Vector Autoregressive Models (VAR)

The vector autoregressive models (VAR) are systems of simultaneous equations that capture the existence of interdependence relations among variables, and allow to assess the impact of stochastic shocks on a specific system variable. As mentioned in the literature, despite limitations such as the absence of any structural or economic relations and the relatively high number of parameters to be estimated, VAR models are important instruments of analysis and forecasting, especially for short and medium term horizons.

The Central Bank, like the vast majority of its international peers, uses VAR models as an analytical tool and, especially, for inflation forecasting since the implementation of the inflation targeting framework, in June 1999. The information provided by the VAR models, along with the ones generated by other econometric tools, are important inputs to the decision-making process of the Monetary Policy Committee (Copom). Thus, considering that the economic system is dynamic, the models used in the Central Bank's inflation forecasts are constantly subject to improvements. The box "Autoregressive vector models" in the March 2008 *Inflation Report*, presented the VAR models used at the time. Considering recent developments, the aim of this box is to provide updated information on the set of VAR models currently in use by the Central Bank.

According to the well-known World's theorem, all stationary time series have a moving average representation[1], which can be approximated by an initial condition and an accumulation of new events, i.e., by an autoregressive representation. Unlike simple autoregressive models, autoregressive vectors have more explained variables (one per equation).

1/ For more details, see Diebold (1998).

In the system of equations that defines the VAR, the variables are treated symmetrically and, for each variable considered, includes an equation containing its lags and also lags of the other system variables. Sims (1980) introduced the autoregressive vectors as an alternative to structural macroeconomic models, which were formed mostly by a large amount of equations with theoretical restrictions difficult to be tested and that resulted in inaccurate forecasts. Having simple operation and well-evaluated forecasts, VAR models had good acceptance in the academic world as well as in central banks.

VAR models examine the relations among the variables by imposing few restrictions on the structure of the economy, which are, basically, the choice of variables and lags. All other characteristics of the models are determined, generally, by the sample considered. In fact, in general, the choice of lags is based on statistical tests. Currently, the VAR models used by the Central Bank are divided into two big groups: VARs with economic fundamentals and purely statistical VARs. In both cases, the VAR models generate inflation forecasts for free prices. Inflation forecasts for the full National Consumer Price Index (IPCA) are obtained by combining the inflation forecasts of free prices from the VARs with projections of administered prices inflation, estimated independently. These projections, together with information from small and medium size structural models, are intended to assist the decision-making of the Monetary Policy Committee.

VARs with economic fundamentals

VARs with economic fundamentals are, in turn, divided into two subgroups: (1) models estimated with monthly data and (2) models estimated with quarterly data. In each of the subgroups seven models are estimated: three traditional VARs, three Bayesian VARs with the Minnesota prior and a VECM (Vector Error Correction model, a way to estimate non-stationary series). The aggregate inflation forecast of each of the subgroups is given by the median forecast of the models within that subgroup. Except for the variables present in the VECM estimation, all endogenous variables are considered in first difference.

The monthly models, with a larger number of observations, are estimated from the beginning of 2000, avoiding a series of structural breaks, especially the transition of the Real Plan and the introduction of the inflation targeting framework. The nominal interest rate is given by the Selic rate held in the month, whereas the real interest rate discounts the inflation rate measured by the IGP-DI. The monetary aggregate is measured by the end-of-period M1. To select the number of lags the *Akaike* (AIC), *Schwarz* (SC) and *Hannan-Quinn* (HC) information criteria are used. Table 1 shows the set of models estimated with monthly data.

Table 1 – Monthly models

Denomination	Endogenous variables	Seasonal adjustment	Lags
VAR I	free prices, administered prices, exchange rate and real interest rate	yes	2
VAR II	free prices, administered prices, exchange rate, nominal interest rate, industrial production and money	yes	6
VAR III	free prices, nominal interest rate, exchange rate and industrial production	no	1
BVAR I	free prices, administered prices, exchange rate, nominal interest rate, industrial production and money	no	6
BVAR II	free prices, administered prices, exchange rate, nominal interest rate, industrial production and money	yes	6
BVAR III	free prices, administered prices, exchange rate and real interest rate	yes	2
VECM	free prices, nominal interest rate, exchange rate, industrial production and error correction	no	1

Table 2 – Quarterly models

Denomination	Endogenous variables	Seasonal adjustment	Lags
VART I	free prices, administered prices, real interest rate and exchange rate	yes	2
VART II	free prices, administered prices, nominal interest rate, exchange rate, industrial production and money	yes	1
VART III	free prices, administered prices, real interest rate, industrial production and risk premium	yes	1
BVART I	free prices, administered prices, real interest rate and exchange rate	yes	1
BVART II	free prices, administered prices, nominal interest rate, exchange rate, industrial production and money	yes	2
BVART III	free prices, administered prices, real interest rate and exchange rate	yes	1
VECMT	free prices, nominal interest rate, industrial production, exchange rate and error correction	no	2

Table 2 presents the set of models estimated with quarterly data[2]. To avoid small sample problems, these specifications use data beginning in late 1994 and trend *dummies* for the period immediately after the Real Plan. Except for the VECMT and some variables used in the VART III and BVART III models, all endogenous variables are considered in first

2/ The "T" after each acronym indicates that the model is estimated with quarterly data.

difference. In VART III, the risk premium, measured by the Emerging Markets Bond Index (EMBI + Br) as such as the real interest rate variable used in BVART III are treated in level. As in the monthly models, the nominal interest rate is given by the Selic rate held in the month, whereas the real interest rate discounts the inflation rate measured by the IGP-DI from the Selic rate. The monetary aggregate is measured by the end-of-period M1. As in the monthly VARs, the number of lags is chosen based on the *Akaike* (AIC), *Schwarz* (SC) and *Hannan-Quinn* (HQ) information criteria.

Statistical VARs

The statistical VARs do not have structural restrictions (of economic order) either in the short or long term, and use a large number of variables and different lags. The variables are divided into six groups of distinct indicators (economic activity, external sector, financial, pricing, monetary and shocks). An important criterion for the choice of these variables was their correlation with inflation indexes.

The Statistical VARs are estimated with monthly data and divided into two classes, each one estimated with a different approach. In each model of the first class, principal components are extracted from variables from all different groups of indicators, totalling 1,536 different models. In the second class, the models use principal components and selected variables. In this class, only three sets of indicators are used at a time, totalling 1,440 models. The aggregate inflation projection in each class is simply the median of the projections within the class. Table 3 presents the variables used in the estimations.

In the first class, the models use the principal components extracted from each group of indicators based on variables of different subgroups of indicators. For example, in the group of economic activity, two main components of energy are obtained (first and second components), taken from the three indices of electricity energy use, and two principal components of production, extracted from the capacity utilization, real GDP and industrial production. In the second class, models are formed from selected variables and convex combinations between the first two principal components of each group of indicators used.

Table 3 – Statistical models

Group of indicators	Selected variables
Economic activity	retail sales, three electricity use indicators, monthly industrial production, real GDP, capacity utilization, unemployment
External sector	VIX, Embi, exchange rate, U.S. PPI (all commodities), export price index, import price index, export *quantum* index, import *quantum* index
Financial	Selic rate, real Selic rate calculated in four ways (deflated by IGP and IPCA, with 3 and 12-month expectations), spreads over Selic, calculated for individuals, corporations, total and for credits with BNDES interest rate
Prices	administered prices, IGP-DI, IPC-BR, IPC-FIPE, free prices
Money	M1, M2, M3, M4, currency held by the public, monetary base and demand deposits
Shocks	CRB commodities index, electricity, gasoline, IPA-IPC, motor oil and petroleum

Composition of the projection

The final composition of the projections of all VAR models (economic and statistical) is a simple arithmetic average of the following six elements: (1) median of the monthly VAR and VECM models, (2) median of the monthly Bayesian VAR models, (3) median of the quarterly VAR and VECM models, (4) median of the quarterly BVAR models, (5) median of the first class of statistical VARs, and (6) median of the second class of statistical VARs.

Finally, this box shows how VAR models are being incorporated into the decision-making process of the Copom and presents a new class of VAR models, the statistical, indicating that the Central Bank models are under constant updates. In this sense, it contributes to increasing the transparency of the decision-making process and, therefore, the credibility of monetary policy.

References

DIEBOLD, Francis X. (1998). *Elements of Forecasting*. Ed.South-Western College Publishing.

SIMS, Christopher A. (1980). Macroeconomics and reality. *Econometrica* 48, p.1-48.