# Why micro-prudential regulations fail?

## The impact on systemic risk by imposing a capital requirement


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**Abstract** This paper investigates why micro-prudential regulations such as capital requirement fail to maintain the stability of a financial system. With a static model on financial institutions' risk-taking behavior, we quantify the impact on systemic risk in the cross-sectional dimension when imposing a capital requirement. Although imposing a capital requirement can lower individual risk, it enhances the systemic linkage within the system at the same time. With a proper systemic risk measure combining both individual risks and systemic linkage, we show that the systemic risk in a regulated system can be higher than that in a regulation-free system. We discuss a sufficient condition under which the systemic risk in a regulated system is always lower. Since the condition is based on comparing both asset and liability sides of bank balance sheets among all institutions in the system, it can be verified only if all required information are available. This suggests that a macro-prudential framework is necessary for establishing banking regulations towards the stability of the entire financial system.



**Keywords:** Banking regulation; systemic risk; capital requirement; macro-prudential regulation.

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# 1 Introduction

Regulations in financial sector are designed for limiting risk-taking of financial institutions and thus prevent potential financial crises. With the failure of the investment bank Lehman Brothers in 2008, the financial system in the US and the EU came close to a complete meltdown. This raises the questioning on current financial regulation rules, such as the Basel I and II Accords. Current policy debate points to the direction of imposing macro-prudential tools which aim at the stability of the entire financial system. The word "macro-prudential" is considered as the opposite of "micro-prudential" which refers to incumbent regulations that focus on limiting risk-taking behavior of individual financial institutions. Academical research has attempted to document what went wrong with micro-prudential regulations, and consequently provide recommendations to the regulation reform. This paper fits this literature by studying the impact of imposing micro-prudential regulation to the systemic risk in the cross-sectional dimension. We confirm that micro-prudential regulation may fail to limit systemic risk and consequently leads the financial system to an instable scenario.

The general critique on micro-prudential regulation is that it fails to achieve the goal of maintaining the stability of a financial system as a whole. In other words, it fails to limit the systemic risk. Such a critique is valid since regulations at micro level are by definition towards limiting individual financial institution from taking excessive risk, which may not necessarily lower the systemic risk at the same time. Nevertheless, to obtain a complete view on the consequence of imposing micro-prudential regulation, it is necessary to conduct a systemic risk analysis. This helps identify under which circumstance, micro-prudential regulation may fail in reducing systemic risk.

Systemic risk refers to the risk that a large proportion of the financial system fall into crises or distresses, which consequently leads to an adverse impact on the macro economy.[1] We distinguish systemic risk in two dimensions: the time dimension, i.e. the interaction between the financial system and the macro economy, and the cross-sectional dimension, i.e.

[1]For an overview on the systemic risk, we refer to de Bandt and Hartmann (2001) and Allen et al. (2009).

the interconnectedness among financial institutions. On both dimensions, current micro-prudential regulation fails in limiting systemic risk. On the time dimension, the evolution of banks' risk-taking behavior may result in a procyclicality problem. Time-invariant micro-prudential regulation may enhance such a problem and thus leads to a high systemic risk[2]. In recent studies, the other dimension of systemic risk, the cross-sectional dimension, has caught attention. The interconnectedness within the financial system is established from either a direct channel such as interbank lending[3] or an indirect channel that financial institutions share common exposures due to diversification at individual level[4]. This study targets to quantify the impact of micro-prudential regulation on the cross-sectional dimension of systemic risk.

We establish a static model on banks' risk-taking behavior under two scenarios: a system with regulation and a regulation-free system. We model banks' asset decomposition by optimizing their portfolio with respect to a return-downside risk utility. Considering that financial institutions are interconnected because of common risk exposures, the interconnectedness, or in other words, systemic linkage, is then determined by the similarity between their banking activities. The key feature in the model is that when financial institutions rebalance their portfolio in order to obey the same regulation rule, their portfolios turn to be more similar, which enhances the systemic linkage within the system. Therefore, although in the regulated system, the individual risk of each institution is lower, the systemic linkage within the system is higher. With defining a systemic risk measure that combines individual risk with systemic linkage, we compare the systemic risks in the two cases and obtain that, under certain condition, the systemic risk can be higher in the regulated case. Moreover, we discuss a sufficient condition under which the systemic risk in the regulated system is always lower. Since the sufficient condition is based on comparing both the asset and liability sides of balance sheets among all institutions within the system, it can be only verified when

[2]See, for example, Borio et al. (2001), Borio and Zhu (2008), Brunnermeier et al. (2009), Shin (2009), Zhu (2008) and among others.

[3]See, for example, Allen and Gale (2000), Freixas et al. (2000), Dasgupta (2004) and among others.

[4]See, for example, Lagunoff and Schreft (2001), de Vries (2005) and Wagner (2010).

having all relevant information and forming a helicopter view on the entire system. This suggests that a macro-prudential framework is necessary for establishing regulations towards the stability of the financial system as a whole.

This study is comparable with Acharya (2009), which investigates the impact of micro-prudential regulation on the cross-sectional dimension of systemic risk by a multi-period general equilibrium model. Acharya (2009) shows that micro-prudential regulation based only the *own* risk of individual banks can in fact accentuate the systemic risk. We intend to avoid the dynamic on the time dimension, while focusing on the interconnectedness. Thus we consider a static model. Nevertheless, it is sufficient to show similar conclusion as in Acharya (2009). Differently, this model helps identify under which scenario, micro-prudential regulation can effectively reduce the systemic risk.

Another stream of literature that is relevant to this study are the measures for systemic risk. Here we employ the systemic risk measure proposed by Segoviano and Goodhart (2009). However, our qualitative conclusion is not limited to this specific choice because most of systemic risk measures in literature bear the same feature that with strong systemic linkage, the systemic risk is high. For measures on systemic risk and systemic importance of financial institutions, see Adrian and Brunnermeier (2008), Tarashev et al. (2009a), Tarashev et al. (2009b), Huang et al. (2009) and Zhou (2010a).

Our finding on the limitation of micro-prudential regulation has direct policy implications. The model suggests that it is necessary to have a macro-prudential regulator holding all relevant information of all financial institutions and forming a helicopter view on the system. That includes monitoring all banks' asset decompositions as well as their capital structures. From our result, we conclude that when regulating a financial system consisting of institutions with similar banking activities, a micro-prudential regulation can be sufficient for reducing systemic risk. In contrast, the macro-prudential regulation is particularly necessary when regulating a diversified financial system which contains heterogeneous financial institutions focusing on different banking activities. For such a system, it is necessary to identify the systemically important institutions (SIFIs) and impose proper prudential regulations on them. This is crucial for managing the systemic risk in the system.

The paper is organized as follows. Section 2 presents the general setup of the model. Section 3 analyzes the impact on individual risk when imposing a capital requirement regulation. The main results on comparing the systemic risks in the regulated and regulation-free cases are established in Section 4. Section 5 provides policy discussions and potential extensions of this study. Proofs of the results are gathered in the Appendix.

# 2 The Model

We introduce the general setup of our model in Section 2.1. Then we discuss the heavy-tailed feature on asset returns in Section 2.2. Section 2.3 discusses a few assumptions that are useful for simplifying the analysis.

## 2.1 General Setup

Consider a financial system consisting of two banks. Each bank can invest in two risky projects and the risk-free rate. The expected returns of the two projects $R_1$ and $R_2$ are $\mu_1$ and $\mu_2$ respectively. Without loss of generality, we assume that the risk-free rate is zero and $\mu_2 > \mu_1 > 0$. Moreover, the two projects are independent.

From the bank side, suppose Bank $j$ holds a portfolio $P_j = w_{j1}R_1 + w_{j2}R_2$, $j = 1, 2$. For simplicity, short selling is not allowed, i.e. $w_{ji} \geq 0$ and $w_{j1} + w_{j2} \leq 1$, for $j = 1, 2$. The portfolio holding is optimized according to a mean-downside risk utility. Suppose the two banks have different levels of risk aversion $\lambda_j$, $j = 1, 2$. Without loss of generality, we assume that $\lambda_1 \leq \lambda_2$, i.e. Bank 1 is less risk averse. More precisely, the utility function of Bank $j$ is given as

$$U_j = w_{j1}\mu_1 + w_{j2}\mu_2 - \lambda_j D(w_{j1}, w_{j2}), \tag{2.1}$$

where $D(w_{j1}, w_{j2})$ is a measure of the downside risk. An example for such a risk measure $D$

is the variance of the portfolio. Then the utility function turns to be a usual mean-variance approach. In the regulation-free case, the portfolio holding of each bank is determined by maximizing the utility in (2.1).

Next, we consider imposing a micro-prudential regulation: capital requirement as in Basel II. In its elementary form, a capital requirement is calculated from the Value-at-Risk (VaR) of the portfolio holding and multiplied by a risk-weight appointed by the regulator. Financial institutions are required to hold sufficient equity capital to achieve the level of the requirement. In our model, instead of requiring a certain amount of capital holding, we regard the capital structure of a bank as a non-adjustable characteristic in short term, while allow banks to adjust their portfolios in order to obey the regulation rule. This setup is in line with the situation in financial crisis: raising new capital is extremely difficult or very expensive during a crisis; instead, financial institutions choose to fire sale their assets. Under such a framework, the capital requirement regulation turns to be a restriction on the VaR of the portfolio held by a bank.

For a given probability level $p$, the VaR of $P_j$, $VaR_j(p)$, is defined by the relation $P(P_j < -VaR_j(p)) = p$. With the VaR of $P_j$, the capital requirement for Bank $j$ is $I_j VaR_j(p) d_j$, where $I_j$ is the total investment on the portfolio, and $d_j$ is a multiplier chosen by the regulator. The capital requirement should be covered by the total (equity) capital raised by the bank, denoted by $E_j$. Hence, we get the restriction as $I_j VaR_j(p) d_j \leq E_j$, for $j = 1, 2$. It can be rewritten as

$$VaR_j(p) \leq T_j := \frac{Q_j}{d_j}, \tag{2.2}$$

where $Q_j := E_j / I_j$ is the equity ratio of the bank.

As discussed above, we regard the equity ratios as fixed within a short period. Moreover, the regulator chooses the regulatory probability level $p$ and the bank specific multiplier $d_j$ ex ante. Hence the threshold $T_j$ in the capital requirement rule (2.2) is regarded as a characteristic of Bank $j$ which is determined ex ante. By fixing the threshold $T_j$, the capital requirement rule (2.2) should be read as a restriction on the VaR of the portfolio held by each bank. When a capital requirement is imposed, banks rebalance their portfolios to obey the rule. Therefore, they solve the constrained utility maximization problem, that is to maximize the utility in (2.1) with the constrain (2.2).

## 2.2 The heavy-tailed feature on asset returns

Recall that the expected returns of the two projects are $\mu_1$ and $\mu_2$, where $\mu_2 > \mu_1 > 0$, i.e. $R_2$ is more profitable than $R_1$. For the downside risk of the two projects, we consider the heavy-tailed feature on the downside distributions of the returns.

The heavy-tailedness of the downside distribution of financial assets has been well documented in the empirical literature, see, e.g. Jansen and De Vries (1991) and Embrechts et al. (1997). Mathematically, it is assumed as follows. For sufficiently large $t$, the left tail of the distribution function of $R_i$ is given as

$$P(R_i < -t) = A_i t^{-\alpha}(1 + o(1)),$$

where $A_2 > A_1 > 0$, i.e. $R_2$ is more risky than $R_1$. The parameter $\alpha$ is called the *tail index*, while $A_i$ is called the *scale*. Moreover, the right tails of the two asset returns are assumed to be thinner than the left tails, i.e.

$$P(R_i > t) = o(t^{-\alpha}).$$

This ensures that when constructing a portfolio based on $R_1$ and $R_2$, the downside risk of the portfolio is dominated by the downside risks of the two asset returns, and the right tails do not intervene. Here we assume equal tail indices $\alpha$ for the two assets. Theoretically, this is the only case in which diversification on risky assets is beneficial and non-trivial, see Zhou (2010b). Finally, since we assume the existence of a finite mean, it implies that $\alpha > 1$. We remark that all above theoretical assumptions on the tail properties of the asset returns have been justified by empirical literature; see, e.g., Jansen and De Vries (1991).

With the equal tail indices among all risky assets, the scale is then a downside risk measure, which is similar to the variance in the Gaussian framework, see Zhou (2010b). The difference is that the scale only measures the risk in the downside tail, while the variance measures the variation around the mean level, which is potentially driven by both the downside and upside tails. Since the portfolio optimization problem considers a mean-downside risk utility, we use the scale as the measure of the downside risk. From the properties of aggregating independent heavy-tailed risks (see, e.g. Feller (1971)), the left tail of the portfolio return held by Bank $j$, $P_j = w_{j1}R_1 + w_{j2}R_2$ is also heavy-tailed and

$$P(P_j < -t) = A_{P_j} t^{-\alpha}(1 + o(1)),$$

where the scale of the left tail is $A_{P_j} = w_{j1}^{\alpha} A_1 + w_{j2}^{\alpha} A_2$. We define the downside risk measure as

$$D(w_{j1}, w_{j,2}) = \frac{1}{\alpha}(w_{j1}^{\alpha} A_1 + w_{j2}^{\alpha} A_2).^5$$

With the explicit utility function, it is thus possible to solve the portfolio optimization problem in both the unconstraint and constraint cases.

## 2.3 Assumptions

We make a few assumptions in order to simplify the analysis on the portfolio optimization problems. Those assumptions are mainly for simplification in the analysis: they are not essential for obtaining the stylized results. It is possible to omit those assumptions while having a full discussion on all scenarios. However, scenarios that are out of those assumptions are either similar, or trivial.

**Assumption 1** *In the regulation-free case, the optimal portfolios held by the banks are not "corner solutions" which assign all portfolio weights to one asset.*

[5] The denominator $\alpha$ is imposed for simplifying the calculation. THi sis similar to the multiplier $1/2$ in the mean-variance approach. It has no impact on the optimization problems.

**Assumption 2** *In the regulation-free case, the optimal portfolios held by the banks are not "partial investment solutions" which assign positive weight to the risk-free asset.*

**Assumption 3** *Any fully invested risky portfolio can not obey the regulation rule.*

We explain why those assumptions are useful in simplifying the analysis, but not essential. Assumption 1 implies that the risk aversion levels of the banks are not too low. If omitted, it implies that banks are in favor of the riskier asset. Then in the regulation-free case, only the riskier asset is held by both banks, while imposing a regulation will simply change their holding on this asset. This makes the systemic risk analysis trivial and meaningless. Assumption 2 implies that the risk aversion levels are not too high. This assumption can be always guaranteed by lowering the risk-free rate, which has no impact on the systemic risk analysis. Assumption 3 implies that the thresholds $T_j$ are sufficiently low such that the regulation rule is effective. Together with Assumption 2, the optimal portfolio in the regulation-free case can not satisfy the regulation requirement. Hence, banks must adjust their investment strategy in order to obey the regulation rule. Without Assumption 3, banks may simply keep the optimal portfolio in the regulation-free case while still obeying the capital requirement. In that case, there is nothing to compare between the regulated case and regulation-free case. Hence the situation is again trivial. Making such an assumption is also reasonable, because $T_j$ is partially determined by the regulator by choosing a proper $d_j$, in other words, the regulator can make sure that $T_j$ is sufficiently low such that the regulation is effective. To summarize, the scenario in which all assumptions are imposed is a non-trivial and representative case for analyzing the impact of imposing capital requirement on systemic risk.

# 3 Capital requirement and individual risk

To study the impact of imposing a capital requirement, we start by analyzing the risk-taking behaviors of individual financial institution, and compare the two cases: with and without a regulation.

In the regulation-free case, the optimal portfolio holding of each bank is given by solving the the unconstrained utility maximization problem. With the downside-risk measure giving in Section 2.2, the utility function of Bank $j$ is then

$$U_j = w_{j1}\mu_1 + w_{j2}\mu_2 - \frac{\lambda_j}{\alpha}(w_{j1}^{\alpha}A_1 + w_{j2}^{\alpha}A_2). \tag{3.1}$$

The solution of the optimal portfolio is given in the following proposition. The proof is postponed to the Appendix.

**Proposition 3.1** *With Assumptions 1 and 2 on the risk aversion levels, the solution of the unconstrained utility maximization problem in the regulation-free case, $(w_{j1}^*, w_{j2}^*)$, is given by firstly solving the equation*

$$(w_{j2}^*)^{\alpha-1}A_2 - (1 - w_{j2}^*)^{\alpha-1}A_1 = \frac{\mu_2 - \mu_1}{\lambda_j}, \tag{3.2}$$

*and then taking $w_{j1}^* = 1 - w_{j2}^*$.*

Notice that Assumption 1 and 2 ensure the existence of a unique solution of equation (3.2).

Combining the facts that $\frac{\mu_2-\mu_1}{\lambda_2} \leq \frac{\mu_2-\mu_1}{\lambda_1}$ and the left hand side of (3.2) is an increasing function of $w_{j2}^*$, we get that $w_{12}^* \geq w_{22}^*$. Intuitively, since Bank 1 is less risk averse, it assigns higher weight on the riskier asset $R_2$.

Next, we consider the regulated case with a capital requirement. Thus VaR constrain in inequality (2.2) is now effective. Under the heavy-tailed framework, the calculation of VaR is convenient thanks to the explicit expansion of the tails. Since the left tail distribution of the portfolio return $P_j$ is a heavy-tailed distribution with tail index $\alpha$ and scale $A_{P_j} = A_1 w_{j1}^{\alpha} + A_2 w_{j2}^{\alpha}$, we get that

$$VaR_j(w_{j1}, w_{j2}; p) \approx \left(\frac{A_1 w_{j1}^{\alpha} + A_2 w_{j2}^{\alpha}}{p}\right)^{1/\alpha}. \tag{3.3}$$

Here the approximation is for low level of $p$.

With a capital requirement, the optimal portfolio construction for each bank is then determined by the constrained utility maximization problem. The following proposition gives the solution to that. The proof is in the Appendix.

**Proposition 3.2** *Denote* $e_i = (\mu_i/A_i)^{\frac{1}{\alpha-1}}$ *for* $i = 1, 2$, *and*

$$c_j = \frac{T_j p^{1/\alpha}}{((e_1)^\alpha A_1 + (e_2)^\alpha A_2)^{1/\alpha}}.$$

*With Assumption 1-3, the constrained utility maximization problem is solved by* $(\tilde{w}_{j1}, \tilde{w}_{j2})$ *as*

$$\tilde{w}_{j1} = e_1 c_j, \quad \tilde{w}_{j2} = e_2 c_j. \tag{3.4}$$

From the optimal solution in (3.4), we get that

$$\frac{\tilde{w}_{j1}}{\tilde{w}_{j2}} = \frac{e_1}{e_2},$$

which is irrelevant to $j$. In other words, the relative portfolio decomposition of the two banks are the same, the only difference is on the total risky investment, which is constrained by the regulation rule. This is due to the common regulation rule applied to them: the capital requirement imposes similar constrains for different institutions, which (partially) overrides their heterogeneity on risk aversion. Such an intuition applies in general to all micro-prudential regulation and is not limited to the capital requirement.

Assumption 1-3 ensures that the VaR of the optimal portfolio in the regulated case is lower than that in the regulation-free case. This reflects the intuition that micro-prudential regulation targets limiting the individual risks. Thus, without analytical comparison, we can conclude that imposing a capital requirement help reduce the risk of individual banks, i.e. $A^*_{P_j} > \tilde{A}_{P_j}$ for $j = 1, 2$, where $A^*_{P_j}$ and $\tilde{A}_{P_j}$ indicates the scales of the downside distributions of the optimized portfolios in the regulation free and regulated cases.

# 4 The impact on systemic risk

From Proposition 3.2, we obtain that with the static model, under the capital requirement regulation, the relative portfolio decomposition are the same across different banks. This implies that the two banks in the system are highly systemically linked: the portfolio returns of the two banks are completely dependent. More specifically, any dependence measure, such as correlation coefficient or tail dependence measure (see, e.g. de Vries (2005) and De Jonghe (2010)), achieves its maximum when analyzing the dependence of the two banks in this case. Hence, the impact of imposing a micro-prudential regulation is two-folded: although imposing a micro-prudential regulation can reduce individual risk as it intends to, at the same time it overrides the heterogeneity on individual risk aversions. As a consequence, financial institutions tend to hold more similar portfolios which results in a higher systemic linkage.

To evaluate the tradeoff at the systemic risk level, it is necessary to consider a systemic risk measure that combines individual risk with systemic linkage. For instance, a fully interconnected system with no individual risk should be regarded as having no systemic risk. Only with a proper systemic risk measure, it is possible to evaluate the tradeoff between reducing individual risk and increasing systemic linkage and further assess whether a regulated system corresponds to a lower level of systemic risk.

Since our simple static model consists of two banks, an example of a proper systemic risk measure is the probability that both banks are insolvent. With the notation in Section 2, the measure is given as

$$SR := P(P_1 < -Q_1, P_2 < -Q_2), \tag{4.1}$$

where $Q_j$ is the capital ratio for Bank $j$. This measure is a special case of the banking stability index discussed in Segoviano and Goodhart (2009).

Apparently, the probability measure in (4.1) is associated to both individual risks of $P_1$ and $P_2$ and the dependence between them. Moreover, $Q_j = d_j T_j$ is higher than the threshold

$T_j$ in (2.2) because the regulators usually set a multiplier $d_j > 1$. Thus the probability in (4.1) must be at an extremely low level, much lower than the probability $p$ used in the regulation rule. Hence it is a probability of a tail event. We use Extreme Value Theory (EVT) to make approximate calculation on such a tail probability.

With the affine portfolio model, and the heavy-tail feature of asset returns, the left tail of the bank portfolio returns $(P_1, P_2)$ follows a bivariate EVT setup, and exhibits tail dependence, see e.g. de Vries (2005). For details on multivariate (or bivariate) EVT, see de Haan and Ferreira (2006). The following lemma shows how to calculate $SR$ given the portfolio structure.

**Lemma 4.1** *Suppose Bank $j$ holds a portfolio $(w_{j1}, w_{j2})$ for $j = 1, 2$. Then the systemic risk measure in* (4.1) *is calculated as*

$$SR \approx A_1 \left( \frac{w_{11}^\alpha}{Q_1^\alpha} \wedge \frac{w_{21}^\alpha}{Q_2^\alpha} \right) + A_2 \left( \frac{w_{12}^\alpha}{Q_1^\alpha} \wedge \frac{w_{22}^\alpha}{Q_2^\alpha} \right). \tag{4.2}$$

From (4.2), when increasing the capital ratio of a bank, the systemic risk may decrease or remain at the same level due to the minimum feature in the formula.

A modification of the formula on $SR$ is that

$$SR \approx A_1 \frac{w_{21}^\alpha}{Q_1^\alpha} \left( \frac{w_{11}}{w_{21}} \wedge \frac{Q_1}{Q_2} \right)^\alpha + A_2 \frac{w_{22}^\alpha}{Q_1^\alpha} \left( \frac{w_{12}}{w_{22}} \wedge \frac{Q_1}{Q_2} \right)^\alpha. \tag{4.3}$$

From this representation, we observe that for calculating $SR$, it is necessary to compare $\frac{w_{11}}{w_{21}}$ and $\frac{w_{12}}{w_{22}}$ with $\frac{Q_1}{Q_2}$.

With the formula (4.2), we compare the systemic risk measures in the regulation-free and the regulated cases. The result is presented in the following proposition.

**Proposition 4.2** *Consider systemic risk measured by the SR measure in* (4.1). *Denote the systemic risk measures in the regulation-free and regulated cases as $SR^*$ and $\widetilde{SR}$ respectively.*

*From the solution of the optimal portfolio in the regulation-free case, $w_{ji}^*$, $i = 1, 2$ and*

*$j = 1, 2$, we define two thresholds as*

$$\begin{cases} l(\lambda_1, \lambda_2; \mu_1, \mu_2, A_1, A_2) := \frac{w^*_{11}}{w^*_{21}} \\ r(\lambda_1, \lambda_2; \mu_1, \mu_2, A_1, A_2) := \frac{w^*_{12}}{w^*_{22}} \end{cases} \tag{4.4}$$

*It is clear that $l < 1 < r$, provided by $\lambda_1 < \lambda_2$.*

*If $\frac{Q_1}{Q_2} \leq l$ or $\frac{Q_1}{Q_2} \geq r$, we have that $SR^* > \widetilde{SR}$, i.e. in the regulated system,the total systemic risk is lower.*

*If $l < \frac{Q_1}{Q_2} < r$, with suitable choices of the parameters $\lambda_j, d_j$, $j = 1, 2$ and $\mu_i, A_i$, $i = 1, 2$, it is possible to have $SR^* < \widetilde{SR}$, i.e. the systemic risk in the regulated case can be higher than that in the regulation-free case.*

We summarize the impacts of imposing a capital requirement on individual risk, systemic linkage and systemic risk in the following theorem.

**Theorem 4.3** *Under the affine portfolio model of banking activities, when imposing a capital requirement, compared to the regulation-free case, we have that 1) the individual risk of each bank is lower; 2) the systemic linkage within the banking system is higher; 3) the systemic risk within the banking system is lower if the capital ratios of the two banks are sufficiently distinguished, i.e. $\frac{Q_1}{Q_2}$ is out of the range $(l, r)$, where $l$ and $r$ are determined by the risk aversion levels of the two banks as in (4.4). If $l < \frac{Q_1}{Q_2} < r$, it is possible that the systemic risk in the regulated system is higher.*

From Theorem 4.3, the systemic risk in the regulated system may achieve a higher level compared to that in the regulation-free case. In other words, the higher systemic linkage within the system imposed by micro-prudential regulation lead to a "cost" in terms of increasing systemic linkage, which offsets the "benefit" gained from lower individual risks. Whether the tradeoff at the systemic risk level is beneficial depends on whether $\frac{Q_1}{Q_2}$ is out of the range $(l, r)$. Hence, this can be regarded as a sufficient condition for having a successful micro-prudential regulation. We further discuss this sufficient condition.

Firstly, the validation of the sufficient condition requires a macro-prudential view on the system. By definition, both $l$ and $r$ are determined by the portfolio holding strategies of the banks, i.e. the asset side of the balance sheet. Meanwhile, $\frac{Q_1}{Q_2}$ is a comparison between the capital ratios of the two banks, i.e. the liability side of the balance sheet. Hence, the condition whether $\frac{Q_1}{Q_2}$ lies in between $l$ and $r$ is a comparison between the asset and liability sides of the balance sheets of the two banks. It can not be verified by having information on only one of the two banks or only one side of the balance sheets. Therefore, Theorem 4.3 demonstrates the potential limitation of micro-prudential regulation based on limited information. To overcome such a limitation it is necessary to have a helicopter view on the strategies and the liability compositions of all banks in the system. In other words, it is necessary to have a macro-prudential approach.

Secondly, we analyze when the sufficient condition holds. It is not difficult to verify that

$$\frac{\partial l}{\partial \lambda_1} > 0, \frac{\partial r}{\partial \lambda_1} < 0, \frac{\partial l}{\partial \lambda_2} < 0, \frac{\partial r}{\partial \lambda_2} > 0.$$

Thus, fixing $\lambda_2$, an increase in $\lambda_1$ would increase $l$ but decrease $r$. Notice that $\lambda_1 < \lambda_2$, increasing $\lambda_1$ is in fact reducing the heterogeneity between the risk aversion of the two banks. Similar result can be observed when fixing $\lambda_1$ and varying $\lambda_2$. We thus conclude that when reducing the heterogeneity between $\lambda_1$ and $\lambda_2$, the range of $(l, r)$ will be reduced. With a narrower range of $(l, r)$, it is more likely that the ratio $\frac{Q_1}{Q_2}$ falls out of the range. Hence, when the two banks are more homogeneous in terms of risk aversion, the capital requirement regulation may be more effective in reducing systemic risk. Such an observation comes from the following intuition. When the two banks are more similar in their asset allocations, their systemic linkage in the regulation-free case would be at a high level. Imposing the capital requirement increases the systemic linkage further. However, that is a relatively minor effect compared to the reduction on individual bank risk caused by imposing the regulation. Therefore, the tradeoff is eventually on the beneficial side: the systemic risk in the regulated case will be lower. Conversely, when the two banks are more heterogeneous in terms of asset decomposition, their systemic linkage in the regulation-free case would be at a low level. Then imposing a capital requirement regulation might increases the systemic risk because the impact on enhancing systemic linkage may dominate the impact on individual risk reduction.

Thirdly, we investigate an alternative way on reducing systemic risk, when the sufficient condition holds. If $\frac{Q_1}{Q_2} < l$, from (4.3), we get that

$$SR^* \approx A_1 \frac{(w_{21}^*)^\alpha}{Q_1^\alpha} \left(\frac{Q_1}{Q_2}\right)^\alpha + A_2 \frac{(w_{22}^*)^\alpha}{Q_1^\alpha} \left(\frac{Q_1}{Q_2}\right)^\alpha = \frac{A_1 (w_{21}^*)^\alpha + A_2 (w_{22}^*)^\alpha}{Q_2^\alpha}.$$

Symmetrically, when $\frac{Q_1}{Q_2} > r$,

$$SR^* \approx \frac{A_1 (w_{11}^*)^\alpha + A_2 (w_{12}^*)^\alpha}{Q_1^\alpha}.$$

Therefore, when $\frac{Q_1}{Q_2}$ falls out of the range $(l, r)$, the systemic risk in the regulation-free case mainly stems from the risk of one bank. In other words, one of the two banks is more *"systemically important"* than the other. In such a case, imposing a strict regulation to reduce the individual risk of the systemically important bank is effective in reducing systemic risk, even if the riskiness of the other bank stays at its initial level. Hence, even if a micro-prudential regulation is effective in reducing systemic risk, it is useful to identify systemically important financial institutions (SIFI) and imposing more strict rule on them. Obviously, to identify the SIFI in our model, it is again necessary to have an overview on both banks' asset and liability composition.

# 5 Policy discussions and extensions

This paper studies why imposing a micro-prudential regulation may not reduce systemic risk and maintain the stability of a banking system as it intends to. As an example of a micro-prudential regulation tool, we consider the capital requirement rule as in Basel II. With the static model, we demonstrate the impact of imposing a capital requirement: although it effectively reduces the individual risks, it also enhance the systemic linkage. The tradeoff may result in a higher systemic risk in the regulated system than that in a regulation-free system. If the liability sides of the balance sheets of the two banks are more heterogeneous than their asset sides, the systemic risk in the regulated system is always lower than that in the regulation-free system.

Throughout the paper, we consider capital requirement as the micro-prudential regulation rule. A system with such a regulation may have a higher systemic risk, because the regulation rule can override the risk appetite of individual financial institutions in guiding the formation of their portfolio holdings, and thus leads to a higher systemic linkage. This intuition is not limited to capital requirement regulation. It applies to all micro-prudential approaches based on a unified rule that applies to all financial institutions in a system. Therefore, we stress that the potential drawback raised in this study is a drawback of all micro-prudential regulations, rather than that of a particular micro-prudential tool.

Policy wise, the findings in this paper have the following implications. Firstly, our result shows the limitation of micro-prudential regulation and the necessity of having a macro-prudential regulation framework. Particularly, the model suggests that it is necessary to have a macro-prudential regulator holding a helicopter view on all financial institutions in the system. That includes monitoring banking activities as well as liability compositions of all financial institutions. Such a macro-prudential framework helps justify whether a (micro-prudential) regulation tool indeed help reduce systemic risk. It is worth mentioning that although a macro-prudential framework is necessary, we may not have to construct new "macro-prudential tools". With carefully monitoring the financial system from a macro-prudential view, the micro-prudential tools such as capital requirement may act as the practical tool for implementing regulations. In the end, a proper regulation scheme may consist of macro-prudential framework and micro-prudential tools: "macro-prudential" should be regarded as a general guidance on how to implement regulation tools, while the practical regulation tools can still be the same as in "micro-prudential" regulations.

Secondly, we also provide policy advice for regulating different types of financial systems. When regulating a system consisting of similar institutions, or in other words, the system is highly interconnected, considering a micro-prudential regulation can be sufficient for reducing the overall systemic risk. In contrast, the macro-prudential regulation is particularly important when regulating a diversified financial system consisting of heterogeneous financial institutions focusing on different banking activities. In both cases, strict regulations on SIFIs are effective in reducing systemic risk. A macro-prudential regulator is again necessary in order to identify SIFIs in the system.

We consider a few potential extensions of our model.

Firstly, the model is static, i.e. it only considers systemic risk in the cross-sectional dimension, without addressing the potential impact of micro-prudential regulation on the time dimension. This is due to the fact that we intend to focus on the cross-sectional dimension. On the time dimension, it has been well documented that current micro-prudential regulations may have a procyclicality problem. A counter-cyclical regulation is thus favored, i.e. the multiplier $d_j$ may vary according to macroeconomic environment. Considering such a variation may partially address some impact of imposing counter-cyclical, or in general time-varying, regulation. Notice that when $d_j$ is in a very low level, the regulation rule may not be effective, i.e. Assumption 3 may not be valid. Increasing $d_j$ to a high level makes the regulation rule effective. Therefore, increasing $d_j$ may mimic the procedure of imposing the regulation rule to a regulation-free system. According to our result, this may actually impose higher systemic risk. From the calculation of the systemic risk, when $d_j$ is sufficiently high for which the regulation is effective, further increasing $d_j$ will always reduce the systemic risk. Nevertheless, even in the latter case, the systemic risk is still possible to be higher than that in the regulation-free case. Therefore, the problem we raised based on systemic risk in the cross-sectional dimension can not be solved by considering counter-cyclical regulations.

It is thus necessary to analyze the overall impact of time-varying regulation rule on systemic risk in both dimensions. This is left for future research.

Secondly, our model assumes that the capital ratios are fixed, at least in short term. This assumption implies that in order to obey the regulation, a bank must adjust its portfolio holding. In reality, financial institutions may raise more capital to achieve the same goal. To relax this assumption, the corresponding extention of our framework is then to allow changes of $Q_1$ and $Q_2$. From (4.2), increasing capital ratio will decrease or maintain the level of systemic risk. Thus if possible, it is indeed better off increasing the capital ratios. However, changing $Q_1$ and $Q_2$ will correspondingly change the ratio $\frac{Q_1}{Q_2}$. A potential outcome is that the value of $\frac{Q_1}{Q_2}$ can move from out of the range $(l, r)$ to the inner part of $(l, r)$, or vice versa. This may change the stylized property on whether the systemic risk is lower for the regulated case than the regulation-free case. If financial institutions raise capital such that their liabilities compositions are similar, then although the systemic risk is reduced in absolute level, the systemic risk under regulation is still possible to be higher than that in the regulation-free case. Particularly, if both banks follow a minimal capital requirement as in Basel I, i.e. $Q_1 = Q_2$, since $l < 1 < r$, the sufficient condition for a lower systemic risk in regulated system is always violated. With such a regulation rule, it is likely that the systemic risk is higher in the regulated case.

The last potential extension is to consider the impact of fire sales. In this model, we do not consider the relation between banks' portfolio rebalance behavior and the value of the assets. When banks fire sale their assets to rebalance their portfolios, it is likely that the assets are further devalued which corresponds to a distributional change in the asset returns. Such an effect will enhance the systemic risk in a system with banks having similar asset allocation. Since we have shown that the systemic linkage in a regulated system is stronger than that in a regulation free system, taking fire sales effect in to consider will further enhance our result. For simplicity, we do not impose such a feature in the current study.

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# Appendix: Proofs

### Proof of Proposition 3.1

To solve the unconstrained utility maximization problem, we first find the explicit boundaries for the risk aversion levels under Assumption 1 and 2. It is presented as in the following lemma.

**Lemma A.1** *Assumption 1 and 2 are equivalent to the following inequality*

$$\frac{\mu_2 - \mu_1}{A_2} < \lambda_j < \left( \left( \frac{\mu_1}{A_1} \right)^{1/(\alpha-1)} + \left( \frac{\mu_2}{A_2} \right)^{1/(\alpha-1)} \right)^{\alpha-1}, \tag{A.1}$$

*for* $j = 1, 2$.

### Proof of Lemma A.1

For Bank $j$ with a portfolio $(w_{j1}, w_{j2})$, the marginal utility on asset $i$ is calculated as

$$MU_{j,i}(w_{j1}, w_{j2}) := \frac{\partial U_j}{\partial w_{ji}} = \mu_i - \lambda_j w_{ji}^{\alpha-1} A_i.$$

It is clear that, in case $w_{j1} = 1$, $w_{j2} = 0$, $MU_{j,1}(1,0) < MU_{j,2}(1,0)$. Thus, if the optimal portfolio weights correspond to a corner solution, it must be a corner solution with $w_{j1} = 0$ and $w_{j2} = 1$, i.e. the optimal portfolio assigns all weights to the more risky asset $R_2$. That implies $MU_{j,1} \leq MU_{j,2}$ for all $w_{j1} + w_{j2} = 1$. Due to the monotonicity of the two marginal utilities, we only need to check $MU_{j,1} \leq MU_{j,2}$ at the point $w_{j1} = 0$ and $w_{j2} = 1$, which leads to

$$\mu_1 \leq \mu_2 - \lambda_j A_2,$$

i.e. $\lambda_j \leq \frac{\mu_2 - \mu_1}{A_2}$. Therefore, the assumption that there is no corner solution is equivalent to $\lambda_j > \frac{\mu_2 - \mu_1}{A_2}$, which verifies the lower bound in (A.1).

For the upper bound, we consider the solution for utility maximization without restrictions on $w_{j1}, w_{j2}$, i.e. the solution of $MU_{j,1} = MU_{j,2} = 0$. That is

$$\bar{w}_{ji} = \left(\frac{\mu_i}{\lambda_j A_i}\right)^{1/(\alpha-1)}, \quad \text{for} \quad i = 1, 2.$$

Then, Assumption 2 implies that $\bar{w}_{j1} + \bar{w}_{j2} > 1$, which gives exactly the upper bound of $\lambda_j$ as in (A.1). □

From the proof of Lemma A.1, we get that with Assumption 1 and 2, or equivalently under condition (A.1), it is not possible to achieve $MU_{j,1} = MU_{j,2} = 0$ within the area $w_{j1}+w_{j2} < 1$. Thus, we consider the constrained utility maximization problem with $w_{j1} + w_{j2} = 1$. By the Lagrange multiplier method, we maximize

$$U'_j := U_j - K(w_{j1} + w_{j2} - 1).$$

Denote

$$MU'_{j,i} := \frac{\partial \tilde{U}'_j}{\partial w_{ji}} = \mu_i - \lambda_j w_{ji}^{\alpha-1} A_i - K.$$

By taking $MU'_{j,1} = MU'_{j,2} = 0$, we get that

$$\mu_1 - \lambda_j (w^*_{j1})^{\alpha-1} A_1 = \mu_2 - \lambda_j (w^*_{j2})^{\alpha-1} A_2.$$

Together with $w^*_{j1} + w^*_{j2} = 1$, we get that the optimal solution is given by first solving (3.2) and then taking $w^*_{j1} = 1 - w^*_{j2}$. Notice that the condition on $\lambda_j$, (A.1), ensures that there exists a unique solution $w^*_{j2}$ in $(0, 1)$. □

**Proof of Proposition 3.2**

Denote the optimal solution of the VaR-constrained utility maximization problem as $(\tilde{w}_{j1}, \tilde{w}_{j2})$. We first show that the optimal solution matches VaR constrain, i.e. $VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) = T_j$.

From Assumption 3, it is clear that $\tilde{w}_{j1} + \tilde{w}_{j2} < 1$. Suppose $VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) < T_j$, then a small variation on $\tilde{w}_{j1}$ can still obey the regulation rule. In order to have $(\tilde{w}_{j1}, \tilde{w}_{j2})$ as the optimal solution, the marginal utility $MU_{j,1}$ must be zero at $(\tilde{w}_{j1}, \tilde{w}_{j2})$. Similarly, we get $MU_{j,2} = 0$. According to the proof of Lemma (A.1), this can not be achieved in the area $w_{j1} + w_{j2} < 1$. Thus, by contradiction, we proved that

$$VaR_j(\tilde{w}_{j1}, \tilde{w}_{j2}) = T_j. \tag{A.2}$$

With Assumption 3, (A.2) automatically implies that $\tilde{w}_{j1} + \tilde{w}_{j2} < 1$. Thus, the VaR-constrained utility maximization problem turns to be a maximization problem on $U_j$ with the restriction (A.2) only. By the Lagrange multiplier method, we maximize

$$\tilde{U}_j := U_j - K'(VaR_j - T_j),$$

where $VaR_j$ is given in (3.3). Denote

$$\widetilde{MU}_{j,i} := \frac{\partial \tilde{U}_j}{\partial w_{ji}} = \mu_i - w_{ji}^{\alpha-1} A_i \left( \lambda_j + \frac{K'}{p^{\frac{1}{\alpha}} \left( A_1 \tilde{w}_{j1}^{\alpha} + A_2 \tilde{w}_{j2}^{\alpha} \right)^{1-\frac{1}{\alpha}}} \right).$$

By taking $\widetilde{MU}_{j,1} = \widetilde{MU}_{j,2} = 0$, we get that the optimal solution $(\tilde{w}_{j1}, \tilde{w}_{j2})$ should satisfy

$$\frac{\mu_1}{\mu_2} = \frac{A_1 \tilde{w}_{j1}^{\alpha-1}}{A_2 \tilde{w}_{j2}^{\alpha-1}}.$$

With the notation $e_i = (\mu_i / A_i)^{\frac{1}{\alpha-1}}$ for $i = 1, 2$, we get that

$$\frac{\tilde{w}_{j1}}{\tilde{w}_{j2}} = \frac{e_1}{e_2}.$$

Hence, the relative proportion between the two risky assets, $\frac{e_1}{e_2}$, is irrelevant to the risk aversion level $\lambda_j$, thus is bank irrelevant. The restriction on VaR determines the total investment, which results in the final solution as stated in the proposition. $\square$

**Proof of Lemma 4.1**

The calculation stems from a generalized version of the Feller convolution theorem (see Feller (1971) and Zhou (2010a)), which gives the tail property on the aggregation of independent heavy-tail distributed random variables. We sketch the calculation as follows. For extremely low $Q_1, Q_2$, i.e. $Q_j = O(VaR_j(p))$ as $p \to 0$, we have that

$$\begin{aligned} SR =& P(w_{11}R_1 + w_{12}R_2 < -Q_1, w_{21}R_1 + w_{22}R_2 < -Q_2) \\ \sim& P(w_{11}R_1 \wedge w_{12}R_2 < -Q_1, w_{21}R_1 \wedge w_{22}R_2 < -Q_2) \\ \sim& P\left(R_1 < -\left(\frac{Q_1}{w_{11}} \bigvee \frac{Q_2}{w_{21}}\right) \quad \text{or} \quad R_2 < -\left(\frac{Q_1}{w_{12}} \bigvee \frac{Q_2}{w_{22}}\right)\right) \\ \sim& A_1 \left(\frac{w_{11}}{Q_1} \wedge \frac{w_{21}}{Q_2}\right)^{\alpha} + A_2 \left(\frac{w_{12}}{Q_1} \wedge \frac{w_{22}}{Q_2}\right)^{\alpha} \square. \end{aligned}$$

**Proof of Proposition 4.2**

Firstly, in the regulated case, from Proposition 3.1, we get that

$$\frac{\tilde{w}_{11}}{\tilde{w}_{21}} = \frac{\tilde{w}_{12}}{\tilde{w}_{22}} = \frac{c_1}{c_2} = \frac{T_1}{T_2}.$$

In case $d_1 \geq d_2$, we have that $\frac{Q_1}{Q_2} \geq \frac{T_1}{T_2}$. From Lemma 4.1, the systemic risk measure in the regulated case is

$$\widetilde{SR} \approx A_1 \frac{\tilde{w}_{11}^{\alpha}}{Q_1^{\alpha}} + A_2 \frac{\tilde{w}_{12}^{\alpha}}{Q_1^{\alpha}} = d_1^{-\alpha} p.$$

Similarly, for the case $d_1 \leq d_2$ we have that $\widetilde{SR} \approx d_2^{-\alpha} p$. In all, we get that, for the regulated case,

$$\widetilde{SR} \approx (d_1 \vee d_2)^{-\alpha} p. \tag{A.3}$$

It implies that with imposing a capital requirement, the systemic risk measure is linked to the tail probability level considered in the regulation, $p$, and the maximum of the multipliers applied to the two banks.

Secondly, we calculate the systemic risk measure for the regulation-free case, $SR^*$. This is more complicated due to the lack of an explicit expression on $w^*_{ji}$ for $i, j = 1, 2$. However, because the solutions are in the regulation-free case, $\frac{w^*_{11}}{w^*_{21}}$ and $\frac{w^*_{12}}{w^*_{22}}$ are independent from $\frac{Q_1}{Q_2}$. Since $\frac{w^*_{11}}{w^*_{21}} < 1 < \frac{w^*_{12}}{w^*_{22}}$, we consider the three different cases.

**Case 1)** $\frac{Q_1}{Q_2} \leq \frac{w^*_{11}}{w^*_{21}} =: l$

In this case, we get that

$$SR^* \approx A_1 \frac{(w^*_{11})^\alpha}{Q_1^\alpha} + A_2 \frac{(w^*_{12})^\alpha}{Q_1^\alpha}.$$

Notice that the portfolio $(w^*_{11}, w^*_{12})$ does not satisfy the regulation rule. It implies that

$$\frac{A_1(w^*_{11}) + A_2(w^*_{12})^\alpha}{T_1^\alpha} > p.$$

Thus, $SR^* > d_1^{-\alpha} p$. Comparing with $\widetilde{SR}$ in (A.3), we get that $SR^* > \widetilde{SR}$. Hence the systemic risk is lower in the regulated case.

**Case 2)** $\frac{Q_1}{Q_2} \geq \frac{w^*_{12}}{w^*_{22}} =: r$

Similar to Case 1), we have in this case $SR^* > d_2^{-\alpha} p \geq \widetilde{SR}$. The systemic risk is also lower in the regulated case.

**Case 3)** $l < \frac{Q_1}{Q_2} < r$

In this case, we get that

$$SR^* \approx A_1 \frac{(w^*_{11})^\alpha}{Q_1^\alpha} + A_2 \frac{(w^*_{22})^\alpha}{Q_2^\alpha}.$$

We show that it is possible to have $SR^* < \widetilde{SR}$ by choosing particular values of the parameters.

Consider the case Bank 1 is extremely risk seeking and Bank 2 is extremely risk averse, i.e. $\lambda_1$ and $\lambda_2$ reach the lower bound and upper bound for $\lambda$ respectively. Then $(w^*_{11}, w^*_{12})$ is the riskiest corner solution $(0, 1)$ and $(w^*_{21}, w^*_{22})$ is the unrestricted solution of maximizing the utility as

$$w_{22}^* = \frac{\left(\frac{\mu_2}{A_2}\right)^{\frac{1}{\alpha-1}}}{\left(\frac{\mu_1}{A_1}\right)^{\frac{1}{\alpha-1}} + \left(\frac{\mu_2}{A_2}\right)^{\frac{1}{\alpha-1}}} = \frac{1}{1 + \left(\frac{\mu_1}{\mu_2}\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-1}}}.$$

For simplicity, we consider $d_1 = d_2 = d$. Then, the systemic risk measure is given as

$$SR^* \approx d^{-\alpha}\frac{A_2}{T_2^\alpha}\left(\frac{1}{1 + \left(\frac{\mu_1}{\mu_2}\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-1}}}\right)^\alpha.$$

Next, we find upper bound for $T_j$. From Assumption 3, for any $w_{j1} + w_{j2} = 1$,

$$\left(\frac{A_1 w_{j1}^\alpha + A_2 w_{j2}^\alpha}{p}\right)^{\frac{1}{\alpha}} > T_j.$$

It is not difficult to verify that the minimum of $A_1 w_{j1}^\alpha + A_2 w_{j2}^\alpha$ with constrain $w_{j1} + w_{j2} = 1$ is achieved at

$$w_{ji} = 1 - \frac{A_i^{\frac{1}{\alpha-1}}}{A_1^{\frac{1}{\alpha-1}} + A_2^{\frac{1}{\alpha-1}}},$$

for $i = 1, 2$. Thus we get the upper bound of $T_j$ as

$$T_j < \frac{(A_1 A_2)^{1/\alpha}}{p^{1/\alpha}\left(A_1^{\frac{1}{\alpha-1}} + A_2^{\frac{1}{\alpha-1}}\right)^{\frac{\alpha-1}{\alpha}}},$$

for $j = 1, 2$. We make further assumption that $T_2$ reaches its upper bound. Then,

$$\frac{1}{T_2^\alpha} = p\frac{\left(A_1^{\frac{1}{\alpha-1}} + A_2^{\frac{1}{\alpha-1}}\right)^{\alpha-1}}{A_1 A_2} = p\frac{\left(1 + \left(\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-1}}\right)^{\alpha-1}}{A_2}.$$

Lastly, we make assumption on the parameters $\mu_1, \mu_2, A_1, A_2$ as $\frac{\mu_1}{\mu_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha}}$. We get that

$$SR^* \approx d^{-\alpha} p \frac{\left(1 + \left(\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-1}}\right)^{\alpha-1}}{\left(1 + \left(\frac{A_2}{A_1}\right)^{1/\alpha}\right)^{\alpha}}.$$

Notice that $\frac{A_2}{A_1} > 1$. Thus for $\alpha > 1$, we have

$$\left(1 + \left(\frac{A_2}{A_1}\right)^{\frac{1}{\alpha-1}}\right)^{\alpha-1} < \left(1 + \left(\frac{A_2}{A_1}\right)^{\frac{1}{\alpha}}\right)^{\alpha}.$$

Together with (A.3), we get that $SR^* < d^{-\alpha} p = \widetilde{SR}$. Therefore, in the case $\frac{Q_1}{Q_2}$ is in $(l, r)$, it is possible that the systemic risk in the regulated system is higher than that in the regulation-free case. $\square$