suggest the focus of the stress-testing analysis be shifted from the usual conditional mean to the conditional tail. The conditional right tail represents what worse may still happen to the credit risk outcome in light of an assumed harmful macro scenario and is the relevant part of the distribution for determining the *ex-post* solvency probability of the system derived from the scenario. This conditional tail focus is not without precedents in the applied financial risk literature. Adrian and Brunnermeier (2008), for example, propose a risk measure, named *covar*, that is similar to the risk concept used in this paper but conceived there for the analysis of systemic risk. The conditional tail focus in stress testing exercises could be further motivated based on the presumption that credit risk conditional right tails are more robust to deviations from historical patterns than the remaining parts of the conditional distribution, precisely because the former are likely to have been generated under those deviations.

Consistently with the conditional tail focus, our alternative approach of stress-testing, is based on a quantile regression model (QR) for the macro-credit risk link (Koenker and Xiao, 2002). Contrary to Wilson model, that, although generating the whole credit risk distribution is still a model focused on the conditional mean, the quantile regression explicitly models the tail of the conditional distribution. Further, the QR approach has a feature that is appealing to stress-testing: the relative importance of the macro variables changes according to the quantile of credit risk distribution, therefore partly addressing the criticism of Sorge and Virolainen (2006) about the potential instability of reduced-form parameters. In particular, macros that have small relative effect on the median of the distribution may gain relevance in explaining a high quantile of the credit risk indicator. Also, as a semi-parametric model, QR relaxes a normality central assumption used in Wilson model. Non-normality is more realistic for stress-testing exercises[1].

The preference for using a particular stress-testing approach is usually subjectively grounded. Because of the typically long credit risk horizon (months or years), few data is usually available to conduct statistically meaningful back-testing exercises of credit risk models. That point is further aggravated in the stress-testing context, because the macro data that would be more relevant for back-testing here corresponds to macro crises, which are rare by definition. Consequently, the policy maker is likely to work with a set of stress-testing approaches, rather than a single tool, and analyze their results at its own judgment. A discussion on how to analyze these several results and how to compare different approaches should then be of great interest but is largely absent in the applied literature[2]. Thus, besides the proposed QR macro-credit risk link, this paper contributes to the stress-testing literature by proposing a novel measure for the analysis and comparison of stress-testing results, namely, the vertical distance between the tails of the unconditional and the distressed conditional credit risk cumulative distributions. It has the interesting interpretation as the change in solvency probability due to the occurrence of the distressed scenario and is investigated in the paper through the use of pp-plots.

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[1] Though not explored in this article, QR approach also lends itself to reverse engineering techniques, in which the set of macro variables that produce specific quantiles of the conditional distribution can be easily recovered.

[2] Here, comparison of approaches refers to comparison of their outputs in the form of credit risk distributions.

A final note about macro scenarios is due: stress-testing exercises usually take them as given. The construction of severe, yet plausible, and economically consistent macro scenarios is an important preliminary step for stress-testing tasks, but is not within the main interest of this paper.[3] We use a rather simple econometric model to build our macro scenarios, that are plugged in both Wilson and QR models in the same fashion. In many central banks, the macro scenario is built by means of a macroeconomic model (e.g. DSGE model) that projects distressed macro variables given more fundamental shocks (e.g. oil price shocks)[4]. Therefore, given the simple nature of our scenarios, the results of this paper should not be interpreted in an absolute way but rather illustrative of the stress-testing approaches employed.

This paper is organized as follows. Section 2.1 discusses the properties and the estimation of Wilson and QR models for the macro-credit risk link. Section 2.2 explains the use of the estimated link models to perform stress-testing and discusses how to analyze stress-testing results. Section 3 introduces the macro and credit data used in the estimations. Section 4.1 estimates and interprets different specifications of the macro-credit risk link for the Brazilian household sector, while section 4.2 presents and analyzes the results of the stress testing exercises. Section 5 concludes.

2. Methodology

2.1 Models for the macro-credit risk link

Structural and reduced-form credit risk models have usually equations and parameters conceived at the level of borrowers or economic sectors. Many studies of central banks or supervisory agencies that investigate credit risk on a system level apply these models on system-aggregated data, implicitly making the interpretation of the parameters more system-like. Van den End et al. (2006) and Boss (2003) follow, for example, Wilson (1997a) in specifying their model for the macro-credit risk link and

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[3] On this issue, see for example Breuer *et. al.* (2009).
[4] Most of these macroeconomic models have, however, no representation of a financial sector or of financial risk, so that the transmission of macro distress to the system credit risk remains carried out in the same fashion as here, through a reduced-form macro-credit risk link (e.g. Wilson).

their stress-testing approach. This paper also explores extensively Wilson model, whose formulation fits within the following more general structure[5]:

$$
\begin{cases}
CRI_t = \dfrac{1}{(1+\exp(-y_t))} \quad (or\ y_t = \ln\left(\dfrac{CRI_t}{1-CRI_t}\right)) & (1) \\[3mm]
y_t = \alpha_0 + \sum_{i=1}^{p}\alpha_i y_{t-i} + \gamma_0 \mathbf{z_t} + \sum_{j=1}^{q}\gamma_j \mathbf{z_{t-j}} + u_t & (2) \\[3mm]
\mathbf{z_t} = \boldsymbol{\mu} + \sum_{k=1}^{m}\mathbf{A_k}\mathbf{z_{t-k}} + \boldsymbol{\varepsilon_t},\quad m>q & (3) \\[3mm]
(u_t,\boldsymbol{\varepsilon_t}) \sim N(\mathbf{0},\boldsymbol{\Sigma}),\ \boldsymbol{\Sigma} = \begin{pmatrix}\Sigma_{u,u} & \Sigma_{u,\varepsilon} \\ \Sigma_{\varepsilon,u} & \Sigma_{\varepsilon,\varepsilon}\end{pmatrix} & (4)
\end{cases}
$$

where:

y_t is the logit transformation of an observable credit risk indicator $CRI_t \in [0,1]$,,

$\mathbf{z_t}$ is a vector of macroeconomic variables at time t,

u_t is a normal error, homoscedastic and independent with regard to past information

and ε_t is a normal white noise.

Equation (2) is the macro-credit risk link, that relates the (transformed) real-valued credit risk indicador y_t contemporaneously to the macro vector $\mathbf{z_t}$, besides potentially also to lags of the macros and auto-regressive terms. The macro variables follow a kind of vector autoregressive system (VAR), according to equation (3)[6]. The model is complemented by the assumption (4), in which the residual terms of the link and of the VAR are jointly normal and correlated through the parameter $\Sigma_{u,\varepsilon}$, so that macro-economic surprises affect the macro-credit-risk relation, adding a stress-testing flavour to the model.

Notice that if $\Sigma_{u,\varepsilon} \neq \mathbf{0}$, macro variables affect CRI both through their total levels but also through the economic surprises these levels represent. From $\boldsymbol{\Sigma}$, the partial regression coefficient between u_t and surprise $(\varepsilon_t)_k$, the k-th coordinate of ε_t, is recovered as $(\Sigma_{u,\varepsilon}\Sigma^{-1}_{\varepsilon,\varepsilon})_k$. Ideally, we would like to have this coefficient with the same

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[5] More precisely, equation (2) generalizes Wilson (1997a) in incorporating lags of the macro and credit variables. It belongs to the class of autoregressive distributed lag (ADL) models (see Davidson and MacKinnon, 1993, p.682).

[6] Because of the presence of $\Sigma_{u,\varepsilon}$, system (3) is not exactly a VAR, but we take this freedom of terminology along the text, for the sake of brevity.

sign of the corresponding $(\gamma_0)_k$. In other words, if one expects a macro variable to be (partially) negatively contemporaneously related to credit risk, so it may also be expected to happen with the unexpected part of it. Further, provided macro surprises that have the same relation with CRI tend to move on the same direction in $\Sigma_{\varepsilon,\varepsilon}$, it will be even reasonable to expect the (total) covariance $\Sigma_{u,\varepsilon}$ to have the same sign as γ_0, respectively for each macro variable. This argument will help us choose an intuitive specification for (2) at section 4.1.

Estimation of system (1)-(4) is rarely discussed in the stress-testing literature. If one believes or wants to leave open the possibility of $\Sigma_{u,\varepsilon} \neq 0$, then $\mathbf{z_t}$ should be treated as endogenous in the link equation (2), because $cov(u_t,\mathbf{z_t})= \Sigma_{u,\varepsilon}$. This makes maximum likelihood estimation (MLE) more complicated than usual. Because of that and because MLE is dependent on the normality (or other known distribution) in (4), an assumption we will relax in our alternative QR model, we prefer to estimate system (1)-(4) by instrumental variables, without making distributional assumptions. Instrument candidates for $\mathbf{z_t}$ come naturally from the m-q lags that appear in (3) but not in (2), provided m>q, a condition we have assumed before and that is also economically reasonable to expect[7]. The general method of moments (GMM) is applied in section 4.1, using those instruments for $\mathbf{z_t}$ at (2).

The main criticism of Wilson model is generally addressed towards the specification of u_t. Frequently, data does not confirm the residual of the link (2) so well behaved, as specified in (4). It is common to find evidence or to make arguments for heteroskedasticity, as well as non-normality on u_t[8] In this regard, notice also that, although estimation can be conducted in a robust fashion to these elements, homoskedasticity and normality are still needed for the simulation of Wilson model for stress-testing purposes, in which the distribution of u_t is required[9]. Further, if the focus of the stress-testing analysis is on the tail conditional on $\mathbf{z_t}$, it is reasonable to argue that the parameter $\Sigma_{u,\varepsilon}$ coupled with the normality assumption for u_t may be a too restricted modeling strategy for the uncertainty of macro-credit risk link. In this respect, notice in

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[7] Technically, these instruments are assured to be valid only if system (3) includes the p auto-regressive terms of y_t. We expect, however, their coefficients to be zero, since preliminary estimations indicated no feedback effect from credit risk indicators to the macro variables.

[8] Our estimations also point to this direction. A potential source of heteroskedasticity is the disregard of variability in the coefficients of the macros (e.g., Lima and Néri, 2006), a feature present in our alternative model.

[9] The simulation procedure is explained in the next section.

(5) that the relative importance of any two contemporaneous macros, say k and l, measured by the ratio of their marginal effects on the CRI τ-quantile, does not depend on τ. This represents a limitation for conditional tail focused stress-testing.

$$\frac{\partial Q(CRI_t, \tau \mid \Im_t)}{\partial z_t^k} \bigg/ \frac{\partial Q(CRI_t, \tau \mid \Im_t)}{\partial z_t^l} = \frac{\partial E(y_t \mid \Im_t)}{\partial z_t^k} \bigg/ \frac{\partial E(y_t \mid \Im_t)}{\partial z_t^l} \qquad (5)^{[10]}$$

$$= \frac{\gamma_0^k + \partial E(u_t \mid \Im_t)/\partial z_t^k}{\gamma_0^l + \partial E(u_t \mid \Im_t)/\partial z_t^l} = \frac{\gamma_0^k + (\boldsymbol{\Sigma}_{u,\boldsymbol{\varepsilon}} \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}}^{-1})_k}{\gamma_0^l + (\boldsymbol{\Sigma}_{u,\boldsymbol{\varepsilon}} \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}}^{-1})_l},$$

where $\Im_t = \{(y_i, \mathbf{z}_i), i \leq t\}$ and $Q(.,\tau)$ denotes the τ-quantile function.

The previous arguments motivate us to consider an alternative model to Wilson. The conditional tail focus and (5) suggest the proposal of a model in which the weights γ_0 of the contemporaneous macros are dependent on the level τ of the conditional quantile[11]. There is also an economic reasoning for introducing variability in γ_0. If link equation (2) were defined on the level of a single borrower, so that y_t measured its creditworthiness, then contemporaneous macro sensitivity γ_0 would make part of the channel through which default correlations across different borrowers arise (e.g. Wilson, 1997a,b, Kouluoglu and Hickman, 1998). Credit portfolios models normally assume these correlations depend only on the economic sectors in which the borrowers belong. When estimating equation (2) on system aggregated data over time, many different borrowers with different macro sensitivities (and consequently different pair-wise correlations) are being implicitly considered in the estimation of γ_0. For the household sector in particular, it is reasonable to expect that macro variables affect the credit risk stance of different families in complete diverse ways: the household sector is far from a unified economic sector. Furthermore, the pool of families with credit granted in the economy varies along time.[12] Therefore, a modeled uncertainty in γ_0

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[10] The first equality follows from the fact that, conditionally on $\Im_t$, CRI_t is a transformation of the normally distributed y_t, so the CRI_t conditional quantile can be expressed as a function of the conditional mean and conditional variance of y_t, besides the level τ. Further, due to the assumed normality, the conditional variance of y_t is a function of Σ and, therefore, does not depend on z_t.

[11] Smaller modifications of Wilson model are also possible: in a slightly different setup, Simons and Rolwes (2009) adopt a fat-tail t-distribution for u_t.

[12] Particularly in Brazil, that has experienced a sharp recent development of the household credit market.

estimated from system-aggregated data incorporates the notion of varying default correlations (or macro sensitivities) across borrowers and along time.

Our alternative model for the macro-credit risk link replaces (2) with a quantile regression (QR) model (Koenker and Xiao, 2002). Below, $Q(y_t, \tau | \mathfrak{I}_t)$ denotes the τ-quantile of the conditional distribution of y_t.

$$Q(y_t, \tau | \mathfrak{I}_t) = \alpha_0(\tau) + \sum_{i=1}^{p} \alpha_i(\tau) y_{t-i} + \gamma_0(\tau)\, \mathbf{z_t} + \sum_{j=1}^{m} \gamma_j(\tau) \mathbf{z_{t-j}}; \qquad \tau \in [0,1] \qquad (6)\,{}^{[13]}$$

Equation (6) explicitly models the tail of the conditional distribution, in contrast to Wilson's conditional mean formulation (2). The coefficients of the macros and auto-regressive terms vary according to the quantile of the conditional distribution of y (or, equivalently, CRI). In particular, the relative importance of contemporaneous macros k and l changes according to τ, through the derivatives of the function $\gamma_0(.)$ (see equation 7 below[14]). This function is not restricted in any important sense *a priori* and is estimated from the data based on quite mild assumptions. It is possible that in extreme quantiles, macro k, say, have a greater relative importance than in the median relation (i.e., $\tau=0.5$).

$$\frac{\partial Q(CRI_t, \tau | \mathfrak{I}_t)}{\partial z_t^k} \bigg/ \frac{\partial Q(CRI_t, \tau | \mathfrak{I}_t)}{\partial z_t^l} = \frac{\partial Q(y_t, \tau | \mathfrak{I}_t)}{\partial z_t^k} \bigg/ \frac{\partial Q(y_t, \tau | \mathfrak{I}_t)}{\partial z_t^l} = \frac{d\gamma_0^k(\tau)/d\tau}{d\gamma_0^l(\tau)/d\tau} \qquad (7)$$

In the setup of equation (6), it is assumed that the stochastic process of y_t can be represented by:

$$y_t = \alpha_0(U_t) + \sum_{i=1}^{p} \alpha_i(U_t) y_{t-i} + \gamma_0(U_t)\mathbf{z_t} + \sum_{j=1}^{m} \gamma_j(U_t)\mathbf{z_{t-j}}, \qquad (8)$$

in which U_t follows a standard uniform distribution, without loss of generality, and is independent with regard to past information.

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[13] Regarding the inference of the QR model, according to the classical paper of Koenker and Basset (1978), quantile estimation of a model $y_t = \mathbf{x_t'}\theta(U_t)$ involves the solution to the problem $\hat{\theta} = \arg\min_{\{\theta \in \mathfrak{R}^s\}} \sum_{t=1}^{T} \rho_\tau(y_t - \mathbf{x_t'}\theta(\tau))$ where $\rho_\tau(.)$ is defined by the authors as $\rho_\tau(u) = \begin{cases} \tau u & , u \geq 0 \\ (\tau-1)u & , u < 0 \end{cases}$ and $s=\dim(\theta)$. The estimator $\hat{\theta}(\tau)$ does not have an explicit form (as the OLS estimator), but the problem above can be solved by linear programming techniques.

[14] In (7) we have used the property that $Q(CRI_t, \tau | \mathfrak{I}_t) = g(Q(y_t, \tau | \mathfrak{I}_t))$, where g is the monotonic logit function.

At (8) the coefficients are functions of a random variable and therefore random variables themselves. The fact that a uniform distribution is used represents just a conventional way to represent the distribution of the coefficients. Important however, in contrast to the normality of u_t in Wilson model, is that these distributions are not required to be normal nor symmetric in any sense[15]. Equation (8) is the convenient form of equation (6) for simulation purposes.

Notice that in equations (6) and (8) all the m lags of the VAR system (3) were included. That is purposefully done to make it feasible to assume independency between U_t and ε_t, in contrast to the dependency structure of Wilson contained in (4). Recognizably, that entangles the effects of the macro variable levels with those of the macro surprises, making interpretation of the coefficients less clear and not directly comparable to Wilson model. However, estimating a dependency structure between U_t and ε_t (and the macro coefficients), while maintaining only the q lags in (6), is not straightforward, due to the resulting endogeneity on the contemporaneous macros. Nevertheless, we briefly suggest a way forward at the conclusion section. For the remainder of the text, however, the QR stress-testing approach consists of equations (1), (3), (8), besides the independency assumption just mentioned. The results of the QR approach will be compared with those of Wilson approach (system (1)-(4)).

2.2 Stress testing

Suppose we are at the end of period (T-1), with the information set available $\Im_{T-1}=\{(\mathbf{z_t},CRI_t),\ t\leq T-1\}$. Suppose we now assume the realization of a (typically harmful) macro scenario within forecasting horizon $H \subset \{t \geq T\}$, to be represented by S and upon which we want to perform stress testing. The new information available becomes $\Im^T = \Im_{T-1}$ U S. The set S could be an historical macro occurrence, that is assumed to happen again, or a hypothetical macro realization possibly conceived with the help of a separate macroeconomic model.

If S fixes completely the macro realizations over horizon H of the stress exercise, so that S= $\{\mathbf{z_T}\}$ when H=T, then equations (2) and (6) with $\mathbf{z_T}$ plugged-in

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[15] Also crutial for the QR model is that these distributions are functions of the same random variable U_t.

provide the transformed CRI outcome of the next period given S (and $\mathfrak{I}_{T-1}$), according to Wilson and QR models, respectively. Equation (6) of QR model already provides, up to a transformation, the right tail (τ-quantiles for high τ's) of the CRI_T conditional distribution, which is the focus of this paper as discussed in the introduction. The coefficient vector $\gamma_0(\tau)$ measures the linear relation between the assumed distressed $\mathbf{z_T}$ values and the τ-quantile of y_T or the non-linear relation between $\mathbf{z_T}$ and the τ-quantile of CRI_T. On the other hand, Wilson model needs a simulation step to arrive at the same τ-quantile. First, $\mathbf{z_T}$ is decomposed into an expected part, which results from the VAR system (3), and the remaining unexpected surprise $\boldsymbol{\varepsilon_T}$. Next, the error term u_T is simulated conditionally on $\boldsymbol{\varepsilon_T}$, according to their joint normal distribution (4). The right tail of simulated CRI_T distribution is the estimate of main interest.

The previous discussion assumed all explanatory variables in the forecasting horizon H of the stress exercise have been fixed. Lehman and Manz (2006), for example, conduct such a procedure. It is very common, however, that S is only partly specified in comparison to H (e.g. Boss, 2003, Jiménez and Mencía, 2007, van den End et al., 2006), either because the exercise fixes just part of the cross-section scenario (e.g. univariate shock: H=T but S={$(\mathbf{z_T})_1$} and/or because the time horizon H is longer than the time period encompassing the assumed macro realizations (e.g. S={$\mathbf{z_T}$} but H>T). In those cases, both Wilson and QR approaches need a simulation step to arrive at the conditional quantiles. First, we use the VAR system (3) to conditionally simulate the remaining unspecified macro variables (e.g. conditionally on $(\mathbf{z_T})_1$ in the case of the univariate shock). Then, given the complete macro realizations over horizon H, we simulate the error term u_T of Wilson model, similarly to the explained in the previous paragraph. Accordingly, the quantile model needs that we simulate U_H from a standard uniform distribution to produce a simulated y_H according to (8)[16]. In both approaches, extreme realizations of CRI_H will derive from bad outcomes of the unspecified macros together with bad transmissions of the macro-credit risk link (high u_H or U_H, depending on the model). As before, our main interest lies on the right tails of the simulated conditional CRI_H distributions.

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[16] In the actual implementation, each simulated U_H is rounded to the closest element of a grid of quantile levels for which estimation of equation (8) is produced. We use a grid of 9 levels.

Generally, the results of a probabilistic stress testing exercise are the distressed CRI_H distribution, conditional on an assumed harmful macro scenario S, and an unconditional CRI_H distribution, based solely on $\Im_{T-1}$. Analyzing the results of the stress test involves comparing these two distributions. Apart from the comparison of their means ($E(CRI|\Im_S)$ *versus* $E(CRI|\Im_{T-1})$), the conditional tail focus of this study naturally suggests comparing their tails (that represent what worse may still happen beyond S). We analyze the horizontal distance between the tails ($Q(CRI,\tau|\Im_S)$ *versus* $Q(CRI,\tau|\Im_{T-1})$, for varying τ), which is somewhat standard in the applied literature[17] (e.g. Foglia (2009), Sorge and Virolainen (2006)), as well as we propose to investigate the vertical distance between the tails ($Prob(CRI < Q(CRI,\tau|\Im_{T-1})|\Im_S)$ *versus* τ, for varying τ). The vertical distance has an interesting interpretation in the context of financial risk. One can view $Q(CRI,\tau|\Im_{T-1})$ as the amount of the banking system own funds set *ex-ante* as a buffer to credit losses at a confidence level τ[18]. Then $Prob(CRI < Q(CRI,\tau|\Im_{T-1})|\Im_S)$ represents the *ex-post* solvency probability of the system with that buffer, given the occurrence of S. We use pp-plots to examine the change in solvency probabilities due to S along τ. To the best of our knowledge, the vertical distance analysis is a novelty in the applied stress testing literature.

Comparing two stress-testing approaches, as those we have in this paper, is less straightforward than analyzing the results of a single methodology, because there are now four distributions involved, the unconditional and the conditional of each approach. Ideally, one would like that the two methodologies differ only in their distressed conditional distributions, not in their evaluations of the unconditional case. However, that is not normally the case, because the different structures that each model proposes to better reflect the consequences of macro shocks also have a bearing on their generated unconditional distributions. Therefore, we explore two methods of comparing stress-testing approaches, an absolute and a relative one, both consistent with our tail focus. The first compares the approaches through the horizontal distance between their conditional tails. It answers how the approaches differ in measuring the absolute impact of the macro scenario S on the CRI scale. The second method compares the vertical

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[17] The horizontal distance is usually called the quantile treatment effect in econometrics literature.
[18] More specifically, as a buffer to cover both expected and unexpected losses (maintaining a system solvency probability equal to τ).

distances between the conditional and unconditional tails across the approaches. It answers how the approaches differ in the variation of solvency probabilities resulted from the scenario and provides a probabilistic relative comparison. While the former makes use only of the conditional distributions, the latter uses information on all the four distributions. There is no reason to expect *a priori* that both methods will lead to the same conclusion.

3. Data

Macro-credit risk link models (2) and (6) are estimated for the credit granted by the Brazilian private financial system to the household sector (excluding mortgages), based on quarterly data from 1995:I to 2009:III (59 observations)[19]. Earlier periods are not considered given the very different high-inflation regime that prevailed before the adoption of the "Real" stabilization plan in 1994, in comparison to the more stable macroeconomic environment thereafter. We use non-performing loans (NPL), measured by the proportion, at the end of each quarter, of loans past due between 2 and 6 months to the total of loans either performing or past-due up to 6 months, as our credit risk indicator (CRI) dependent variable. As noted by Jiménez and Mencía (2007), the introduction of an upper past-due threshold (in our case, 6 months) reduces the persistency of the NPL series. In our case, past-due loans included in one quarter will generally be considered at most in only one more quarter. This NPL indicator is the longest feasible credit risk indicator series available to this study. The alternative natural candidate, the LLP (loan loss provision) series, suffers a structural break in 1999, when new provisioning rules were implemented by the Central Bank of Brazil.

However, our NPL indicator also presents limitations. As a stock measure, it captures the performance of loans granted at different points in time instead of following a fixed set of loans over time. Therefore is likely to be affected by changes in credit granted volumes and loan maturities. As those changes are not necessarily credit

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[19]The household sector is chosen for the presentation of results of this paper on the basis of more robust and higher quality results found for the estimated macro credit risk link, compared to other types of credit (e.g. commercial credit). Mortgages being excluded is not very restrictive since they have comprised historically just a small fraction (less than 10%) of the credit granted by the private financial system to the household sector. The remaining loans have an average maturity ranging from one semester to one year and a half along the sample (according to estimates based on other sources).

risk related, the NPL could become a distorted measure of credit risk. That limitation is equally present in many central banks' and supervisory agencies' studies that make use of NPL accounting data. Figure 1 shows the evolution of our NPL measure over the time span of the data. Since mid 2004, NPL oscillates around 8% and it was slightly decreasing since mid 2006, until the arrival of the impact of the recent international financial crisis in Brazil, in 2008:IV, at the end of our sample. Besides this last crisis, the NPL series also displays a period of stress manifestation at the end of 1995 (consistent with a sharp fall in real GDP growth at figure 3, related to the effect of the Mexican crisis in Brazil – see Minella (2001) and Sobrinho (2010)) and, also, reveals a pronounced plateau during the period 2002-2004 (consistent, among other factors, with an increase of unemployment in 2001-2002 and a peak of inflation in 2002, consequences of several shocks that hit the Brazilian economy and of the lack of confidence in the macroeconomic policy of the new elected Brazilian government in 2002 - see Minella *et al.* (2003)).

Regarding the macro dataset, the following variables are initially considered in preliminary estimations of the macro-credit risk link: real GDP growth rate, industrial production growth rate, unemployment rate, CPI inflation rate (IPCA), short and long-term interest rates (Selic and TJLP, respectively), Embi+Br, real exchange rate, and net public debt-GDP ratio[20]. Notwithstanding the relative difficulty or ease in thinking about the theoretical impact of each variable on credit risk (e.g. inflation, as discussed in the next paragraph), the mentioned set represent variables that the monetary authority is generally used to monitor or work with and, therefore, is useful as a starting point for the purposes of deriving practical stress testing approaches. We consider also the quarterly change of credit volume, in order to capture the influence of the recent development of the Brazilian household credit market[21]. In order to avoid collinearity among regressors, the variables industrial production and long-term interest rate are

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[20]The data sources are the Central Bank of Brazil (BCB), Institute of Applied Economic Research (IPEA), Brazilian Institute of Geography and Statistics (IBGE) and Bloomberg. For the unemployment rate, we adopt the (seasonally adjusted) time series UN2 of Da Silva Filho (2008), due to the 2002 methodology change in the computation of the unemployment rate series of IBGE (PME). Real GDP growth rate refers to the growth from the corresponding quarter of the previous year to the current quarter. Inflation and interest rates are expressed in quarterly rates. Excepting for NPL, all series are log-transformed, i.e., ln (1+rate/100).

[21] Here credit volume stands for the outstanding credit amount granted by the Brazilian private financial system to the household sector that is less than 6 months past-due at the end of the quarter.

discarded.[22] In addition, the effects of real exchange rate, Embi+Br and net public debt-GDP are found to be little robust across different specifications of the models and therefore also dropped out from the analysis. This way, the variables considered in the final specifications of equations (2) and (6) are, therefore, real GDP growth, unemployment, inflation, short-term interest and credit volume[23].

Table 1 shows the correlation matrix among these final variables and displays a maximum absolute correlation of 0.57. Consistent with section 2.1, the VAR system (3) includes this same remaining group of explanatory variables. Unit root tests generally indicate the variables are stationary at table 2. The time series of these variables are plotted in figures 2 and 3. In recent years, it is possible to observe a movement towards higher real output growth, lower unemployment and lower interest rates. In fact, monetary policy has consistently gained credibility since the adoption of the inflation targeting regime in 1999, allowing for an improvement in macroeconomic fundamentals and lower levels of the policy interest rate (e.g. Bevilaqua et al., 2008).

It is possible to conjecture about the correct signs of the contemporaneous macro effects on the conditional mean (2) or median ((6) with $\tau=0.5$) of the (logit transformed) NPL. It is more reasonable to interpret the contemporaneous effects from the point of view of credit demand: the expression of borrowers' credit risk after their credit was granted. Macro variables directly related to the business cycle fluctuation, such as the real GDP growth, and to some extent credit volume growth, are expected to be negatively correlated to the NPL time series[24]. A positive sign is expected for the unemployment rate, even more so in the household sector, in which unemployment has a direct bearing. The higher the short-term interest rate, the higher may be the cost of *post*-set rate loans and, therefore, a positive contemporaneous response on NPL might be expected, but this type of loan has become less common with the consolidation of the Brazilian macroeconomic stability. Higher inflation during the life of the loans may decrease the real cost of the debt but it is also likely to diminish net resources available for payment in the case of families with little savings, that make most of the Brazilian household sector. Kalirai and Scheicher (2002) present a comprehensive discussion

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[22] For instance, we have, in the sample, correlation (real GDP growth; industrial production) = 0.8543, and correlation (short-term interest rate; long-term interest rate) =0.8984.
[23] The estimations are discussed in the next section.
[24] Notice that our expectation for a negative sign for credit volume growth comes from a contemporaneous specification, whereas in other studies (e.g. Borio and Drehman, 2009) the variable is used as an early warning indicator and there a positive sign is generally found.

about the expected signs of the average macro effects on credit risk. On the other hand, the signs and magnitudes of the macro effects on the extreme quantiles of credit risk (e.g. τ=0.9 in (6)) are not clear *a priori*[25].

4. Results

4.1 Estimation of Wilson and QR approaches

In order to guide us in the macro lag specification of the link models (choice of q in (2) and (6)), the estimation of the VAR system (3) is first investigated in search for the appropriate number of lags m. Standard information criteria (Schwarz and Hannan-Quinn) indicate m=1 as the best specification, according to table 3.[26] Table 4 presents the estimated coefficients and t-statistics for the VAR(1) model. Notice the quite good fit (adjusted R-squared) for the unemployment and short-term interest rate equations, despite using a single-lagged specification. Each lagged variable is mostly significant in explaining the corresponding contemporaneous variable, except for credit volume. On the other hand, credit volume is affected strongly and significantly by lagged interest rate. Lagged interest rate also shows significant effects in all the equations.

The choice of m=1 suggests we initially consider the specification of Wilson equation (2) with only contemporaneous macros (q=0) and equation (6) with also one-lags on the explanatory variables[27]. In both cases, an autoregressive term is included in order to capture some persistence of the credit risk indicator (p=1). Table 5 reports, for the sake of completeness, the estimates for the only-contemporaneous and one-lagged specifications of both Wilson and QR link models (2) and (6). Only-contemporaneous specification of model (2) is estimated together with the other five equations contained in system (3) (with m=1) by GMM, to take into account the potential effect of the macroeconomic surprises ε_t on realized credit risk and the resulting so produced

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[25] Due to the limited sample size, we do not produce estimates for rather extreme quantiles, such as at the 99.9 % level, common in the credit risk literature. Our estimates go as far as to the 90% quantile.

[26] All the inverse roots of the AR characteristic polynomial are inside the unit circle, supporting the stability of the VAR (1) model.

[27] Other specifications based on a higher number of lags were also investigated. They are, however, more difficult to interpret due the several inversions of coefficient signs along the lags.

endeogeneity on the macros at (2)[28]. One-lagged specification of model (2) is estimated by OLS, since m=q=1 makes it reasonable to assume $\Sigma_{u,\varepsilon} = \mathbf{0}$ and, therefore, endogeneity no longer present. The specifications of QR model (6) are estimated according to the footnote 14 of section 2.1 and reported for the median ($\tau=0.5$) and the right tail ($\tau=0.9$). However, these estimates do not encompass any treatment of endogeneity, which is a relevant issue for the only-contemporaneous specification. Newey and West (1987) HAC standard errors are constructed for the conditional mean models (2), whereas a bootstrap procedure is adopted to estimate the covariance matrix of the QR parameter estimators. In addition, we report, for the quantile regressions, the Koenker and Machado (1999) goodness-of-fit measure (pseudo adjusted R-squared).[29]

In all specifications of table 5, the contemporaneous variables, when significant, have the expected signs, according to the discussion of section 3 (although caution should be place when interpreting reduced-form coefficients). GDP and credit volume possess negative coefficients whereas unemployment, short-term interest and inflation show positive signs[30]. Notice, however, that contemporaneous inflation and interest rate are only significant at the GMM estimate and at the right tail of QR model in the only-contemporaneous specification[31]. Finally, note the (logit transformed) NPL indeed seems to be a persistent series, since the autoregressive coefficient situates around 0.6.

It is interesting to note that the conditional median is explained by a similar number of significant variables as the conditional mean, in both only-contemporaneous and one-lagged specifications. However, the magnitude of the coefficients can be very different: see for example the coefficients of contemporaneous credit volume and lagged inflation. The magnitude distance between the conditional mean and median coefficients in the one-lagged specification is a clear sign of asymmetry of the conditional (logit transformed) NPL distribution, which is a property naturally captured

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[28] The one-lagged macros and credit volume are used as instruments for the contemporaneous explanatory variables of equation (2). The explanatory lagged variables of system (3) are treated as exogenous. Recall that, if endogeneity is present, then OLS estimates will be biased and inconsistent. We perform, later in this section, a Hausman test to formally test for endogeneity.

[29] According to Koenker and Machado (1999), unlike the standard R-squared, which measures the relative success of a conditional mean function in terms of residual variance, the pseudo R-squared measures the relative success of the corresponding quantile regression model at a specific quantile in terms of an appropriately weighted sum of absolute residuals. In this way, it constitutes a local measure of goodness-of-fit for a particular quantile, rather than a global measure of goodness-of-fit over the entire conditional distribution.

[30] For inflation, the expectation was less clear a priori. Its effect is further discussed ahead in the section.

[31] In the latter, only interest rate.

in the quantile regression setup. On the other hand, the conditional mean and median coefficients are not supposed to be directly comparable at the only contemporaneous specification. While the former, estimated by instrumental variables, measure the marginal effects holding the unobservable macroeconomic surprises ε_t fixed, the latter, estimated without treatment of endogeneity, adds up the marginal effects of the macro levels and the unobservable macro surprises altogether. The sizable distance between the coefficients of unemployment at the only-contemporaneous specification indicate, for example, that an unexpected increase in unemployment is likely to have a bearing on credit risk, apart from the effect of the already considered contemporaneous unemployment level. This hypothesis is investigated in a slightly modified specification of (2) presented below.

Table 5 also allows the comparison between the median and the extreme right tail estimates of QR link model (6). The variables which are significant in explaining both the median and the extreme tail show in those cases the same signs. However, certain variables are only significant in explaining either the median or the tail (e.g. interest rate is significant only for the tail in the only contemporaneous specification, whereas lagged GDP and unemployment are significant only for the median in the one-lagged specification). Similarly, from the point of view of the coefficients' magnitudes, the macro-credit risk link also behaves distinctly, according to the median or the tail. Notice, for example, in the lagged specification, the sizable distance between the coefficients of unemployment (contemporaneous and lagged), of lagged inflation and of the auto-regressive term, as well as between the coefficients of GDP in the only contemporaneous specification. In particular, lagged inflation increases its relative importance very strongly at the tail (which partially explains the respective decrease of the auto-regressive coefficient). In fact, Wald tests performed to check for slope inequality across quantile estimates (following Koenker and Bassett, 1982a,b) show the coefficients of the explanatory variables (except the intercept) are different from $\tau=0.5$ to $\tau=0.9$. The last line of table 5 presents the results of such tests performed on a jointly basis and represent evidence that our QR model does not encompass unrealistic over-parameterization[32].

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[32] On the other hand, the issue of the importance of the distinction between Wilson and QR models of the macro-credit risk link for stress-testing exercises is examined in the next section.

The estimation of a quantile regression at the tail based on a small sample of 59 observations at table 5 recommends care when interpreting the significance of its coefficients. In fact, table 6 reveals the actual constraints posed by the small sample size: slope equality tests cannot distinguish estimates at quantiles higher than 70% from the 90% quantile regression coefficients. In other words, a large part of the right middle of the distribution is undistinguishable from the point of view of the right high tail. On the other hand, the fact that significance was found in comparing the median to the 90% quantile (the result of table 5) mean that there are sufficient degrees of freedom to capture variation between largely separated quantiles. Moreover, differentiation of the quantile estimates is, in fact, a matter of which part of the credit risk distribution is taken as reference. Table 6, for example, reveals that if focus is given to differentiation from the median, almost the whole right part of the distribution passes the task.

We would like to choose specifications of both Wilson and QR link models to be used for stress-testing. For Wilson model in particular, we prefer a specification that disentangles the effects of the macro levels and the surprises they represent, in line with Wilson approach (1)-(4). However, the only-contemporaneous specification of Wilson model (2) at table 5 is not very suited for story telling because unemployment and credit volume are not significant. Not only one would expect their effects to be significant (particularly unemployment) but also they are significant in all other specifications of table 5.

The inspection of the one-lagged specifications may shed some light on the proposal of an alternative specification for Wilson model, by noting that the only lagged variables significant at the OLS estimates (as well as at the QR estimates) are unemployment and inflation. Since the autocorrelations of unemployment and inflation are empirically much higher than the crossed-lagged correlations with other variables (see table 4), the effects of macroeconomic surprises in unemployment and inflation might have been captured at the one-lagged estimates. For unemployment this is indeed likely to be the case. Unemployment is significant both contemporaneously, with positive sign, and laggedly, with negative sign. These signs indicate, in the framework of Wilson approach (1)-(4), that both total contemporaneous unemployment as well as unexpected unemployment increases credit risk, controlling for the other variables[33].

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[33] When expressing y_t as a function of ε_t (instead of u_t) in (2) specified with q=0 and m=1, the coefficient of the k-th coordinate (e.g. unemployment) of $\mathbf{z_{t-1}}$ is easily seen to be $-\Sigma_{u,\varepsilon}\Sigma_{\varepsilon,\varepsilon}^{-1}\mathbf{A}_{(:,k)} \cong -(\Sigma_{u,\varepsilon}\Sigma_{\varepsilon,\varepsilon}^{-1})_k a_{(k,k)}$.

Besides, the high magnitude of unemployment coefficients at the one-lagged specification, when compared to the only-contemporaneous one, adds further support to the effect of unemployment surprise on NPL. On the other hand, contemporaneous inflation is not significant and the coefficient of lagged inflation is positive, making it unreasonable to conclude that only unexpected inflation is related to credit risk (and in a negative way). Therefore, we prefer to interpret the effect of inflation as simply a lagged positive effect without encompassing any surprise effect. Being only lagged, it may be related not only to aspects of demand (e.g. the decrease of families' net resources available to repayment due to inflation costs) but also to aspects of bank credit supply, although this last transmission channel is less clear (Boyd and Champ, 2003).[34]

The discussion of the previous paragraph suggests an alternative to the only-contemporaneous specification of Wilson model, with lagged inflation replacing contemporaneous inflation, but still allowing for the potential effect of macro surprises, particularly in unemployment. The first column of table 7 contains the GMM estimate of such specification. Notice the significance, with the expected signs, of all variables, but interest rate, and a high adjusted R-squared, even when compared to the specifications with larger number of variables at table 5. Table 9 reports the results of a Hausman (1978) test for such specification which shows that unemployment rate (and only this macro) seems to be endogenous and indicating the positive effect of an unemployment surprise on NPL, in the context of Wilson formulation (1)-(4). An unemployment surprise can be interpreted as the proportion of families that unexpectedly become unemployed and, therefore, the result is highly intuitive[35]. Table 10 shows the estimate of $Corr(u_t, \varepsilon_t)$, built from the GMM estimate of Σ. Consistently with the unemployment endogeneity and in line with the discussion carried out at section 2.1, notice logit(NPL) shows a large positive (total) correlation with unemployment and close to zero (total) correlations with the other macros.

Consistently with the selected alternative specification of (2) (to be used in the stress-testing exercises), the corresponding alternative specification of QR model (6)

As $a_{(k,k)} > 0$, the partial regression coefficient of $(\varepsilon_t)_k$, $(\Sigma_{u,\varepsilon}\Sigma_{\varepsilon,\varepsilon}^{-1})_k$, has the opposite sign of the coefficient of $(z_{t-1})_k$.

[34] Somewhat consistently with the bank credit supply channel interpretation, Jiménez et al. (2009) find that a higher inflation at origination of the loan implies more risk.

[35] One could argue that instead of unemployment level and unemployment surprise, the effect of unemployment would be best captured through the first difference of the unemployment rate. We have tried such specification but the resulting estimate was less intuitive and found other macros insignificant.

contains lagged inflation in place of contemporaneous inflation and also includes lagged unemployment, to encompass the effect of unemployment surprise[36]. The last two columns of table 7 contain the median and extreme right tail estimates of such specification. Notice again the significance, with the expected signs, of all variables, but interest rate in the median and GDP and lagged unemployment in the tail, and the large values for the pseudo adjusted R-squared, even when compared to the specifications at table 5. For the sake of comparison, table 7 also shows the corresponding OLS estimate of this specification (fourth column), as well as the QR estimates corresponding to the specification selected for Wilson model (second and third columns). Similar comments to those of table 5, about the comparison between OLS and QR($\tau=0.5$) estimates and between QR($\tau=0.5$) and QR($\tau=0.9$) estimates are also valid regarding table 7. Slope equality tests (table 8) show that these better fit QR specifications of table 7 have most of the right middle of the distribution distinguishable from the 90% quantile estimates. For these specifications, the small sample size places more constrains on the differentiation from the median. Finally, it is worth noting that, although interest rate is insignificant in the estimates of table 7, it will play an important role in the stress-testing exercises, particularly for horizons H>1, given its strong correlation with future credit volume growth, as previously noted.

Besides the estimation of (6) at certain τ's, the quantile regression model also leads to more refined results. For instance, figure 4 shows the NPL distributions conditional on the macroeconomic observations of 2008:III and 2009:III. They are estimated non-parametrically from the QR previously selected specification, by means of an Epanechnikov kernel over a discrete grid of quantiles (see Schulze (2004, p.36) for further details). The distributions are leptokurtic and platykurtic, respectively, and both negatively skewed. They reveal a type of uncertainty not previously accounted for in the literature on macro stress testing of credit risk and related to the variability of default correlations, as discussed at section 2.1. When using the QR approach to stress-testing, that uncertainty is coupled with the uncertainty ε pertained to the VAR system, shaping the resulting NPL distributions.

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[36] Seeking a parsimonious specification, the other lagged macros are not included. In this regard, recall that lagged unemployment is, by far, the variable the highest correlated with contemporaneous unemployment (see table 4).

4.2 Stress-testing exercises

Because until 2009:III Brazilian real GDP growth had not yet recovered to 2008 pre-crisis figures[37] (figure 3), we take 2008:III, the last quarter before the GDP shock that impacted Brazilian economy, as our base quarter. We consider macro scenarios for the following quarter (T=2008:IV) and examine their consequences from that quarter (H=1) until one year ahead (H=4). Scenarios are built by adding (or subtracting) 1, 2 or 3 standards deviations (σ) to the forecast generated by the macro VAR system (3) for 2008:IV.[38] The macro values z_T fixed by the scenarios are reported at table 11. We consider both univariate scenarios, where only one macro suffers the shock (while the others are simulated conditionally on the former), and multivariate scenarios, in which all macros (but not credit volume) are supposed to jointly suffer bad realizations at 2008:IV. To obtain the unconditional and the distressed conditional NPL distributions, we resort to the simulation methodology explained in section 2.2. Next, we start examining the results from Wilson approach.

Figure 5 shows, for all the scenarios considered and H=1, the distressed conditional NPL densities estimated by Wilson approach. Distress on inflation and on interest rate produce tiny (right) shifts of the unconditional distribution. That is consistent with the absence of contemporaneous inflation and the non-significant, close to zero coefficient of interest rate at table 7. On the other hand, distressed GDP, distressed unemployment and multivariate distress are considerably more harmful. It is worth remarking that the expectations of the 2-σ and 3-σ distressed GDP distributions (respectively 6.96% and 7.08%) are very close to the true NPL observed in 2008:IV (7.00%). That is to be expected, since the true GDP shock suffered by the Brazilian economy in 2008:IV situates between the 2-σ and 3-σ univariate GDP scenarios of this paper. From the densities, it is possible to construct the NPL cumulative distribution functions (CDFs), whose right tails are presented in figure 6 for scenarios based on 2 standard deviations (σ=2). Here we confirm, from a tail point of view, that

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[37] Based on a GDP growth rate in relation to the corresponding quarter of the previous year.

[38] According to the notation of section 2.1, σ is the appropriate element of diag($\Sigma_{\varepsilon,\varepsilon}$). The direction of the shocks (i.e. σ or -σ) is determined in the sense of contemporaneously increasing credit risk. For inflation and interest rate, positive shocks are considered. Admittedly, 3-σ scenarios might not be severe enough, given our short series, but increasing their severity could, instead, introduce significant model risk, particularly for reduced-form equations. That is maybe the reason why most reduced form approaches use these types of scenario (e.g. Cihák (2007), Foglia (2009)).

unemployment and GDP distresses represent, in this order, the most severe univariate scenarios for H=1. As discussed in section 2.2, the impact of the scenarios can be measured by the horizontal distances between the CDFs. At most of the tail ($0.75 \leq \tau \leq 0.95$), the impacts of multivariate, unemployment, GDP and interest rate distresses are each one ahead of the previous by an approximate parcel of 0.2% of NPL.

Figure 6 also allows an investigation of the probability of withstanding the macro shocks. Suppose, for example, that the financial system works at a 95% confidence level for protection during the one-quarter horizon. Then, the amount of own funds that the system should have reserved *ex-ante* is the 95% quantile of the unconditional distribution, which is approximately equal to 7.5% of NPL, according to Wilson approach (see arrows AB and BG of figure 6).[39] However, given the occurrence of a 2-σ unexpected unemployment shock, the respective distressed unemployment distribution translates now the possible NPL outcomes, in place of the former unconditional distribution. The probability of withstanding the unemployment shock in that same quarter, with the 7.5% NPL buffer set *ex-ante*, decreases to approximate 78% (arrows BD and DF). Similarly, a 2-σ GDP shock makes the solvency probability fall from 95% to approximate 87.5% (arrows BC and CE). Carrying out these computations for several initial confidence levels, we arrive at the pp-plots of figure 7.

For the pp-plots shown in this section the horizontal-axis measures the system confidence level set *ex-ante*, while the vertical axis represents the *ex-post* confidence level, or, equivalently, the probability of withstanding the macro scenario. The straight blue line is always the identity function, corresponding to the unconditional distribution, whereas the other pp-plots represent the different macro scenarios. The vertical distances between the latter and the identity are exactly the vertical distances between the conditional and unconditional CDF tails proposed in section 2.2. Figure 7 shows, for example, that distressed unemployment makes a confidence level a priori of 95% fall approximately to 91%, for a 1-σ shock, to 78%, for a 2-σ shock, and rather below 75% (out of the figure), for a 3-σ shock, suggesting the latter case is not easily absorbed in only one quarter[40]. On the other hand, even a 3-σ GDP scenario has a good chance of being absorbed in one quarter: solvency probability decreases from 95% to still 82%

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[39] The analysis refrains from regulatory requirement considerations.
[40] If a 3-σ unemployment shock deserves protection at H=1, a higher unconditional quantile should be set *a priori* as a buffer.

approximately. Finally, note the multivariate scenarios are not easily absorbed for σ=2 and yet more unlikely for σ=3.[41]

Figure 8 shows the tail pp-plots estimated by the QR approach. Although qualitatively the graphs are very similar to figure 7, there are some important differences. At σ=1, the decrease in solvency probability brought about by distressed unemployment is less acute, making distressed GDP the most severe univariate macro scenario by a tiny margin. For σ≥2, distressed unemployment is still the most severe univariate shock, but GDP shocks are now more severe than in Wilson approach. A 3-σ GDP shock reduces solvency probability from 95% to approximate 70%, a 12% larger decrease than in Wilson. Indeed, at σ=3, multivariate distress, distressed unemployment and distressed GDP are clearly not easily absorbed. Figure 9 shows the CDFs for the 2-σ scenarios estimated by the QR approach. In comparison to figure 6, notice QR CDFs can be less parallel at the tails than in Wilson. For example, the horizontal distance between distressed unemployment and distressed GDP distributions at τ=0.75 is three times the corresponding distance at τ=0.95. Since the GDP CDF is also quite parallel to the unconditional CDF, the consequence is that the relative impact of distressed unemployment in relation to distressed GDP is more acute at the third quartile than at the extreme tail. That result follows the spirit of property (7) of the QR link model[42].

We now turn to a more formal comparison between Wilson and QR approaches (but still initially restricted to H=1). Figure 10 superposes the unconditional and conditional densities of the two approaches for σ=2. A closer investigation of the simulated data reveals that the mean and median of both approaches are almost identical but, while Wilson densities are all positively skewed, QR densities are mostly negatively skewed (except for the multivariate and unemployment ones), a consequence of the negative skewness of the QR macro credit-risk uncertainty shown in figure 4. Figure 10 shows further that QR densities are higher on the center and, more importantly, present narrower tails for all macro scenarios. That makes the horizontal distances between Wilson and QR CDF tails positive (see figure 11), leading to the conclusion that, on the absolute NPL scale, Wilson approach views the shocks more

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[41] Whether probability figures should be considered sufficiently high or low (shock not easily absorbed) is largely a subjective matter. At this section, we tend to view one-quarter conditional probability figures below 75% as too low.
[42] However, it is not a direct consequence of (7), because at (7) the macro variables not distressed are hold fixed, while, in our stress-testing exercises, they are simulated.

severely. Finally, note that the bimodality of the QR multivariate distress density, which is also present for σ=1 and σ=3, represents a feature only possible to be captured in the QR approach.[43]

Although Wilson distressed right tails are to the right of the corresponding QR tails, Wilson unconditional tails are to the right too (see first graphs of figures 10 and 11). That is the reason why we have seen before, on the relative probabilistic scale of the pp-plots, that GDP shocks were more severe under the QR approach. That indeed holds (though generally by a minor extent) for basically all other 2-σ and 3-σ univariate macro scenarios at H=1. Figures 12 and 13 display the variation in solvency probabilities across the two approaches. QR pp-plots are mostly below Wilson pp-plots for all macro scenarios, but especially for multivariate distress, distressed GDP and distressed unemployment (that one mostly for σ=3). Notice, also, that the comparison between the two approaches may also depend on the *ex-ante* confidence level τ, as indicated by the crossing of pp-plots at the 2-σ distressed unemployment scenario. To conclude, our analysis illustrates how the results of the task of comparing two stress-testing approaches depend on the method of comparison employed. Indeed, for H=1, we have seen the impact of the macro scenarios are greater on the absolute NPL scale according to Wilson approach, but more harmful on the relative probabilistic scale according to QR approach.

It is worth noting that the most harmful effect of distressed unemployment (figures 7 and 8), in comparison to the other univariate scenarios, is directly related to the largest unemployment coefficients, among the contemporaneous coefficients, in the selected specifications of table 7 (besides the highest correlation of unemployment with logit(NPL) at the first column of table 10, for Wilson model). Nonetheless, the supremacy of unemployment distress should be dampened when the time horizon of the stress exercises is widened, due, for example, to the high lagged coefficients of inflation in table 7. Figure 14 shows Wilson pp-plots of all 2-σ macro scenarios for time horizons from H=1 (the case so far analyzed) until H=4 (one year ahead). The results, which are qualitatively similar to QR approach and to different σ's, indicate how the order of severity of the shocks at the tails varies with the horizon[44]. At transition from H=1 to

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[43] QR densities show generally larger kurtosis too.
[44] It is easy to see that the order of severity of the scenarios in a particular stress-testing approach is identical whether investigated by pp-plots or CDFs.

H=2, distressed inflation becomes the most severe univariate scenario, whereas interest rate distress surpasses GDP in severity. Moving to H=3, distressed interest rate becomes more severe than unemployment and stands out as the second most severe univariate scenario behind inflation. At H=4, it finally assumes the lead in univariate severity, while at the other extreme the effect of distressed GDP vanishes[45]. Finally, as expected, multivariate distress is, by far, the most harmful scenario, particularly for H>1, with low chances of being withstood even for H=3 (solvency probability close to 50% for the 95% *ex-ante* confidence level). However, unless we consider catastrophic events, the build-up of multivariate scenarios is likely to happen along many consecutive quarters rather than on a single shot, so that the multivariate distress scenario of this paper should be viewed mainly as of theoretical interest and as a base of comparison with respect to the univariate scenarios.

The same pp-plots of figure 14 are aggregated by macro scenario at figure 15. That shows the time evolution of the macro shocks on the tails until one year ahead of their occurrences. As H goes to infinity we expect the impacts of the shocks to vanish and the respective pp-plots to return to the unconditional identity line. That is already the case for distressed GDP at H=4. Being GDP the variable whose impact more quickly vanishes is consistent with the short-lived impact of the true GDP shock suffered by the Brazilian economy at 2008:IV (see figure 3). The impact of distressed unemployment similarly decreases continuously since H=1, but is still present at H=4, reflecting a more persistent distress. Interest rate distress has the opposite behaviour, increasing its impact continuously until H=4, at least. Finally, the impacts of multivariate and inflation distresses start increasing, assume their largest magnitude at H=2, and reverse their trajectories onwards. The results of QR approach are qualitatively similar.

The results of figures 14 and 15 may help the supervisory authority or central bank in customizing the point of start and the duration of a regulatory response to a particular shock that had occurred. Such actions could include, among others, the release of regulatory capital buffers or the support of short-term liquidity lines to the system. More generally, however, the identification of the macro vulnerabilities of the system by stress tests (e.g. variations in solvency probabilities as depicted in our

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[45]As previously noted, the effect of distressed interest rate on NPL is a rather indirect one, transmitted through the other macro variables and credit volume.

pp-plots) should foster more focused supervision *ex-ante* to check whether banks perceives the same vulnerabilities and whether appropriate contingency plans have been formulated.

The differences between Wilson and QR densities are generally smaller for H >1 than in figure 10, because the uncertainty about the macro variables z_H, modelled in the same fashion in both approaches, increases with H and dominates the uncertainty of the macro-credit risk link, that has a different form according to each approach (u_t or U_t). Results not displayed show Wilson and QR tail pp-plots very close for all macro scenarios, but multivariate distress for every H and distressed inflation for H=2 and H=3. In those exceptions, QR approach perceives the shocks more severely from the relative view of the pp-plots, similarly to the results of H=1.[46] On the other hand, the gaps between Wilson and QR tails are less neglectable on the absolute NPL scale, with QR tails located more to the right, in contrast to the case of H=1. Anyway, caution should be placed on results for H>1, since the precision of tail estimation is likely to be poor in those cases. Nevertheless, the mentioned observations serve to illustrate the point that the time horizon may also affect the comparison between different stress-testing approaches.

5. Conclusion

This papers estimates the macro-credit risk link on the credit granted by the Brazilian private financial system to the household sector by both the traditional Wilson (1997) model and an alternative proposed quantile regression method (Koenker and Xiao, 2002). Appropriate specifications of Wilson and QR models show negative significant effects on credit risk (measured by NPL) of real GDP growth and credit volume growth and positive significant effects of unemployment rate and lagged inflation rate. Further, Wilson model estimates find evidence of an additional positive effect of unexpected unemployment variation on NPL, while QR estimates indicate that the relative importance of the macro variables varies along the conditional credit risk distribution. That variation can be conceptually related, on a micro level, to uncertainty

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[46] Further, at $2 \leq H \leq 3$, the 2-σ or 3-σ distressed inflation scenario becomes not easily absorbed, particularly according to the QR pp-plots.

in default correlations. Each link model leads to a respective stress-testing approach. Although QR link model is richer in parameters and precludes a normality assumption, Wilson and QR stress-testing approaches produce not so different results qualitatively. According to both approaches, at the one-quarter horizon, unemployment rate distress produces the most harmful univariate effect, followed by GDP distress, whereas distressed inflation and distressed interest rate show higher impacts at longer periods. The impact of distressed interest rate scenarios is brought about indirectly, through the transmission on the other macro variables and on credit volume.

The stress-testing exercises of this paper focus on the tails of the conditional credit risk distributions. These tails represent what worse may still happen to the credit risk outcomes in light of the assumed bad macro scenarios and are the relevant parts of the NPL distributions for determining the *ex-post* solvency probabilities of the system. Pp-plots comparing the distressed conditional and unconditional tails show the variations in solvency probabilities due to the occurrence of the scenarios. For example, a 3-standard deviation GDP shock reduces solvency probability at the same quarter to 82% in Wilson approach (given the 95% unconditional quantile set *ex-ante* as a buffer) but produces an approximately 12% larger decrease in QR approach. Indeed, our results show that the QR approach generally perceives the scenarios more severely from the relative probabilistic view of the pp-plots, that make use of our proposed measure of vertical distance between the distribution tails. QR being more conservative adds support to the idea that, by capturing the influence of varying default correlations, stress-testing approaches may better capture the macro vulnerabilities of the financial system. On the other hand, the scenarios of this paper have a larger absolute impact on the NPL scale at H=1 according to the traditional Wilson approach, illustrating also that the method of comparison is crucial in determining which stress-testing approach is more conservative or liberal. For practical matters and based on this paper, which approach should be relied upon then? A cautious recommendation, particularly suited to stress testing, could be to analyze each piece of result (e.g. variation in solvency probabilities, horizontal dislocation of tails of the NPL distributions) with special attention to its most conservative estimate between the approaches used.

Three important limitations of this study are worth mentioning. The first refers to the reduced number of 59 observations for the Brazilian NPL. The short time series poses a constraint on the precision of our estimations (particularly in the more

parameterized QR approach) and reduces the robustness of the estimates obtained. One alternative could be to enlarge the cross-section dimension by considering other types of credit (e.g. commercial credit); however, the literature on quantile regression within a panel data setup is relatively new and quite scarce (e.g. Galvão and Mones-Rojas, 2010)[47]. Second, the NPL indicator is a *stock* measure of credit risk and, therefore, not directly comparable to the banks' capital, usually understood as a buffer to cover a *flow* of losses over a long horizon. Therefore, our conclusions of the stress-testing exercises are based on the NPL unconditional tails, rather than on the actual system capital[48]. The third limitation is common to every system-wide stress-testing exercise of credit risk that uses aggregated data. Working only at the system level could lead to an underestimation of systemic risk, because the failure or difficulties in one bank can propagate through the chain of bilateral interbank exposures (e.g. see discussion in Sorge and Virolainen, 2006). In spite of these points, we believe the estimates shown in this paper and the underlying discussion can be of great utility to the policy maker or supervisor in the need for pragmatic, but still enough versatile, tools of macro stress testing of credit risk.

A methodological extension of this paper could be to model the macro-credit risk link in the QR style, but explicitly recognizing the potential effect of macroeconomic surprises. This could be carried out similarly to Wilson model, by endowing the QR approach with a joint distribution for (U_t, ε_t)). Besides disentangling the surprise effect, that modeling strategy would also have the advantage of relating the variation of the macro sensitivity of credit risk, or, say, of default correlations, to the macro economy itself. In this way, it would introduce arbitrary forms of non-linearity at the macro-credit risk link, which is a feature very much debated in the stress-testing literature, but usually thought of in an *ad-hoc* fashion (e.g. Misina and Tessier, 2008). On the other hand, as noted in section 2.1, that strategy would also introduce endogeneity on the contemporaneous macros at the macro-credit risk link, rendering traditional QR estimation inappropriate. The alternative estimator candidate could be based on the instrumental variable quantile regression method (IVQR), as proposed by

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[47] Besides, our estimations of the QR macro credit risk link solely on commercial credit did not yield robust results.

[48] Thus, the vulnerabilities identified are intrinsic to the uncertainty pertained to the evolution of the economy and the macro-credit risk link themselves and not to any miscalculation or wrong incentives of banks.

Chernozhukov and Hansen (2008). Nevertheless, whether real data indeed corroborates that modeling strategy is an open issue. Additional research is advised on its exploration.

6. Appendix

6.1 Tables

Table 1: Contemporaneous correlations among variables

	Logit_CRI	Real GDP growth	Unemployment rate	Inflation rate IPCA	Short-term interest rate Selic	Credit volume, quarterly change
Logit_CRI	1.00	-	-	-	-	-
Real GDP growth	-0.43	1.00	-	-	-	-
Unemployment rate	0.50	-0.25	1.00	-	-	-
Inflation rate IPCA	0.27	0.22	-0.09	1.00	-	-
Short-term interest rate Selic	0.28	-0.02	-0.01	0.57	1.00	-
Credit volume, quarterly change	-0.40	0.30	-0.04	-0.44	-0.44	1.00

Table 2: Unit root tests

Test	Logit-transformed NPL	Quarterly change in credit volume	Real GDP growth rate	Unemployment rate	Inflation rate (IPCA)	Interest rate (Selic)
ADF	0.0179	0.0001	0.0106	0.0465	0.0024	0.0124
PP	0.0320	0.0001	0.0065	0.1577	0.0050	0.0548
KPSS	0.2020	0.2071	0.1522	0.2509	0.3296	0.1215

Notes: p-values are presented for the ADF and Phillips-Perron (PP) tests (in which the null hypothesis = variable has a unit root) and the LM statistic for the KPSS test (null hypothesis = variable is stationary). Asymptotic critical values of the KPSS test for 1%, 5% and 10% level are (respectively) 0.739000, 0.463000 and 0.347000. We use quarterly data, with a sample period from 1995:I until 2009:III (59 observations).

Table 3: VAR Lag order selection criteria

Lag	AIC	SC	HQ
0	-25.87118	-25.6887	-25.80062
1	-29.9169	-28.82200*	-29.49349*
2	-29.84323	-27.83589	-29.06698
3	-29.95108	-27.03132	-28.82198
4	-30.18165*	-26.34947	-28.69972

Notes: * indicates lag order selected by the criterion. AIC: Akaike information criterion; SC: Schwarz information criterion; and HQ: Hannan-Quinn information criterion.

Table 4: VAR estimation

	Real GDP growth rate (% p.a.)	Unemployment rate (%)	Inflation rate IPCA (% per quarter)	Interest rate Selic (% per quarter)	Credit volume change (% per quarter)
Real GDP growth rate (t-1)	0.69428	-0.041557	0.085334	0.065707	0.606921
	[6.56358]	[-2.12704]	[1.27252]	[1.42432]	[1.77601]
Unemployment rate (t-1)	0.346787	0.940967	0.057601	0.060267	0.483938
	[1.66088]	[24.3994]	[0.43516]	[0.66182]	[0.71742]
Inflation rate IPCA (t-1)	-0.021981	-0.004239	0.369838	-0.059224	-0.237726
	[-0.09776]	[-0.10208]	[2.59446]	[-0.60392]	[-0.32725]
Interest rate Selic (t-1)	-0.257289	0.045564	0.219065	0.928307	-1.380831
	[-1.76835]	[1.69549]	[2.37497]	[14.6294]	[-2.93762]
Credit volume, quarterly change (t-1)	0.018625	-0.005253	0.001986	0.024429	0.090625
	[0.40875]	[-0.62417]	[0.06877]	[1.22931]	[0.61564]
Intercept	-0.014518	0.004894	-0.007798	-0.005549	0.040296
	[-0.63951]	[1.16706]	[-0.54183]	[-0.56040]	[0.54942]
Adjusted R-squared	0.5268	0.9257	0.3541	0.8394	0.2685

Note: t-statistics in [ ]. Standard LM tests indicate no serial correlation in the VAR residuals.

Table 5: Estimation of macro-credit risk link models Wilson and QR

Dependent variable: y(t) = logit (NPL(t))	Only contemporaneous specification						One-lagged specification					
	Wilson(GMM)		QR (τ=0.5)		QR (τ=0.9)		Wilson(OLS)		QR (τ=0.5)		QR (τ=0.9)	
Intercept	-0.9723	(***)	-1.1357	(***)	-1.0844	(***)	-1.2913	(***)	-1.4564	(***)	-2.3196	(***)
y (t-1)	0.6609	(***)	0.6294	(***)	0.6137	(***)	0.5579	(***)	0.5029	(***)	0.2210	(***)
Real GDP growth rate (t)	-1.0759	(***)	-1.0334	(***)	-1.7339	(***)	-0.4381		-0.4626		-0.4457	
Unemployment rate (t)	1.4062		2.2799	(*)	2.5186	(*)	11.1237	(***)	11.1535	(***)	4.8274	(***)
Inflation rate IPCA (t)	1.4299	(**)	1.6429		-0.5709		0.0982		0.4560		-2.0462	
Interest rate Selic (t)	1.0163	(*)	1.0108		1.9969	(***)	-0.4642		-0.6884		1.1016	
Credit volume, quarterly change (t)	-0.2313		-0.3274	(*)	-0.5729	(**)	-0.3628	(*)	-0.5321	(***)	-0.6639	(***)
Real GDP growth rate (t-1)							0.1782		0.4149	(*)	-1.6190	
Unemployment rate (t-1)							-8.9612	(***)	-8.6094	(**)	0.7137	
Inflation rate IPCA (t-1)							3.5469	(***)	2.9456	(***)	6.6561	(***)
Interest rate Selic (t-1)							0.2673		0.5060		-0.6870	
Credit volume, quarterly change (t-1)							0.0028		0.0306		-0.1279	
Adjusted R-squared	0.7408		-		-		0.8103		-		-	
Pseudo adjusted R-squared	-		0.5010		0.5371		-		0.5598		0.6602	

Quantile slope equality test		
Ho: theta (tau=0.5) = theta (tau=0.9)		
Wald test for all regressors, except intercept		
Chi-squared statistic	11.82	1039.36
degrees-of-freedom	6	11
p-value	0.066	0.000

Notes: Sample 1995:I - 2009:III. Variables that are statistically significant at 1, 5 or 10% are marked by (***), (**) or (*) respectively. The GMM column refers to a GMM system of six equations: the credit risk equation (presented above) and other five equations related to the macroeconomic environment (including the credit volume). These five equations take the form of a VAR(1): X(t) = alpha+beta*X(t-1)+eps(t), in which X={real GDP growth rate, unemployment rate, inflation rate, interest rate, credit volume (quarterly change)}. All these six equations are jointly estimated via GMM, based on a set of instruments composed of one lags of the macroeconomic variables and the credit volume. We use GMM-HAC estimates (Bartlett kernel), which are robust to heteroskedasticity and autocorrelation. TJ tests support the validity of overidentifying restrictions. Only the credit risk equation results are presented above. The pseudo adjusted R-squared is a goodness-of-fit measure of Koenker and Machado (1999). Quantile slope equality test follows Koenker and Bassett (1982 a,b).

Table 6: Quantile slope equality test (p-values)

Quantile Regression - Model	Only contemporaneous specification		One-lagged specification	
Wald test for all regressors, except intercept				
Ho : theta (tau=0.50) = theta (tau=0.55)	0.947		0.888	
Ho : theta (tau=0.50) = theta (tau=0.60)	0.239		0.997	
Ho : theta (tau=0.50) = theta (tau=0.65)	0.004	(*)	0.000	(*)
Ho : theta (tau=0.50) = theta (tau=0.70)	0.000	(*)	0.000	(*)
Ho : theta (tau=0.50) = theta (tau=0.75)	0.000	(*)	0.000	(*)
Ho : theta (tau=0.50) = theta (tau=0.80)	0.000	(*)	0.000	(*)
Ho : theta (tau=0.50) = theta (tau=0.85)	0.005	(*)	0.000	(*)
Ho : theta (tau=0.50) = theta (tau=0.90)	0.066		0.000	(*)
Ho : theta (tau=0.60) = theta (tau=0.90)	0.008	(*)	0.000	(*)
Ho : theta (tau=0.70) = theta (tau=0.90)	0.824		0.513	
Ho : theta (tau=0.80) = theta (tau=0.90)	0.747		0.520	
Ho : theta (tau=0.85) = theta (tau=0.90)	0.991		0.592	

Note: (*) indicates rejection of the null hypothesis at a 5% significance level.

Table 7: Estimation of final specifications of Wilson and QR macro-credit risk link models

Dependent variable: y(t) = logit (NPL(t))	With lagged inflation						With lagged inflation and unemployment surprise					
	Wilson (GMM)		QR (τ=0.5)		QR (τ=0.9)		Wilson(OLS)		QR (τ=0.5)		QR (τ=0.9)	
Intercept	-1.4581	(***)	-1.5723	(***)	-2.2083	(***)	-1.2970	(***)	-1.3627	(***)	-1.9698	(***)
y (t-1)	0.5184	(***)	0.4772	(***)	0.3104	(***)	0.5524	(***)	0.5187	(***)	0.3773	(**)
Real GDP growth rate (t)	-0.7268	(**)	-0.9281	(***)	-1.4105		-0.3403		-0.4518	(**)	-0.9822	
Unemployment rate (t)	2.9462	(***)	3.2472	(***)	6.7377		11.0591	(***)	8.7679	(***)	10.7525	(**)
Unemployment rate (t-1)	-		-		-		-8.9607	(**)	-6.6386	(***)	-4.8536	
Inflation rate IPCA (t-1)	3.9212	(***)	3.4060	(***)	7.0244	(***)	3.7153	(***)	3.5470	(***)	7.0526	(***)
Interest rate Selic (t)	0.0337		0.2932		-0.9360		-0.1856		-0.2006		-1.4299	
Credit volume, quarterly change (t)	-0.4601	(**)	-0.5842	(***)	-0.8340	(*)	-0.3610		-0.5360	(***)	-0.7694	(***)
Adjusted R-squared	0.7973		-		-		0.8250		-		-	
Pseudo adjusted R-squared	-		0.5531		0.6298		-		0.5902		0.6449	

Quantile slope equality test		
Ho: theta (tau=0.5) = theta (tau=0.9)		
Wald test for all regressors, except intercept		
Chi-squared statistic	50.47	30.95
degrees-of-freedom	6	7
p-value	0.000	0.000

Notes: Sample 1995:I - 2009:III. Variables that are statistically significant at 1, 5 or 10% are marked by (***), (**) or (*) respectively. The GMM column refers to a GMM system of six equations: The credit risk equation (presented above) and other five equations related to the macroeconomic environment (including the credit volume). These five equations take the form of a VAR(1): X(t) = alpha+beta*X(t-1)+eps(t), in which X={real GDP growth rate, unemployment rate, inflation rate, interest rate, credit volume (quarterly change)}. All these six equations are jointly estimated via GMM, based on a set of instruments composed of one lags of the macroeconomic variables and the credit volume. We use GMM-HAC estimates (Bartlett kernel), which are robust to heteroskedasticity and autocorrelation. TJ tests support the validity of overidentifying restrictions. Only the credit risk equation results are presented above. The boxed specifications are the ones used in the stress-testing exercises. The pseudo adjusted R-squared is a goodness-of-fit measure of Koenker and Machado (1999). Quantile slope equality test follows Koenker and Bassett (1982 a,b).

Table 8: Quantile slope equality test (p-values)

Quantile Regression - Model	With lagged inflation		With lagged inflation and unemployment surprise	
Wald test for all regressors, except intercept				
Ho: theta (tau=0.50) = theta (tau=0.55)	0.970		0.998	
Ho: theta (tau=0.50) = theta (tau=0.60)	0.553		0.912	
Ho: theta (tau=0.50) = theta (tau=0.65)	0.778		0.983	
Ho: theta (tau=0.50) = theta (tau=0.70)	0.354		0.000	(*)
Ho: theta (tau=0.50) = theta (tau=0.75)	0.000	(*)	0.001	(*)
Ho: theta (tau=0.50) = theta (tau=0.80)	0.000	(*)	0.000	(*)
Ho: theta (tau=0.50) = theta (tau=0.85)	0.000	(*)	0.023	(*)
Ho: theta (tau=0.50) = theta (tau=0.90)	0.000	(*)	0.000	(*)
Ho: theta (tau=0.60) = theta (tau=0.90)	0.000	(*)	0.000	(*)
Ho: theta (tau=0.70) = theta (tau=0.90)	0.000	(*)	0.000	(*)
Ho: theta (tau=0.80) = theta (tau=0.90)	0.007	(*)	0.000	(*)
Ho: theta (tau=0.85) = theta (tau=0.90)	0.052		0.226	

Note: (*) indicates rejection of the null hypothesis at a 5% significance level.

Table 9: Hausman test on Wilson selected specification

Dependent variable: y(t) = logit (NPL(t))	OLS	
Intercept	-1.1204	(**)
y (t-1)	0.5925	(***)
Real GDP growth rate (t)	-0.2351	
Unemployment rate (t)	1.5430	
Inflation rate IPCA (t-1)	3.3165	(**)
Interest rate Selic (t)	-0.3336	
Credit volume, quarterly change (t)	-0.8323	
residual_GDP_growth	-0.1879	
residual_unemployment_rate	9.5544	(**)
residual_interest_rate	-0.1098	
residual_credit_volume_change	0.4629	
Adjusted R-squared	0.8143	

Notes: Sample 1995.I - 2009.III. Inflation rate (t-1) is considered exogenous. In this paper, we adopt the version of the Hausman test suggested by Davidson and MacKinnon (1989, 1993), which is based on two OLS regressions: In the first one, we regress the suspect variable on instruments and all exogenous variables and retrieve the residuals. Then, in the second OLS regression, presented above, we re-estimate the credit risk equation now including the residuals from the first regression as additional regressors. If there is no endogeneity then the coefficient on the first stage residuals should not be significantly different from zero. In our case, unemployment rate seems to be endogenous.

Table 10: Estimation of Correlation(ut;et) - Wilson selected specification

Residual correlation matrix	Logit (NPL)	Real GDP growth rate	Unemployment rate	Inflation rate IPCA	Interest rate Selic	Credit volume
Logit (NPL)	**1.000**	-0.054	0.340	-0.049	-0.012	0.104
Real GDP growth rate	-0.054	**1.000**	-0.356	0.281	0.070	0.026
Unemployment rate	0.340	-0.356	**1.000**	-0.147	0.161	0.050
Inflation rate IPCA	-0.049	0.281	-0.147	**1.000**	0.201	-0.373
Interest rate Selic	-0.012	0.070	0.161	0.201	**1.000**	-0.172
Credit volume, quarterly change	0.104	0.026	0.050	-0.373	-0.172	**1.000**

Note: estimation of Corr(ut,et) is built from the GMM estimate of sigma.

Table 11: Scenarios for stress testing

Macroeconomic variable	Real GDP growth rate (%)	Unemp. Rate %	Inflation rate % (IPCA)	Interest rate % (Selic)
Last observation (2008.III)	6.58	7.72	1.07	3.22
1 standard deviation shock	3.08	7.96	2.46	4.22
2 standard deviations shock	1.25	8.31	3.63	5.04
3 standard deviations shock	-0.55	8.67	4.81	5.87

6.2 Figures

Figure 1 – Credit risk indicator (Non-performing loans - NPL, household sector)
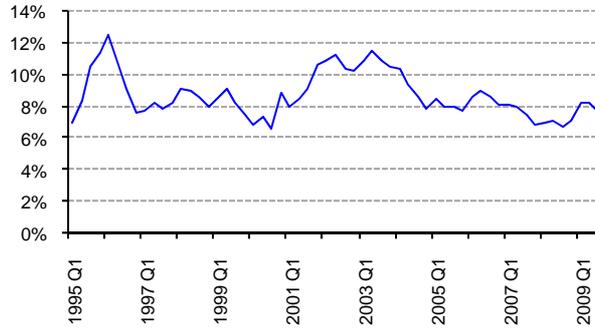
Figure 2 – Credit volume growth, household sector (quarterly change)
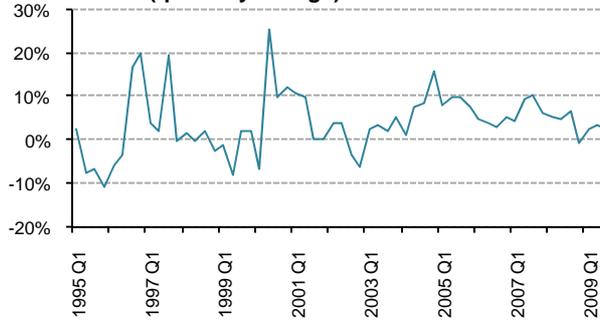
Figure 3 – Macroeconomic Variables

Real GDP growth (annual rate)

Unemployment rate

IPCA inflation (quarterly rate)

Short-term interest rate - Selic (quarterly rate)
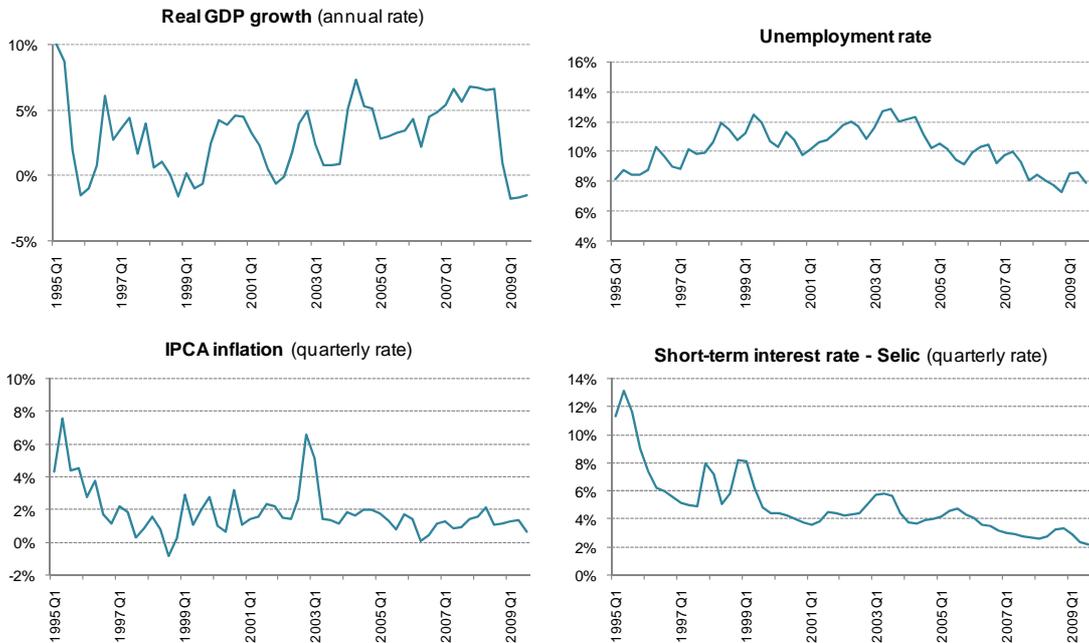
Notes: Change in real GDP is from a given quarter of the previous year to the same quarter of the year indicated. Unemployment rate series is obtained from Da Silva Filho (2008).

Figure 4 – NPL conditional distributions – QR selected specification

(i) conditional on 2008.III

(ii) conditional on 2009.III
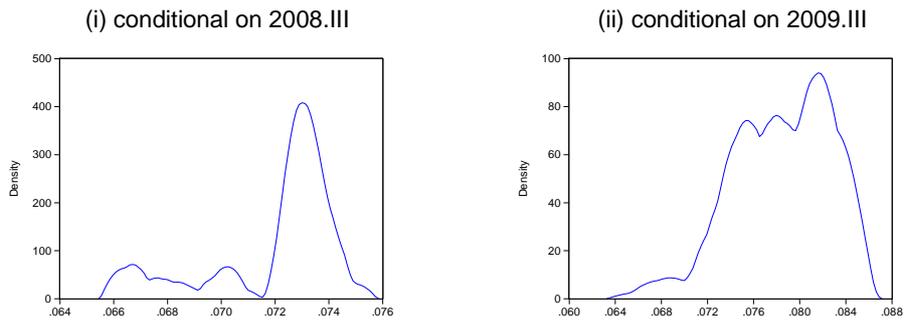
Note: The conditional distribution (evaluated at the last observation) is nonparametrically estimated through an Epanechnikov kernel.

Figure 5 – Wilson distressed NPL densities - H=1
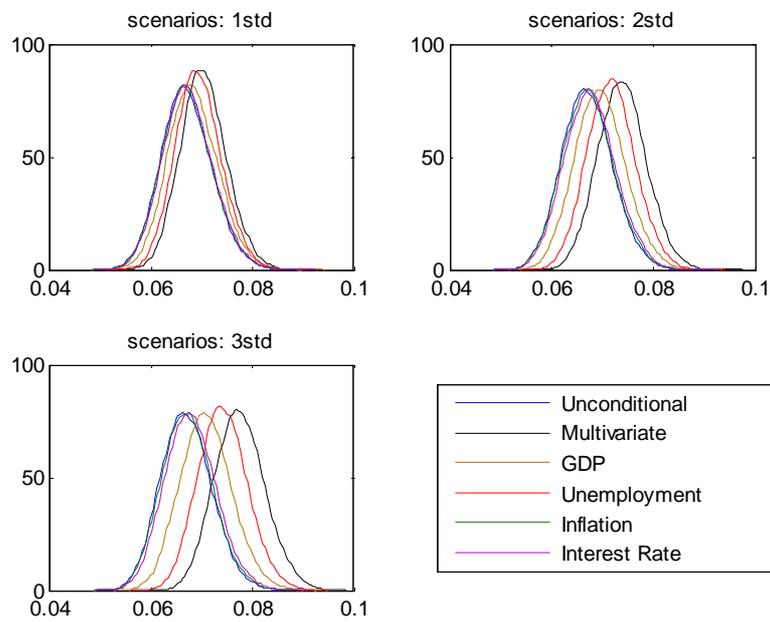
Note: Figure above shows for all considered scenarios (H=1) the distressed conditional NPL densities estimated by the Wilson approach.

Figure 6 – Wilson distressed NPL cumulative distribution tails - H=1,σ=2

Empirical CDF tails of Wilson - h=1, sigma=2
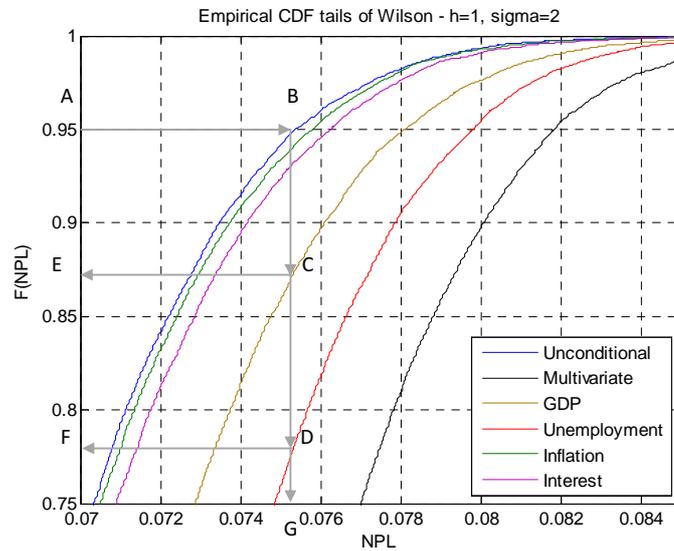
Figure 7: Tail pp-plots estimated by Wilson - H=1

scenarios: 1std

scenarios: 2std

scenarios: 3std

Unconditional	
Multivariate	
GDP	
Unemployment	
Inflation	
Interest	
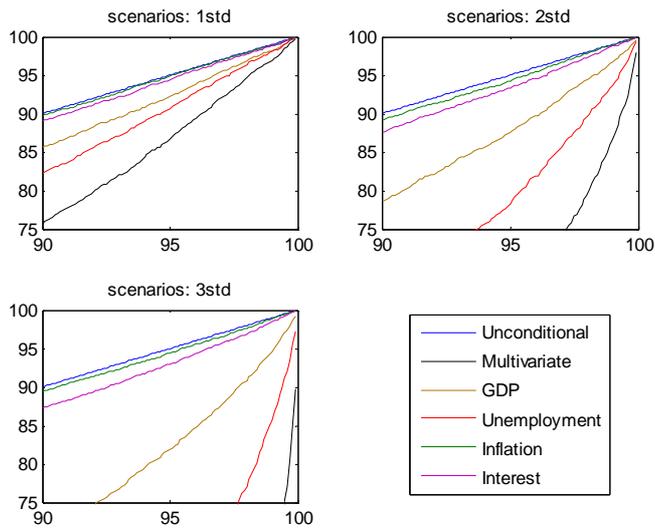
Figure 8 - Tail pp-plots estimated by QR - H=1

scenarios: 1std

scenarios: 2std

scenarios: 3std

Unconditional	
Multivariate	
GDP	
Unemployment	
Inflation	
Interest	
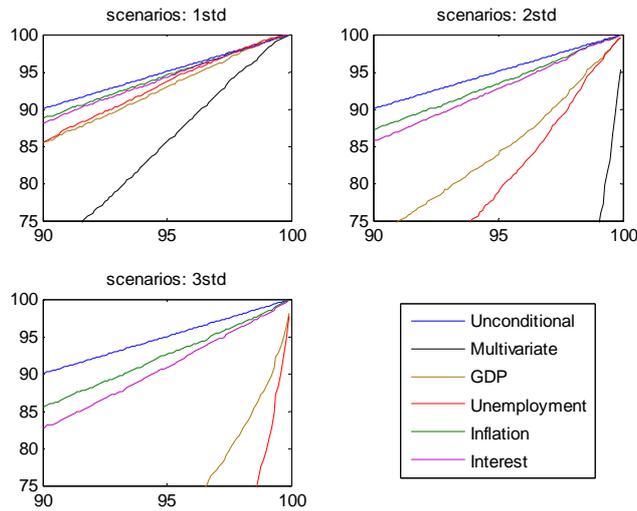
Figure 9 – QR distressed NPL cumulative distribution tails - H=1,σ=2

Empirical CDF tails of qr - h=1, sigma=2

F(NPL)

NPL

Unconditional	
Multivariate	
GDP	
Unemployment	
Inflation	
Interest	
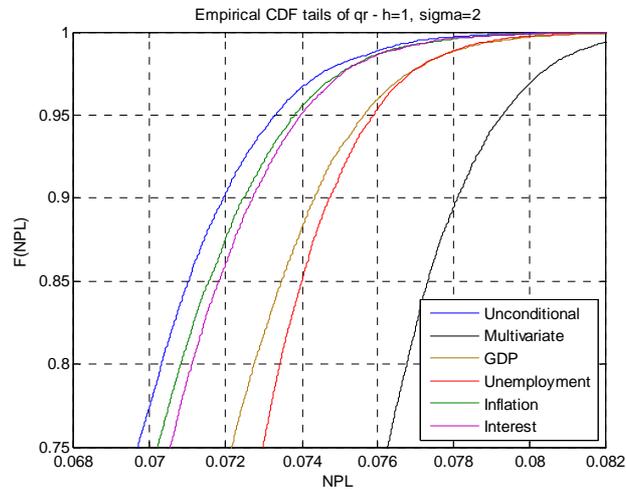
Figure 10 - Wilson and QR distressed NPL densities - H=1, σ=2
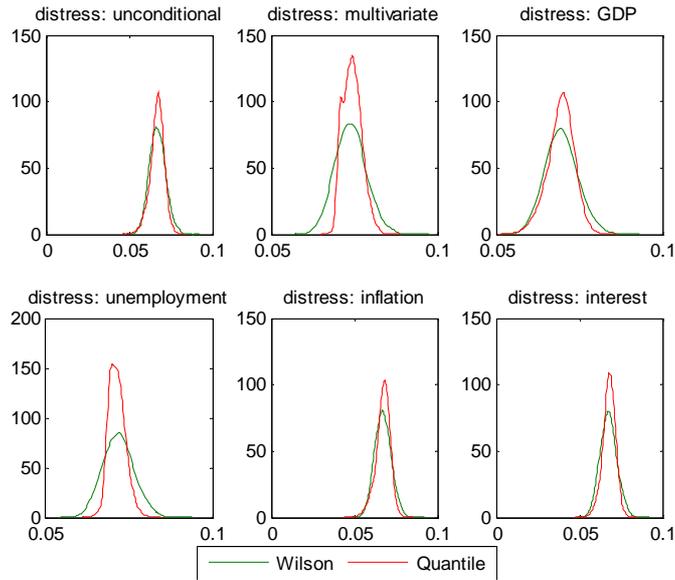
Figure 11 – Wilson and QR distressed NPL cumulative distribution tails - H=1, σ=2
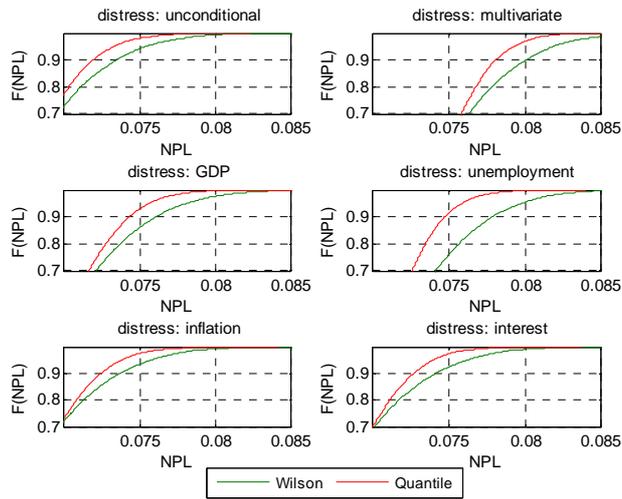
Figure 12: Tail pp-plots by Wilson and QR - H=1, σ=2

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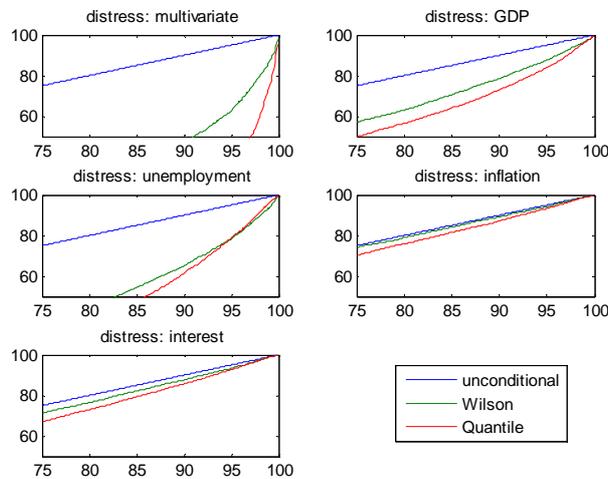
Figure 13: Tail pp-plots by Wilson and QR - H=1, σ=3
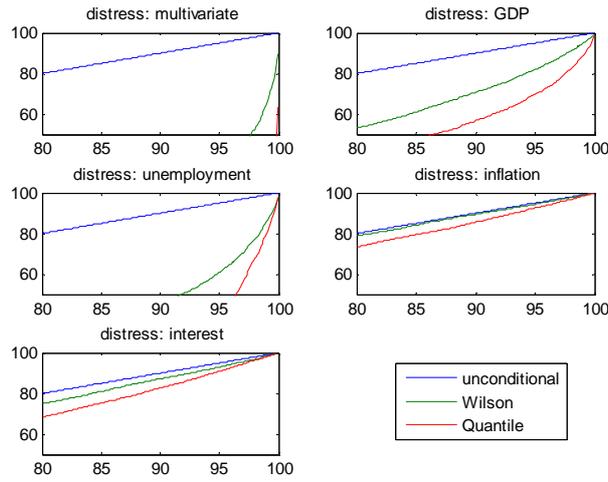
Figure 14: Order of severity of 2-σ scenarios through pp-plots – Wilson
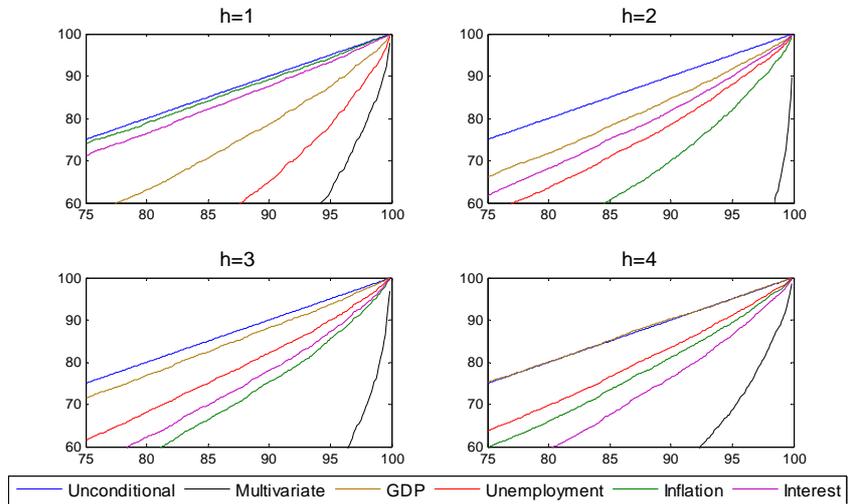
Figure 15: Time evolution of 2-σ scenarios through pp-plots – Wilson
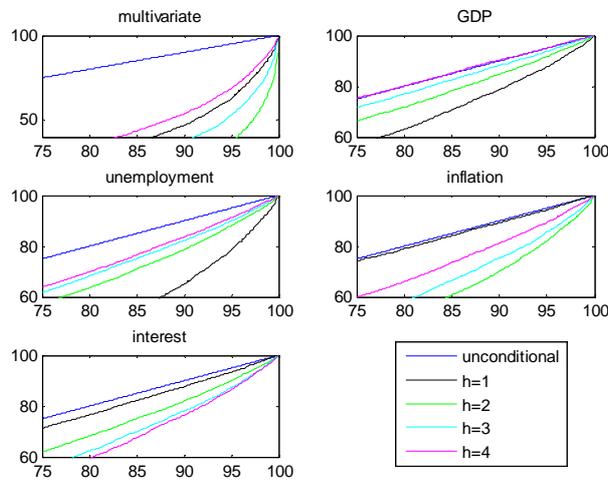
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