

Variance Decomposition of Rent-Installment Ratio in the Brazilian Real Estate Market

Waldery Rodrigues Júnior*

Senior Economist at IPEA (waldery.rodriques@ipea.gov.br), Professor at the Master in Finance Program-UnB

Pedro Henrique Melo Albuquerque

Researcher at IPEA (pedro.melo@ipea.gov.br), Professor at the Administration Department-UnB

Abstract

The interaction of spatial equilibrium model and financial economics results seems a very promising strand of research. In particular an integration of a financial-no-arbitrage condition (the indifference decision of a buyer-investor regarding real estate and alternative financial instruments) with a spatial-no-arbitrage condition (related to the indifference decision of a seller-renter regarding housing units) can lead to interesting results and clarification of some stylized facts in real estate markets. One quantitative approach used by Glaeser & Gyourko (2009) and others that has been used in this discussion is the decomposition of the rent-price ratio (R/P) variance very much in a similar way that Campbell-Shiller (1988,a,b) did for the stock and bond markets using the dividend/price (d/P) ratio. Our paper provides a first order Vector Auto-Regressive (VAR) for a close variable to R/P ratio. We work with the rent-installment (R/I) ratio. The installment variable is used in place of housing price given availability of data in the Brazilian case (there is no broad data base related to housing prices in Brazil for a substantive period of time). Besides the empirical feasibility aspect the way we model the R/I ratio gives an interesting theoretical opportunity to analyze the (no-arbitrage) decision of a buyer-renter while keeping most of the framework used to study the P/R ratio. The results of the R/I variance decomposition for Brazil points to a dominance of housing premia influence vis-a-vis other factors (interest rate, rent grow, income, employment). This is true for both studied periods (1982-1993 and 1996-2006). In line with results reported by Campbell, David, Gallin and Morris (2009) we find a small level of predictability (housing) returns compared to reported results in the financial literature. Our work is to the best of our knowledge the first to provide such decomposition for R/I in the Brazilian real estate market. Throughout the paper we compare our results to the relevant empirical papers on the issue of variance decomposition for R/P .

JEL Codes: R31 - Housing Supply and Markets, R12 - Size and Spatial Distributions of Regional Economic Activity, G12 - Asset Pricing; Trading volume; Bond Interest Rates, C32 - Time-Series Models

Key Words: Real Estate Markets, Spatial Elements, No Arbitrage Condition, Rent-Price Ratio, Rent-Installment Ratio, Variance Decomposition, VAR, Housing Premia, Brazilian Economy, Interest Rate.

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1 Introduction

The study of housing markets is a promising field for merging a financial and spatial approach to the problem. It can be stated that the analysis is incomplete if it is done with a narrow financial approach that did not take space into consideration and narrow spatial approach that do not account for financial aspects of housing assets. There is much scope for a merging between Space-Urban Economics and Financial Economics. In particular an integration of a financial-no-arbitrage condition (the indifference decision of a buyer-investor regarding real estate and alternative financial instruments) with a spatial-no-arbitrage condition (related to the indifference decision of a seller-renter regarding housing units) can lead to interesting results and clarification of some stylized facts in real estate markets. Regardless to say the housing markets is a very important one because it encompasses the dominant decision in the household portfolio.

The financial literature about the dynamics of housing prices is plenty of good references. Case and Shiller (1987, 1989, 1990) may pointed as the first to study housing price dynamics including statement of financial No-Arbitrage-Condition (NAC) when investors make a decision between investing in housing or other (financial) assets with equally risk-adjusted returns. NAC between the renter-owner was studied by Poterba (1984) and Henderson and Ioannides (1982).

In a spatial NAC Alonso (1964) and Rosen (1979) studied investment decision when there are similar gains from owning housing asset in different places.

Both strands of literature have come to show flaws of at least an incomplete analysis of the dynamics of the housing markets. For example the spatial equilibrium models for real estate imply that housing prices should be bigger for place with more amenities (such as a pleasant climate) but the models are not capable of stating where we are paying the fair value (are you overpaying or underpaying?).

By its turn the financial literature applied to the issue has its weakness. The predictability of housing prices has been documented since the work by Case and Shiller (1989) where finding of excess returns for housing investments goes against the Efficient Markets Hypothesis (EFM). However the P/R ratio predicted by the buyer-renter NAC may vary with some difficult to measure (unobserved factors) such as: the level of risk aversion (γ), the maintenance and depreciation involved in housing investments (δ), the growth rate of housing prices and rents, income volatility and other characteristics of the housing units (those used for renting are substantively different than those used for owning). For example, the modeling by Glaeser and Gyourko (2009) reports that itemizing owner-occupiers are willing to pay circa 40% more than landlords for the same property for a given level of (maintenance and depreciation) costs. In summary the models may fail to represent the fact that owned units and rented units as well as individual owners and renters are quite different elements/subjects.

These observation should make clear that there are implications for the analysis of rent-price (R/P) variance decomposition: demand schedules for owners and renters are related but not precisely comparable (not to mention they

cannot be assumed to be cointegrated series without careful analysis). The correlation between rents and prices over time may not be that high (as usually treated in the specialized literature).

The financial literature have not coped with the problems of limits to arbitrage, much in the sense proposed by Shleifer and Vishny (1997). Since equilibrium prices (for housing units) will be obtained when arbitrage conditions are ruled away the presence of limits to arbitrage may dampen the importance of current financial models prescription to real estate. The "postponing" strategies used for arbitrageurs in real estate (postponing a home purchase and remaining a renter, postponing a home sale and remaining a renter) may be severely limited by factors such as risk aversion or income or wealth volatility. Sinai (1997) reports that only 4% of owners actually become renters and uses the arbitrage channel (equating the owning returns to renting returns). The postponing strategy may not arbitrage away random shocks to housing markets.

Passing to the issue of variance decomposition it may be conjectured that similar to stock and bond markets where one can observe too much high frequency positive serial correlation in price changes and too much variation in stock prices relative to the fundamentals (dividends), the housing markets may shows too much variation in (high frequency) housing prices (P) and installment values (I) relative to the fundamentals (rents).

1.1 The Rent–Installment (R/I) Ratio Analysis

When come to the detailed quantitative analysis of housing markets some variables appear as key: housing prices, rent values, interest rates, risk premia, risk aversion for housing markets. In particular some quantitative analysis have studied the patter for the Rent-Price (R/P) Ratio. This is done very much in a similar way that Campbell-Shiller (1988,a,b) did for the stock and bond markets using the dividend/price (d/P) ratio. Our paper provides a first order Vector Auto-Regressive (VAR) for a close variable to R/P ratio for the Brazilian case.

We work with the rent–installment (R/I) ratio. The installment variable is used in place of housing price given availability of data (there is no broad data base related to housing prices in Brazil for a substantive period of time).

The way we study the R/I ratio gives a very interesting opportunity to analyze the (no-arbitrage) decision of a buyer-renter while keeping most of the framework used to study the P/R ratio.

The role played by data availability in our paper is key. The uses of installment data (I) let us proceed with the research. As a matter of fact it is much celebrated that some institution that will collect data regarding rent and housing prices will be needed for Brazil. See for example Folha de Sao Paulo (2010). One of the pros would be a better management of eventual housing price bubbles. Glaeser and Gyourko (2009) also points that most models will improve if researcher had better data on the income series of potential renters and new rental contracts.

The rest of paper is organized as follow: section 2 presents the model for understanding the relation between housing prices (or installment values) and

rents and the role played by risk aversion. Section 3 uses a dynamic version of the famous Gordon Growth Model to set up a framework for the variance decomposition of the I/R ratio. The next section discussed the Brazilian data used in our paper. Section 5 shows the results and Section 6 points to some preliminary conclusions.

2 Model

2.1 Owner-Renter Equilibrium Model

The essence of owner-renter No-Arbitrage-Condition (NAC) model is to let the decision makers (owners or renters) indifferent about owning a home unit or renting to someone else, i.e., consumers will receive the same return from both decisions. For the time being consider a representative risk neutral individual (risk aversion will be considered later). He/she will pay a property tax (ϱ) times the housing price (P_t) and the income (marginal) tax rate of τ will also be levied on the owner. Both property and interest payments are supposed to be deductible for owners-occupiers. The individual can borrow money at an interest rate of r . Suppose also that the individual earns an income of Y_t .

Condition 1 *The individual housing rents R_t which is a cost to the renter should be equal to the cost of housing, $(1 - \tau)(r + \varrho)P_t - (P_{t+1} - P_t)$*

Let $\delta \cdot H_t$ represents the net unobserved costs of being an owner-occupier (one's own landlord). Hence we have the following equation:

$$R_t = [(1 - \tau)(r + \varrho) + \delta] P_t - \underbrace{E_t(P_{t+1} - P_t)}_{\text{expected capital gain}}$$

This differential equation has a recursive structure.

Lemma 2 *Iterating forward and applying a transversality condition on P_t lead us to the traditional formula in deterministic settings that:*

$$P_t = \sum_{j=0}^{\infty} \frac{R_{t+j}}{[(1 - \tau)(r + \varrho) + \delta]^{j+1}}$$

Proof. Use the geometric series result for infinite sum (with factor < 1) and use the law of iterated expectations. ■

Since the stream of expected future rents ($R_t, R_{t+1}, R_{t+2}, \dots$) is unobserved it commonly used a law for formation of these future events such as $R_{t+1} = (1 + a)R_t$. Hence we are left with the following result.

Lemma 3 *under constant growth rate for rents we have that $P_t = \frac{R_t}{(1 - \tau)(r + \varrho) + \delta - a}$*

Remark 4 *The comparative statics of the equation above lead to reasonable results. In particular $\frac{\partial P}{\partial a} > 0$. Straightforward results can be obtained for $(R_t, \delta, r, \varrho, \tau, a)$.*

Remark 5 This result is similar to those obtained in traditional growth models (such as infinite horizon Ramsey models)

Remark 6 The upper bound \bar{a} for the rate of rent growth is $a < \bar{a} = (1 - \tau)(r + \varrho) + \delta$

Lemma 7 The same equation is obtained if one assumes (the milder condition) that housing prices P are expected to increase at the same rate a (period by period): $EP_{t+1} = (1 + a)P_t$. These means that we do not require individuals to be indifferent with respect to the no-arbitrage-condition owner-renter for their lifetimes (but only from time t to time $(t+1)$) or , in other words, that rents R will grow forever at the fixed rate a .

Hence we can state that $P = P(R_t, \delta, r, \varrho, \tau, a)$.

A baseline-exercise for the U.S. is provided by Glaeser and Gyourko (2009). It shows that assuming $[(\delta, r, \varrho, \tau, a) = (0.025, 0.055, 0.015, 0.25, 0.038)]$ we will obtain the price-rent ratio to be $\frac{P}{R} \simeq 25.3$.

Calibrating the parameters for the Brazilian economy one can obtain:

$[(\delta, r, \varrho, \tau, a) = (0.025, 0.02, 0.015, 0.30, 0.10)]$, $\frac{P}{R} \simeq 13.2$

Lemma 8 This leave us with the result that while one should expect the ratio of rental value to house price to be around 0.32% per month in the U.S. the Brazilian counterpart should be 0.61%

Proof. Let as an exercise to the reader. ■

Note the elasticity of the Price-Rent Ratio with respect to the value of the rent growth (appreciation) is a modest number ($\xi_{\frac{P}{R}, a}$ is around 1).

There is a vast discussion on the influence of the interest rate (r) on $\frac{P}{R}$. See Shiller (2005, 2006), Himmelberg et al (2005) and McCarthy and peach (2004) for further discussion of this issue.

One conjecture that can be checked for P e R series is: since the variance of income shocks for renters is a small fraction of the variance for owners (or for the whole population for that matter) than the rent series P should be more stable than the house prices. In the Brazilian case these should be true if on top of the usual economic conditions one adds the fact the most rent contract are set for a constant value for P in an annual basis.

2.2 No Arbitrage Condition for the Buyer-Investor

Let $\delta_I \cdot H_t$ represents the maintenance costs for the investor in housing units. Hence we have the following equation:

$$R_t + \underbrace{E_t(P_{t+1} - P_t)}_{\text{expected capital gain}} - (r + \varrho + \delta_I)P_t = \underbrace{0}_{\text{zero profit}}$$

3 Variance Decomposition of Installment-Rent (I/R) Ratio

In this section we analyze the variance decomposition used in finance literature and provide some twists for application in the real estate market. After we discuss the relevance of the concept of housing premia (associated with the market risk premium (MPR) of the CAPM model in finance literature). We close the section with the list of macroeconomic variables to be used in the empirical strategy.

3.1 Campbell-Shiller's Variance Decomposition Methodology and Application to the R/I Ratio

Definition 9 Consider the one-period gross real return to housing (φ_t) as $\varphi_{t+1} \equiv \frac{P_{t+1} + R_{t+1}}{P_t}$, where P is the real price of housing and R is housing rents.

As proceeded in Campbell, Davis, Gallin and Matin (2009, **CDGM hereafter**) we use the method of Campbell and Shiller (1988a,b) to rewrite this gross return and use as CDGM did a log-linear approximation as a methodology that gives the log of the rent-price ratio at any given date t .

Hence we have that $\log(R_t/P_t) \equiv r_t - p_t$, represents the expected net present value of all future real rates of return to housing and real growth in housing rents (for dates $t + 1 + j$ where $j = 0, \dots, \infty, j \in \mathbb{N}$)

$$r_t - p_t = k + E_t \left[\sum_{j=0}^{\infty} \rho^j \varphi_{t+1+j} \right] - E_t \left[\sum_{j=0}^{\infty} \rho^j \Delta r_{t+1+j} \right]. \text{ Note that there is}$$

no role for risk aversion in this set up.

$$\rho = \left(1 + e^{\overline{r-p}} \right)^{-1} = \frac{1}{1 + \left(\frac{R}{P} \right)}. \text{ One can note that this can be associated with}$$

the hypothesis of complete markets. Also note that the discount factor ρ is a constant.

$$k = (1 - \rho)^{-1} \left[\ln(\rho) + (1 - \rho) \ln \left(\frac{1}{\rho} - 1 \right) \right] = \frac{\ln \left[\rho \left(\frac{1}{\rho} - 1 \right)^{1-\rho} \right]}{1-\rho} = \frac{(1-\rho) \ln(1-\rho) + \rho \ln(\rho)}{1-\rho}$$

where φ is the log of the gross real return to housing,

r is the log of real housing rents,

ρ is a discount factor related to the average of the rent-price ratio (written as $e^{\overline{r-p}}$), and

k is a constant of linearization. Note that k will influence the variance decomposition in a very straightforward way.

Remark 10 Note that the value for the discount rate ρ is coherent with the value frequently used (around 0.9) when treatment present value models. In our case the value is around 0.5 but this is due to the fact that we are using the installment I instead of price P . As a matter of fact our simulations show that if we used $\varsigma \cdot I = P$ for a constant number of parcel ς with ς assuming value in the interval (70, 120) we obtain a discount factor of about 0.9.

Definition 11 *the real return to housing, φ , is the sum of a real risk-free interest rate, i , and the per-period housing premium over that rate (π).*

$$\varphi \equiv i + \pi$$

Given this representation we can write that the log rent-price ratio ($r - p$) or for that matter the log rent-installment ratio ($r - I$) can be rewritten as the sum of 3 components linked to:

- a) the expected present value of future real risk-free interest rates i_{t+1+j} ,
- b) housing premia π_{t+1+j} , and
- c) rent growth Δr_{t+1+j} .

Then we may write the following equation:

$$r_t - p_t = k + \underbrace{E_t \left[\sum_{j=0}^{\infty} \rho^j i_{t+1+j} \right]}_{=\mathfrak{I}_t} + \underbrace{E_t \left[\sum_{j=0}^{\infty} \rho^j \pi_{t+1+j} \right]}_{=\Pi_t} - \underbrace{E_t \left[\sum_{j=0}^{\infty} \rho^j \Delta r_{t+1+j} \right]}_{=\mathfrak{D}_t} = k + \mathfrak{I}_t + \Pi_t - \mathfrak{D}_t$$

Remark 12 *Very similar formulae may be written for $r_t - I_t$ if we used $\varsigma \cdot I = P$ and make a (linear) relation to $r_t - p_t$. The empirical part will require the estimation of the parcel ς . Besides that note the mathematical statistics is straightforward.*

Remark 13 *This setup can be compared to the traditional classic Gordon growth model where the growth rates (Δr) and the rates of return ($\varphi \equiv i + \pi$) are constant. This imply that $\frac{R}{P} = i + \pi - \Delta r$.*

Regarding real estate finance applications one can rewrite the above equation (for the level of the rent-price ratio) as $\frac{R_t}{P_t} = i_t + \pi - g_{t+1}$

where i_t is some current real interest rate, π is an assumed **constant** housing premium over this rate and g_{t+1} is the expected capital gain or loss to housing over some future horizon.

3.2 Construction of the Empirical Strategy

Let use data on house prices, rents, interest rates, and various macroeconomic variables (x_t) to construct time-series estimates of the vector $(\mathfrak{I}_t, \Pi_t, \mathfrak{D}_t)$ using a first order VAR as done in CDGM paper and others (see Campbell (1991) and Campbell and Ammer (1993)).

Criterion 14 $Z_t = (i_t, \pi_t, \Delta r_t, x_t')'$

where $x_t = (Y_t, L_t, N_t)$ is a column vector that contains other macro variables such income Y and employment L (in CDGM paper they also used population N. See a note on this below). Those macro variables are for predicting the vector $(i_t, \pi_t, \Delta r_t)$.

Since $Z_t \sim$ first-order VAR we have:

$$Z_t = Z_{t-1} + \varepsilon_t$$

The estimates of the discounted present values of the vector $(\mathfrak{S}_t, \Pi_t, \mathfrak{D}_t)$ are the first three elements of expression

$$\widehat{A} \left(I - \rho \widehat{A} \right)^{-1} Z_t$$

where:

- a) The symbol $\widehat{}$ denotes estimation,
- b) The Identity Matrix is represented by $I_{n \times n}$.
- c) the discount factor ρ is a constant.

The estimates obtained with the VAR strategy is represented by henceforth by $(\widehat{\mathfrak{S}}_t, \widehat{\Pi}_t, \widehat{\mathfrak{D}}_t)$.

Criterion 15 *The dynamic accounting identity represented by $r_t - p_t = k + \mathfrak{S}_t + \Pi_t - \mathfrak{D}_t$, lead us to state that the vector of variables $(\mathfrak{S}_t, \Pi_t, \mathfrak{D}_t)$ can fully describe the rent-price ratio $r_t - p_t$ up to a constant of linearization (k). Similar result can be stated regarding the dynamic path for $r_t - I_t$ given the linear relation involved in $\varsigma.I = P$.*

In the case of not so perfect foresight a discrepancy (e_t) between the estimated R/P ratio and the actual R/P ratio will be in place. Formally we have that:

Definition 16 *The forecast discrepancy e_t is implicitly defined as $\widehat{r_t - p_t} = r_t - p_t + e_t$ where $\widehat{r_t - p_t} = k + \widehat{\mathfrak{S}}_t + \widehat{\Pi}_t - \widehat{\mathfrak{D}}_t$ and $e_t = \widehat{\Pi}_t - \widehat{\mathfrak{D}}_t$*

Condition 17 *(Residual Condition): The present value of future rent growth $\left(E_t \left[\sum_{j=0}^{\infty} \rho^j \Delta r_{t+1+j} \right] \right)$ is obtained as a residual.*

This is in line with what is common practice in the specialized literature and allow for comparison of our results to the results obtained for stock and bond markets.

Definition 18 *Let $\xi_t = \widehat{\mathfrak{D}}_t + e_t$.*

Hence we can obtain that $r_t - p_t = k + \widehat{\mathfrak{S}}_t + \widehat{\Pi}_t - \xi_t$. This will be used

henceforth in the empirical strategy (and it will give consistent results due to the use of the accounting identity for housing premia).

Accordingly the variance decomposition for $r_t - p_t$ is given by:

$$var(r_t - p_t) = var(\widehat{\mathfrak{S}}_t) + var(\widehat{\Pi}_t) + var(\xi_t) + 2cov(\widehat{\mathfrak{S}}_t, \widehat{\Pi}_t) - 2cov(\widehat{\mathfrak{S}}_t, \xi_t) - 2cov(\widehat{\Pi}_t, \xi_t)$$

3.3 Housing Premia Definition and Use

Definition 19 *The real return to owner-occupied housing is given by $\varphi_t \equiv \frac{R_t + P_t - P_{t-1}}{P_{t-1}}$*

Note the presence of the term P_{t-1} in the numerator.

The housing premia (sometimes called the excess return to housing investment above an investment in a risk-free government bond with an appropriate maturity) may be stated as $\pi_t = \varphi_t - i_t$

Here we represent:

- i) R_t as the real (inflation-adjusted) rent accruing to homeowners between $t+1$ and t ;
- ii) P_t is the real price of housing,
- iii) i_t as the real return (yield) in the government bond (or note) as of the same maturity (t).

There is no adjustment for taxes or depreciation as is proceeded in the specialized finance literature (which computes pre-tax returns).

We also compute (see the empirical part) the first lag auto-correlation coefficient (also represented by ρ , so be careful for not making a confusion with the constant discount rate).

4 Specification and Estimation of the First Order VAR

For each market, the forecasting equations for $(i_t, \pi_t, \Delta r_t)$ are:

$$\begin{cases} i_t = \delta_0 + \delta_{\Delta r} \Delta r_{t-1}^{US} + \delta_{\pi} \pi_{t-1}^{US} + \delta_i i_{t-1} \pi + \delta_{\Delta Y} \Delta Y_{t-2}^{US} + \delta_{\Delta L} \Delta L_{t-2}^{US} + \delta_{\Delta N} \Delta N_{t-2}^{US} + \varepsilon_t^i \\ \Delta r_t = \gamma_0 + \gamma_{\Delta r} \Delta r_{t-1} + \gamma_{\pi} \pi_{t-1} + \gamma_i i_{t-1} \pi + \gamma_{\Delta Y} \Delta Y_{t-2} + \gamma_{\Delta L} \Delta L_{t-2} + \gamma_{\Delta N} \Delta N_{t-2} + \varepsilon_t^{\Delta r} \\ \pi_t = \beta_0 + \beta_{\Delta r} \Delta r_{t-1} + \beta_{\pi} \pi_{t-1} + \beta_i i_{t-1} \pi + \beta_{\Delta Y} \Delta Y_{t-2} + \beta_{\Delta L} \Delta L_{t-2} + \beta_{\Delta N} \Delta N_{t-2} + \varepsilon_t^{\pi} \end{cases}$$

The necessary information for closing the VAR are:

$$\begin{cases} \left(\underbrace{(\Delta r_t^{US}, \pi_t^{US})}_{\text{current}}, \underbrace{(\Delta Y_t, \Delta L_t, \Delta N_t)}_{\text{current}}, \underbrace{(\Delta Y_t^{US}, \Delta L_t^{US}, \Delta N_t^{US})}_{\text{current}} \right) \\ \left(\underbrace{(i_{t-1}, \Delta r_{t-1}^{US}, \pi_{t-1}^{US})}_{\text{lag 1}}, \underbrace{(\Delta Y_{t-1}, \Delta L_{t-1}, \Delta N_{t-1})}_{\text{lag 1}}, \underbrace{(\Delta Y_{t-1}^{US}, \Delta L_{t-1}^{US}, \Delta N_{t-1}^{US})}_{\text{lag 1}} \right) \end{cases} \text{ and}$$

5 Data for the Brazilian Case

a) Housing Prices and Installment Values: from PNAD-IBGE. The work of extraction price data is the most demanding of all series used in our work. The complete methodology is available upon request to the authors.

b) Income: also from PNAD-IBGE.

c) Originally CDGM used the Risk Free Treasury Bills and Inflation Expectations (of the same maturity). Given the absence of data for inflation expectation

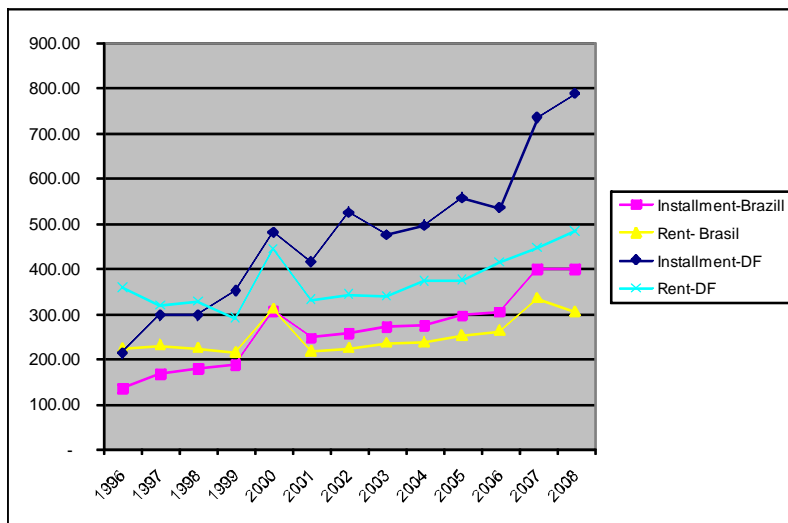


Figure 1:

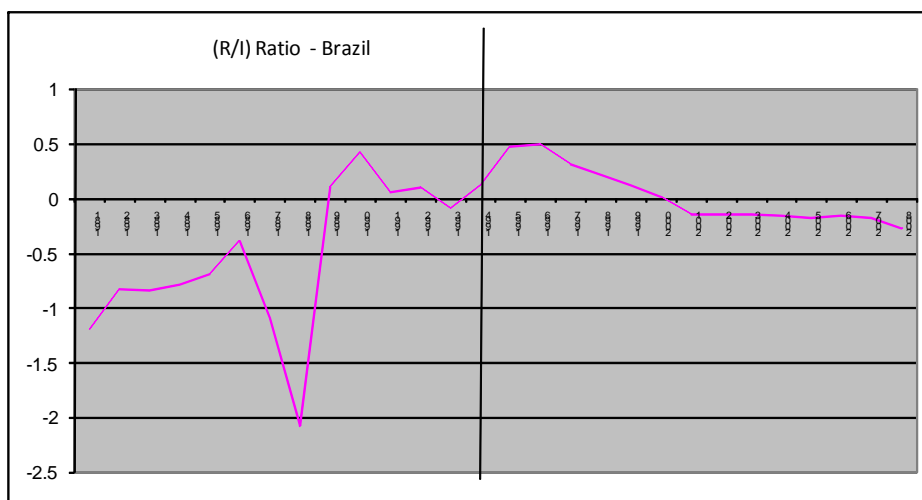


Figure 2:

before year 2000, we worked with the nominal real basic interest rate for Brazil (SELIC)

d) Housing Premia: defined accordingly to CDGM's methodology.

e) Macroeconomic Conditions: we used employment and level of income. The series on population was originally used in Glaeser & Gyourko (2009). Due to high correlation of this series with the employment and income we let population series out of the model.

f) There are many missing data for the year 1993 and 1994. Since the year 1994 is also the starting one for the Real plan (a successful stabilization plan for Brazil) we decided to divide the analysis in two periods: 1982-1993 and 1996-2008

Note: the magnitude of the parameter $\delta = 0.0025$ is assumed to be a good bet for the Brazilian economy during the period under analysis.

6 Results

6.1 Summary of Data and Descriptive Statistics

Below are descriptive statistics (average, standard deviation and auto-correlation coefficient) for the annualized real growth rate of rents Δr_t (or DR), the real annualized return to housing φ_t (PHI), and the excess return to housing or housing premia π_t (PI). Alongside we also have the same descriptive statistics for the macro variables (income Y and employment L).

Shown in columns 4 is the real housing returns in the aggregate averaged (Brazil): 10.37% per year. The excess returns averaged about -0.58%. Compare this to CDGM reported results to the U.S.: 6.47% and 2.99% respectively.

The results across the states varied severely (much more than the results for the U.S.). This is true for both φ_t (PHI) and the excess return to housing or housing premia π_t (PI). As straightforward to add a commentary that the level of uncertainty in the Brazilian economy is a major factor for these results (and the hyperinflation of the 80's is a point in case).

The results for the first-order auto-correlation ρ in the Brazilian economy also is in sharp contrast with the reported results for the American economy: there ρ is stable around the value of (positive) 0.6. In Brazil the value of the persistence coefficient was in general negative (but is rather uniform cross-states!).

6.2 First Order VAR: Brazil

Below we list some of the VAR results for Brazil. For the other 27 sub-national entities the data is available upon request to the authors.

Note that we obtain a smaller level of predictability returns (compared to reported in the financial literature). That can be seen by the values for the ρ parameters (Wald Test).

Location	AVG_I	AVG_D	AVG_P	AVG_I	AVG_D	SD_PI	SD_DR	SD_PHI	SD_DY	SD_DL	rho_PI	rho_DI	rho_DP	rho_DI	rho_PI
Acre	10.74	0.02	19.38	0.02	0.01	72.27	0.23	73.74	0.21	0.01	-0.38	-0.54	-0.44	-0.63	-0.40
Alagoas	-1.56	0.03	18.01	0.03	0.00	23.75	0.26	62.49	0.23	0.00	-0.10	-0.55	-0.47	-0.48	-0.03
Amapá	-1.72	0.04	8.68	0.03	0.01	18.30	0.26	18.93	0.22	0.01	-0.16	-0.53	-0.51	-0.15	-0.21
Amazonas	-2.71	0.02	5.75	0.02	0.00	20.38	0.21	19.61	0.20	0.01	-0.14	-0.47	-0.50	-0.46	-0.23
Bahia	2.65	0.03	18.32	0.03	0.00	40.68	0.25	55.36	0.22	0.00	-0.29	-0.61	-0.47	-0.49	-0.20
Brasil	-0.58	0.03	10.37	0.03	0.00	25.17	0.25	27.97	0.21	0.00	-0.07	-0.52	-0.46	-0.49	-0.10
Ceará	-2.82	0.04	8.96	0.03	0.00	19.52	0.28	25.21	0.23	0.00	-0.05	-0.53	-0.47	-0.35	-0.07
Distrito Federal	1.46	0.04	13.51	0.03	0.00	26.99	0.27	33.38	0.19	0.00	-0.44	-0.58	-0.48	-0.41	-0.36
Espírito Santo	-1.89	0.05	7.25	0.03	0.00	23.38	0.30	23.12	0.21	0.00	-0.04	-0.56	-0.48	-0.48	-0.09
Goiás	96.62	0.02	112.99	0.03	0.00	509.45	0.16	532.20	0.22	0.00	-0.48	-0.51	-0.75	-0.49	-0.48
Maranhão	-0.38	0.04	9.59	0.03	0.00	26.34	0.28	27.67	0.24	0.00	-0.46	-0.54	-0.49	-0.70	-0.47
Mato Grosso	-0.96	0.03	7.89	0.03	0.00	24.74	0.23	25.54	0.20	0.00	-0.07	-0.55	-0.48	-0.35	-0.12
Mato Grosso do Sul	0.23	0.03	10.56	0.03	0.00	30.66	0.25	31.74	0.21	0.00	-0.13	-0.51	-0.48	-0.49	-0.15
Minas Gerais	-1.61	0.04	7.64	0.03	0.00	24.60	0.26	24.56	0.21	0.00	-0.07	-0.52	-0.47	-0.53	-0.12
Paraná	-0.11	0.04	10.30	0.03	0.00	27.03	0.26	28.54	0.21	0.00	-0.05	-0.53	-0.46	-0.59	-0.07
Paraíba	281.77	0.04	282.19	0.03	0.00	1420.59	0.26	1397.42	0.24	0.00	-0.51	-0.51	-0.51	-0.49	-0.51
Pará	4.74	0.03	16.49	0.03	0.00	40.16	0.23	44.91	0.21	0.01	-0.64	-0.54	-0.46	-0.50	-0.57
Pernambuco	0.87	0.04	12.10	0.03	0.00	29.26	0.28	32.72	0.22	0.00	-0.16	-0.54	-0.47	-0.52	-0.16
Piauí	-1.01	0.04	7.89	0.04	0.00	26.77	0.27	26.36	0.24	0.00	-0.12	-0.52	-0.48	-0.44	-0.18
Rio Grande do Norte	0.96	0.04	12.12	0.03	0.00	29.33	0.27	32.42	0.23	0.00	-0.12	-0.50	-0.46	-0.48	-0.13
Rio Grande do Sul	2.44	0.03	14.91	0.03	0.00	33.72	0.23	39.90	0.21	0.00	-0.23	-0.50	-0.46	-0.39	-0.20
Rio de Janeiro	-1.91	0.03	9.28	0.03	0.00	23.96	0.22	27.11	0.21	0.00	-0.13	-0.53	-0.45	-0.28	-0.16
Rondônia	1.40	0.02	14.66	0.02	0.01	28.90	0.23	38.33	0.20	0.01	-0.33	-0.43	-0.47	-0.53	-0.26
Roraima	2.06	0.04	15.20	0.03	0.01	25.65	0.25	35.21	0.20	0.01	-0.23	-0.58	-0.51	-0.42	-0.21
Santa Catarina	10.48	0.04	22.93	0.03	0.00	67.81	0.26	70.23	0.21	0.00	-0.45	-0.51	-0.46	-0.36	-0.46
Sergipe	2.96	0.03	11.72	0.04	0.00	29.11	0.25	30.35	0.22	0.00	-0.44	-0.52	-0.48	-0.19	-0.46
São Paulo	-1.91	0.02	11.92	0.02	0.00	23.49	0.21	35.87	0.20	0.00	-0.09	-0.51	-0.46	-0.28	-0.07
Tocantins	29.37	0.06	0.50	0.06	-0.01	99.43	0.28	1.59	0.26	0.06	0.42	0.07	0.07	-0.03	0.39

Figure 3:

Simple Summary Statistics							
Variable	Type	N	Mean	Standard Deviation	Min	Max	Label
I	Dependent	13	0.01497	0.00377	0.00936	0.02078	Real risk-free interest rate
PI	Dependent	13	-0.23376	0.51868	-0.81341	0.93788	Per-period premium
DR	Dependent	13	0.00757	0.03174	-0.06331	0.06957	Growth rates of the log of real housing rents
DY	Dependent	13	0.01445	0.03594	-0.06873	0.10366	Per-capita income growth
DL	Dependent	13	0.00121	0.00102	-0.00122	0.00285	Employment growth

Figure 4:

Model Parameter Estimates						
Equation	Parameter	Estimate	Standard Error	t Value	Pr > t	Variable
I	AR1_1_1	1.01420	0.09501	10.67	0.0001	I(t-1)
	AR1_1_2	0.00548	0.00290	1.89	0.1011	PI(t-1)
	AR1_1_3	-0.05504	0.08124	-0.68	0.5199	DR(t-1)
	AR1_1_4	-0.01835	0.05938	-0.31	0.7664	DY(t-1)
	AR1_1_5	0.78293	0.78007	1.00	0.3490	DL(t-1)
PI	AR1_2_1	-22.62274	22.79071	-0.99	0.3540	I(t-1)
DR	AR1_2_2	-0.39789	0.69606	-0.57	0.5854	PI(t-1)
	AR1_2_3	12.21705	19.48881	0.63	0.5506	DR(t-1)
	AR1_2_4	-7.93021	14.24451	-0.56	0.5951	DY(t-1)
	AR1_2_5	7.47265	187.12263	0.04	0.9693	DL(t-1)
	AR1_3_1	-0.57263	1.02939	-0.56	0.5953	I(t-1)
	AR1_3_2	-0.03498	0.03144	-1.11	0.3026	PI(t-1)
	AR1_3_3	0.55395	0.88025	0.63	0.5491	DR(t-1)
	AR1_3_4	-0.51158	0.64338	-0.80	0.4526	DY(t-1)
DY	AR1_3_5	7.30218	8.45176	0.86	0.4162	DL(t-1)
	AR1_4_1	-0.35804	1.00498	-0.36	0.7321	I(t-1)
	AR1_4_2	-0.03589	0.03069	-1.17	0.2805	PI(t-1)
	AR1_4_3	1.47124	0.85938	1.71	0.1306	DR(t-1)
	AR1_4_4	-1.35274	0.62812	-2.15	0.0682	DY(t-1)
DL	AR1_4_5	14.82161	8.25134	1.80	0.1155	DL(t-1)
	AR1_5_1	0.07211	0.02855	2.53	0.0395	I(t-1)
	AR1_5_2	-0.00099	0.00087	-1.14	0.2915	PI(t-1)
	AR1_5_3	0.01724	0.02441	0.71	0.5030	DR(t-1)
	AR1_5_4	-0.00899	0.01784	-0.50	0.6297	DY(t-1)
	AR1_5_5	-0.01296	0.23441	-0.06	0.9575	DL(t-1)

Figure 5:

				Contribution (%)		
NAME_	IT	PIT	GT	IT	PIT	GT
IT	0.0567	7.7462	0.0595	0.0%	0.9%	0.0%
PIT	7.7462	1797.9950	15.0010		100.8%	-1.7%
GT	0.0595	15.0010	0.1332			0.0%

Figure 6:

				Contribution (%)		
NAME_	IT	PIT	GT	IT	PIT	GT
IT	3.98E-05	-0.00072	-8.4E-06	0.2%	-5.5%	-0.1%
PIT	-0.00072	0.031351	0.002104		120.0%	-16.1%
GT	-8.4E-06	0.002104	0.00037			1.4%

Figure 7:

6.3 Variance Decomposition (in %): Brazil 1982-1993

Note that the sum of the reported 6 elements in the last 3 columns sums up to 100%.

6.4 Variance Decomposition (in %): Brazil 1996-2008

Note that the sum of the reported 6 elements in the last 3 columns sums up to 100%.

6.5 Main Results Regarding Variance Decomposition for Brazil

Note the contribution of the housing premia: it is the dominant source of variation for both the 1982-1993 and 1996-2008. This is similar to the results obtained by Glaeser & Gyourko (2009) for the pre-boom period therein analyzed.

The results for the forecast discrepancy $e_t = \widehat{\Pi}_t - \widehat{\mathcal{D}}_t$ reported in columns 3 and 6 shows that it plays only a small role. Housing premia is "king". The contribution of the covariances shares are much smaller than those by the variances shares. These two results are in sharp contrast with the results reported for the U.S. Also different than the Glaeser & Gyourko (2000) results it is not true that the covariance always dampens the total volatility of rent-price ratios in the Brazilian economy (it is happened in the U.S. economy).

6.6 Variance Decomposition (in %) for each of the 27 States for both 1982-1993 and 1996-2008

These results are available upon request. They show similar pattern as those reported above for Brazil.

7 Conclusions

It is regarded as highly promising a conjoint study of house markets variables using: i) the spatial approach and ii) the financial approach. The interaction of the two corresponding No-Arbitrage Conditions (NACs) for the owner-seller and the renter-investor may bring valuable insights in the realm of real estate market studies. Some stylized facts in this market may be clarified or even solved but this merger.

From the financial literature some tools have been applied to describe the variance decomposition of the rent-price ratio (R/P) in a way similar to what Campbell-Shiller (1988,a,b) and others have done for the stock and bond markets using the dividend/price (d/P) ratio.

Our paper provided a novice first order Vector Auto-Regressive (VAR) for a close variable to R/P ratio for the Brazilian case. It is the first one to analyze the rent-installment (R/I) ratio. The installment variable is used in place of housing price given availability of data since there is no broad data base related to housing prices in Brazil for a substantive period of time.

The way we study the R/I ratio gives a very interesting opportunity to analyze the (no-arbitrage) decision of a buyer-renter while keeping most of the framework used to study the P/R ratio as is done in papers such as CDGM.

The results of the R/I variance decomposition points to a dominance of housing premia influence for the variance decomposition of the P/R ratio for both studied periods (1982-1993 and 1996-2006).

This can be compare to the results obtained by Campbell et al (2009) where the housing premia was the dominant effect for the variance decomposition of R/P ratio for the pre-boom period in the U.S. and it was a very important factor (accounting for about 20%) of the variance of R/P in the postboom period. Note also that we obtain a smaller level of predictability returns (compared to reported in the financial literature).

The results for the forecast discrepancy $e_t = \widehat{\Pi}_t - \widehat{\mathcal{D}}_t$ showed that it played only a small role. Housing premia is "king" in explaining variance decomposition for R/I ration in Brazil.

The contribution of the covariances shares are much smaller than those by the variances shares. These two results are in sharp contrast with the results reported for the U.S. Also different than the Glaeser & Gyourko (2000) results it is not true that the covariance always dampens the total volatility of rent-price ratios in the Brazilian economy (it is happened in the U.S. economy).

This work is a first one in a role for discussing these important and complex issues. There is a scarce scientific literature in Brazil discussing the main facts

of the real estate markets. That happens for many reasons and one that should be cited is the scarcity of data. It is also noticeable the need for using multidisciplinary tools (drawn from diverse areas of study: urban economics, financial economics, macroeconomics, time series econometrics, etc).

Hence further research in this topic is needed and guaranteed. We can cite two that are straightforward needs:

a) Further Research: formally add some spatial elements in the (Bi-Variate) No-Arbitrage Condition Model. Since we have the information for the 27 sub-national entities it is hard to make a point that one state influence another regarding housing purchase-sale decisions and the correspondent prices. But when come to districts, municipalities and areas within a cities (accordingly to IBGE's classification in "setor censitário") one may hope for getting spatial element to really influence the R/P or R/I ratios for the its variance decomposition.

b) Given the structural long-term path for a (arguably) descending nominal basic interest rate (SELIC) what can one expect for the behavior of the R/P and R/I ratios? Can this be associated with credit availability enhancement (brought about the falling SELIC and other structural factors)?

7.1 Conjecturing on Some Real Estate Variables Patterns

By close analysis of the series used in this paper we can point to a result that is intriguing. The relation between the (R/P) and the growth of per capita income may shows that they are cointegrated. Hence we have the following:

Conjecture 20 *Does $(R/P)/\Delta Income$ and $(R/I)/\Delta Income$ follows a Random Wald in any country?*

For the American economy casual inspection may force the answer to "yes".

Conjecture 21 *The behavior of the Rent/Price or Rent/Installment ratio is likely to follow a Inverted-U-Shape with respect to Income.*

Again for the American economy casual inspection may force the answer to "yes". Above is illustrated the case for the metropolitan statistical area (MSA) of Chicago.

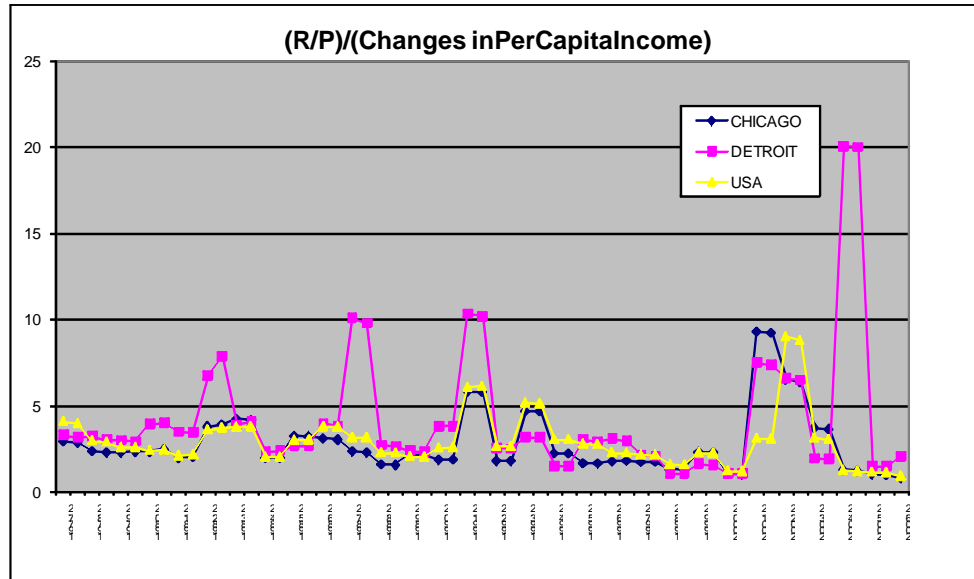


Figure 8:

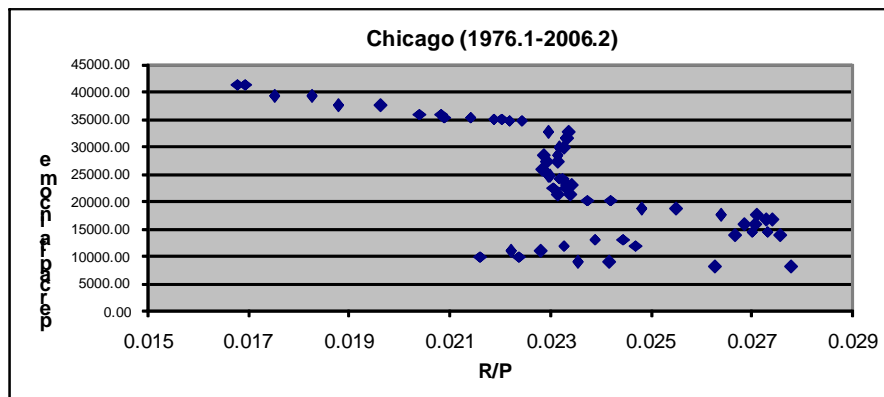


Figure 9:

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9 Appendix: SAS Codes

Below is the SAS code used to extract the information regarding housing rents and installments from PNAD-IBGE.

9.1 Preparation of the Data Base

Note: this code is too long to place here. It is available upon request from the 1st author.

9.2 Estimating the VAR

```
PROC IMPORT OUT= WORK.REAL
  DATAFILE= "C:\Users\Pedro Albuquerque\Desktop\New Folder\Estimação\REAL
  ESTATE 17 04 2010.XLS"
  DBMS=EXCEL REPLACE;
  RANGE="SAS$";
  GETNAMES=YES;
  MIXED=NO;
  SCANTEXT=YES;
  USEDATE=YES;
  SCANTIME=YES;
  RUN;
  DATA REAL;
  SET REAL;
  LABEL VAR01='Unidades da Federação';
  LABEL VAR02='Ano';
  LABEL VAR03='Soma do valor da prestação entre os domicílios que pagam
prestação dos seus imóveis.';
  LABEL VAR04='Soma do valor do aluguel entre os domicílios que pagam
aluguel.';
  LABEL VAR05='Soma dos rendimentos das pessoas';
  LABEL VAR06='Total de ocupados.';
```



```

IF VAR01="&UF" AND TYPE="EST" THEN OUTPUT;
RUN;
DATA A;
SET MATRIZ;
RUN;
DATA ZT;
SET REAL;
IF VAR01="&UF" AND VAR02=&ANO THEN OUTPUT;
KEEP VAR11 VAR12 VAR13;
RUN;
DATA _RHO_;
SET _RHO_;
IF VAR01="&UF" THEN OUTPUT;
RUN;
PROC IML;
USE A;
READ ALL VAR {AR1_1 AR1_2 AR1_3} INTO A;
*PRINT A;
USE ZT;
READ ALL VAR {VAR11 VAR12 VAR13} INTO ZT;
*PRINT ZT;
ZT=ZT';
*PRINT ZT;
USE _RHO_;
READ ALL VAR {RHO} INTO RHO;
*PRINT RHO;
RESULT=A*INV((I(3)-RHO*A))*ZT;
RESULT=RESULT';
*PRINT RESULT;
CREATE RESULTADO FROM RESULT;
APPEND FROM RESULT;
CLOSE RESULTADO;
RUN;
QUIT;
DATA RESULTADO;
SET RESULTADO;
VAR01="&UF";
VAR02=&ANO;
RUN;
DATA RESULTADO;
MERGE RESULTADO _RHO_;
RUN;
DATA RESULTADO;
SET RESULTADO;
R_P=K+COL1+COL2-COL3;
RUN;

```

```

DATA FINAL;
SET FINAL RESULTADO;
RUN;
%END;
%MEND;
DATA FINAL;RUN;
%VAR(Brasil);
%VAR(Acre);
%VAR(Alagoas);
%VAR(Amapá);
%VAR(Amazonas);
%VAR(Bahia);
%VAR(Ceará);
%VAR(Distrito Federal);
%VAR(Espírito Santo);
%VAR(Goiás);
%VAR(Maranhão);
%VAR(Mato Grosso);
%VAR(Mato Grosso do Sul);
%VAR(Minas Gerais);
%VAR(Paraná);
%VAR(Paraíba);
%VAR(Pará);
%VAR(Pernambuco);
%VAR(Piauí);
%VAR(Rio Grande do Norte);
%VAR(Rio Grande do Sul);
%VAR(Rio de Janeiro);
%VAR(Rondônia);
%VAR(Roraima);
%VAR(Santa Catarina);
%VAR(Sergipe);
%VAR(São Paulo);
%VAR(Tocantins);
DATA FINAL;SET FINAL;RUN;IF _N_ EQ 1 THEN DELETE;RUN;
PROC EXPORT DATA= WORK.FINAL
OUTFILE= "C:\Users\Pedro Albuquerque\Desktop\New Folder\Estimação\RESULTADO.xls"
DBMS=EXCEL REPLACE;
RANGE="RESULTADO";
RUN;

```