Forecasting the Yield Curve: a statistical model with macroeconomic variables

Research about term structure of interest rates (TSIR) basically rests on two classes of models, usually known as statistical models and equilibrium models. In the first group the TSIR is constructed through an interpolation process and forecasts are done using time series models. In the second group, the models incorporate equilibrium arguments, such as no-arbitrage, and forecasts are produced by the dynamics implied in the model.

Despite the lack of economic theory grounds, statistical models are preferred in practical problems due to their lesser estimation complexity. Following this trend, this box, based on Leite et al. (2009), presents a statistical model of the yield curve (premium/expectation model) that incorporates three ingredients: macroeconomic information, data collected in market survey and risk premium of forward rates. This model is used to forecast the Brazilian yield curve six months ahead and its performance is confronted with the performance of three other well known methods: the random walk, forward rate and Diebold and Li (2006) model (henceforth, DL model).[1]

Each new day the time to maturity of an ID future contract decreases by one day. Thus, the time to maturity of the yields changes from one observation to the next. In order to eliminate this variability of times to maturity, yields are interpolated yields the Svensson parametric model (1994):

1/ The database used is composed of 1day ID future yields extracted from contracts traded on the Brazilian Mercantile and Futures Exchange (BM&F) and inflation expectations (measured by the National Index of Consumer Prices), collected by the Central Bank, observed in the first working day of each month during the period of December 2002/December 2007.

$$R_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}\right) + \beta_{4t}\left(\frac{1-e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right). \quad (1)$$

Figure 1 – Evolution of spot rates

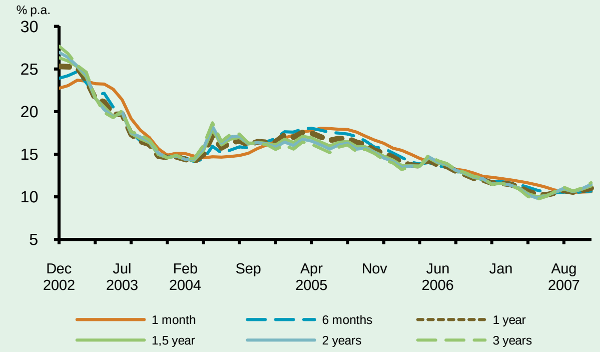
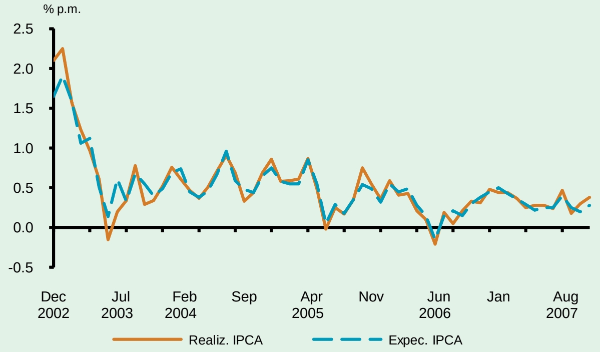


Figure 2 – Evolution of inflation and inflation expectations



The lambda (l) is fixed according to the approach suggested by Almeida et al (2007), which implies l_{1t} = 3.58 and l_{2t} = 7.17, for all *t*. Then, using OLS, beta coefficients (b) are estimated. Figure 1 shows the evolution of spot rates for maturities of 1 month, 6 months, 1 year, 1.5 years 2 years and 3 years over the sample period considered.

Figure 2 shows the evolution of the inflation expectation median of the institutions listed in the Top 5 group, as well as the observed inflation. It should be noted that the inflation rate refers to the month immediately previous to the date of collection of information on the ID future and inflation expectation. The two series present strong positive correlation and a preliminary analysis allows one to infer that inflation expectations would be a good leading indicator of actual inflation.

If R_t (t) is the spot rate at *t* for the time to maturity t and P(t, t) = exp {- Rt (t) t} is the value at *t* of one monetary unit at t + t, then the instantaneous forward rate at *t* for the term *t* is

$$f(t,\tau) = -\frac{\partial \log(P(t,\tau))}{\partial \tau} \ .$$

From Equation 1:

$$f(t,\tau) = \beta_1 + \beta_2 e^{-\lambda_1\tau} + \beta_3\lambda_1\tau e^{-\lambda_1\tau} + \beta_4\lambda_2\tau e^{-\lambda_2\tau}.$$

Considering the Selic rate as the instantaneous short-term rate, the risk premium of the forward rate should be defined as:

$$\eta(t,\tau) = f(t,\tau) - Selic_t$$

Ludvigson and Ng (2007) show that the risk premium return on bonds is strongly related to macroeconomic fundamentals such as price indices. Using this result, a linear relationship between the risk premium of the forward rate and the inflation expectation is specified:

$$\eta(t,\tau) = \gamma_1 IPCA_t + \gamma_0, \quad (2)$$

where the $IPCA_t$ is the inflation expectation measured by the IPCA in t for the first IPCA not yet announced.

The estimations and forecasts were performed using data from December 2002 and the month until the forecasting time. The forecasts for six months ahead start in December 2005 (forecast for June 2006), and extend until June 2007 (forecast for December 2007), with a total of 19 forecasts.

Using Equation 2 and the estimated values of g_0 and g_1, it is possible to forecast the forward rate risk premium six months ahead. To do that, it is necessary to know the expectation of the first unknown IPCA six months ahead. The forecast for the seventh unknown IPCA provided by the Central Bank survey is used as a proxy for this expectation. Then, to forecast yields it is also necessary to know the expectation of the Selic rate six months ahead. Once more, this variable is provided by the Central Bank survey.

Table 1 – Bias for out of sample forecasts (p.b.)

	6 months	1 year	1,5 year	2 years	3 years
PE	47	70	88	99	110
RW	-136	-120	-110	-104	-98
FR	-96	-101	-98	-96	-92
DL	24	51	-77	95	113

Table 2 – Mean-square error for out of sample forecasts

	6 months	1 year	1,5 year	2 years	3 years
PE	68	98	119	133	148
RW	144	139	144	151	162
FR	122	153	167	173	177
DL	129	143	158	169	181

Tables 1 and 2 present, respectively, the six month ahead forecasting bias and the RMSE, both in basis points, from the premium/expectation model and three other competitors: the random walk, the forecasts based on forward rates and the DL model. This latest forecasting technique adjusts the yield curve on each day via a parametric form similar to equation 1, but with only three factors (it does not consider the second curvature). The forecasts are performed by applying an autoregressive process for beta coefficients. The decay factor of the yield curve (l) was set at 3.58 for the DL model.

The results presented indicate that the premium/expectation model outperforms the three other competitors for all maturities when the comparison criterion is the RMSE. However, when bias (observed value minus forecast value) is used as a metric to compare the models, none of the models is clearly superior, despite the slight superiority of the DL model.

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