An application of Bayesian Model Averaging to forecast inflation in Colombia

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May, 2010

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Motivation

- For Central Banks it is important to count with accurate predictions of future inflation in order to determine the monetary policy stance and make appropriate decisions.
  Alternatives:
  1. Select the 'best' model in terms of fit or forecasting? structural or econometrics models? or select a suite of forecasting models?.
  2. Chose among a large set of potential predictors the ones that better help to explain the dynamics of inflation and predict it in the future?
- Need to summarize the available information into a single output that capture all the relevant information from each individual forecast: Forecast combination
- Combined forecast produces smaller forecast error than any individual forecast, (Bates and Granger [1969], Newbold and Granger [1974],)
- The easy way: equal weighted average, but might be biased and affected by extreme values.
- others alternatives of forecast combination

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BMA is

- A procedure of variable and model selection
- Based on uncertainty about the true data generating process
- Based on the Bayes theorem

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Description of the methodology. Continue...

Procedure

- From a set of M models, $\mathbf{M}_1, \ldots, \mathbf{M}_M$.
- with prior belief about the probability of each model being the true one, $P(\mathbf{M}_i)$ for $i = 1, \ldots, M$,
- Given the observed data $\mathbf{Y}$
- Using Bayes theorem, the posterior probability of each model being the true one is given by:

$$P(\mathbf{M}_i/\mathbf{Y}) = \frac{m(\mathbf{Y}/\mathbf{M}_i)P(\mathbf{M}_i)}{\sum_{i=1}^{M} m(\mathbf{Y}/\mathbf{M}_i)P(\mathbf{M}_i)} \tag{1}$$

where $m(\mathbf{Y}/\mathbf{M}_i)$ is the marginal likelihood of model i defined as

$$m(\mathbf{Y}/\mathbf{M}_i) = \int L(\mathbf{Y}/\Theta_i, \mathbf{M}_i)P(\Theta_i/\mathbf{M}_i)d\Theta_i \tag{2}$$

where L is the likelihood and $P(\Theta_i/\mathbf{M}_i)$ is the posterior density of the parameter vector of model i.

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$$P(\Delta/\mathbf{Y}) = \sum_{i=1}^{M} P(\Delta/\mathbf{Y}, \mathbf{M}_i) P(\mathbf{M}_i/\mathbf{Y}) \tag{3}$$

- For a function $g(\Delta)$ its posterior distribution is give by

$$E(g(\Delta)/\mathbf{Y}) = \sum_{i=1}^{M} E(g(\Delta)/\mathbf{Y}, \mathbf{M}_i) P(\mathbf{M}_i/\mathbf{Y}) \tag{4}$$

- For the forecast $\tilde{Y}_{t+h} = E(Y_{t+h}/Y_t)$, the optimal forecast combination is the weighted average of the forecasts generated by each model.

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Description of the methodology. Continue...

- When considering the case of variable selection, the posterior probability that variable j is included in the true model is given by

$$p(X_j/\mathbf{Y}) = \sum_{i=1}^{M} I(X_j \in \mathbf{M}_i)P(\mathbf{M}_i/\mathbf{Y}) \tag{6}$$

where $I(X_j \in \mathbf{M}_i)$ is an indicator variable, taking value of one when variable X_j is in model $\mathbf{M}_i$ and zero otherwise.

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Linear Regression models $$Y = Z\gamma + \epsilon$$
where $Y = \{y_1, \cdots, y_T\}$, $\gamma = (\alpha, \beta')'$, $Z = (1, X)$ contains explanatory variables and $\epsilon \sim N(0, \sigma_\epsilon^2 I)$

- priors for the parameters
  - for the variance the Jeffrey's non-informative prior
$$p(\sigma_\epsilon^2) \propto \frac{1}{\sigma_\epsilon^2} \tag{7}$$
  - The prior distribution for the vector parameter γ/σ_ϵ^2 is the g-prior distribution, (Zellner,1986)
$$p(\gamma/\sigma_\epsilon^2, \mathbf{M}) \sim N_{k+1}(0, c\sigma_\epsilon^2(Z'Z)^{-1}) \tag{8}$$
with
$$c = \begin{cases} K^2 & \text{if } T \leq K^2 \\ T & \text{if } T > K^2 \end{cases} \tag{9}$$
as suggested by Fernandez et al, 2001.

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Priors

- priors for the models

$$P(\mathbf{M}_i) \propto \delta^{k_i}(1-\delta)^{K-k_i} \tag{10}$$

  - K : maximum number of variables allowed in a model
  - k_i : number of variables included in model i
  - δ : is set such that the expected model size is equal to some prior. In particular, when $\delta = 0{,}5$ the prior model probability is the same for each model.

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Marginal likelihood

These set of priors lead to the posterior on the parameters

$$p(\gamma/\mathbf{Y}) \sim t_{k_i+1}(\gamma_1, S_1, M_1, \upsilon_1) \tag{11}$$

where $\gamma_1 = \frac{c}{c+1}\hat{\gamma}$ and $\hat{\gamma}$ is the OLS estimate, $\upsilon_1 = T - 1$

$$S_1 = \frac{c}{c+1}(Y - Z\hat{\gamma})'(Y - Z\hat{\gamma}) + \frac{1}{c+1}Y'Y \tag{12}$$

$$M_1 = \frac{c+1}{c}Z'Z \tag{13}$$

This leads to the marginal likelihood, which is also a multivariate t-distribution

$$m(\mathbf{Y}/\mathbf{M}) \propto (c+1)^{-(k+1)/2} S_1^{(-T/2)} \tag{14}$$

Contents

Predictive likelihood

- Avoid in$-$sample overfitting problem
- Based on the predictive ability of the model

The full sample $(\mathbf{y}_1, \cdots, \mathbf{y}_T)$ is split into two parts:

- Y^* with T_1 observations: used to obtain the posterior distribution on the parameters
- $\tilde{Y}$ with T_2 observations: used to evaluate the model performance.

The posterior predictive likelihood $P(\tilde{Y}/Y^*, \mathbf{M}_i)$ is given by

$$P(\tilde{Y}/Y^*, \mathbf{M}_i) = \int L(\tilde{Y}/\Theta_i, Y^*, \mathbf{M}_i) P(\Theta_i/Y^*, \mathbf{M}_i) d\Theta_i \tag{15}$$

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Predictive likelihood ...

Under the priors above, the predictive density of $\tilde{Y} = (\mathbf{y}_{T_1+1}, \cdots, \mathbf{y}_T)$ is

$$P(\tilde{Y}/\tilde{Z}, Z^*, Y^*, \gamma^*, \sigma_\epsilon^2) \sim N_{T_2}(\tilde{Z}\gamma^*, \sigma_\epsilon^2 I_{T_2}) \tag{16}$$

- $\tilde{Z}$: out-sample matrix of explanatory variables
- γ^* : parameter vector estimated with the training sample Y^*.

The predictive posterior density of $\tilde{Y}$ is a multivariate t-student distribution.

$$\tilde{Y}/\tilde{Z}, Z^*, Y^* \sim \mathbf{t}_{T_2}(\tilde{Z}\gamma_1, S^*, (I_{T_2} + \tilde{Z}(M^*)^{-1}\tilde{Z}')^{-1}, T_1) \tag{17}$$

with density function

$$P(\tilde{Y}/\tilde{Z}, Z^*, Y^*) \propto \frac{{S^*}^{T_1/2} \left|M^*\right|^{1/2}}{\left|M^* + \tilde{Z}'\tilde{Z}\right|^{1/2}} \times [S^* + (\tilde{Y} - \tilde{Z}\gamma_1)'(I_{T_2} + \tilde{Z}(M^*)^{-1}\tilde{Z})^{-1}(\tilde{Y} - \tilde{Z}\gamma_1)]^{-T/2} \tag{18}$$

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Posterior probabilities...

For each model

- Using marginal likelihood

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- The idea is to find a set of M "good" models from a large set of K potential predictors. The model space contains 2^K possible models.
- Restrict the model space considering models containing up to k covariates. The model space contains $(\sum_{j=0}^{k} \binom{K}{j})$ possible models. But still too many models to evaluate.
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RJMCMC algorithm

Procedure

From an initial state of the chain $(\theta_{\mathbf{M}}, \mathbf{M})$

- Propose a jump from model $\mathbf{M}$ to model $\mathbf{M}^*$ with probability $j(\mathbf{M}^*/\mathbf{M})$
- Generate a vector u from a proposal density $\mathbf{q}(u/\mathbf{M}, \mathbf{M}^*)$
- Set $(\theta_{\mathbf{M}^*}, u^*) = g_{\mathbf{M},\mathbf{M}^*}(\theta_{\mathbf{M}}, u)$, where g is a specified invertible function and u, u^* satisfy $dim(u) + dim(\theta_{\mathbf{M}}) = dim(u^*) + dim(\theta_{\mathbf{M}^*})$
- Accept the move with probability

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and set $\mathbf{M} = \mathbf{M}^*$ if the move is accepted.

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From an initial state of the chain $(\theta_{\mathbf{M}}, \mathbf{M})$

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- condidering local moves only:
  1. add or drop jump. $j(M/M^*) = j(M^*/M) = \frac{1}{K}$, with K the number of available variables.
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- If all parameters of the proposed model are generated from a proposal distribution, then
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1. Models of size up to 5 variables are considered. Model space: $\sum_{j=0}^{5}\binom{73}{j} \approx$ 16 million models.
2. $\delta = 0{,}065$

$$Y_{t+h} = \alpha + \sum_{j=0}^{5} Z_{j,t}\gamma_j + \epsilon_{t+h} \tag{25}$$

3. initial state of the chain randomly chosen (model size and explanatory variables)
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- Second stage: Model selection
  1. Include 2 lags of each pre-selected variable to the dataset (60 potential predictors)
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Empirical Application ...

Implementation of BMA

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- For most horizons the forecasting errors are autocorrelated,
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$$\mathbf{e}_{t+h/t} = \varphi_0 + \varphi_1 \mathbf{e}_{t+h-1/t} + \vartheta_{t+h} \tag{27}$$

- Boostrapping samples were drawn over the residuals of that model ϑ_{t+h}.
- Using the residual sample and the parameter estimates, the forecasting error sample series was constructed, $\mathbf{e}^i_{t+h/t} = \hat{\varphi}_0 + \hat{\varphi}_1 \mathbf{e}_{t+h-1/t} + \vartheta^i_{t+h}, i = 1, \cdots, 500.$
- The sample size equals the number of out-sample forecast available for each horizon.

Criterion:

- For each sample, calculate the RMSE relative to the RMSE of the random walk forecast.
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  $AIC = 2\frac{k}{T} - 2\frac{l}{T} \propto 2\frac{k}{T} + \log\left(\frac{\sigma_\epsilon^2}{T}\right)$
  where l is the log likelihood, k is the number of parameters in the model and $\sigma_\epsilon^2 = \frac{1}{T}\sum_{t=1}^{T}\epsilon_t^2$ is the estimate of the variance of the residuals.
- The difference between the AIC criteria of a pair of models is an unbiased estimator of the difference of the KL distance for the two models. The KL distance, (Kullback and Leibler,1951), is defined as

$$\mathbf{I(f,g)} = \int f(x)\log\left(\frac{f(x)}{g(x/\hat{\theta})}\right)dx \tag{28}$$

  where $f(x)$ is the unknown data generating process, $g(x/\hat{\theta})$ is the given model and $\hat{\theta}$ is the estimate of the parameters.

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Information Theoretical Averaging. Continue...

- There are M forecasting models, ranking according to the KL distance.
- $\exp(-\frac{1}{2}\psi_i)$ with $\psi_i = AIC_i - \text{mín}\, AIC_j$, can be interpreted as the weight of model i to be the KL best model given that there is a certain model in set of forecasting models (M) which is the KL best model for the available data.
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Simple Averaging

Equal weights for each individual forecast

$$\hat{\mathbf{Y}}_{T+h} = \frac{1}{M} \sum_{i=1}^{M} \hat{\mathbf{Y}}^{i}_{T+h/T} \tag{31}$$

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Dynamic Factors model

- methodology of forecasting with many predictors
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Variables with higher posterior probabilities - Predictive Likelihood

h=1		h=3		h=6		h=9		h=12	
CPI	0.999	GASAL	0.960	GASAL	0.972	TCNMPROM	0.463	GAOTGA	0.998
RESNETAS	0.334	GAOTGA	0.546	IPI	0.694	GASAL	0.302	AEA	0.992
TERMINTE	0.149	ISNIMAOB	0.441	GAVES	0.316	CDT90DBA	0.247	NCREGUL	0.738
MBISI	0.146	IPI	0.438	GAVIV	0.258	MBISI	0.197	TERMINTE	0.305
NCNOTRAN	0.127	NCTRAN	0.290	TIBPROME	0.225	GAEDU	0.156	GATRAN	0.304
IPI	0.118	RESNETAS	0.237	TCNMPROM	0.225	CHBRUTA	0.153	UECFINAL	0.250
DTFNO90D	0.091	EFECTIV	0.149	MBKMATCO	0.161	DEPCTAHO	0.117	PCNOVIS	0.080
MBCDUR	0.088	GATRAN	0.098	ITCRIPPN	0.143	DEPCTCOR	0.116	CREDBR	0.061
ISNIMAEM	0.084	CRDOBPRI	0.093	MBCNODU	0.111	AEMIN	0.114	EFECTIV	0.054
ISNCOMIN	0.084	MBISI	0.074	GAEDU	0.110	UEFORK	0.112	EXPSITEC	0.049
DEPCTCOR	0.070	UECFINAL	0.072	ITCRIPCT	0.102	PCVIS	0.103	GAVES	0.044
MBKSA	0.067	AEA	0.069	TASACTIV	0.084	GAVIV	0.100	CDT90DBA	0.044
ITCRIPPN	0.063	MBKEQTRA	0.060	CDT90DBA	0.080	TASACTIV	0.098	EXISTEN	0.041
TIBPROME	0.062	VOLACTPE	0.051	UEFORK	0.072	ISNIMAEM	0.096	M3	0.035
MBKSI	0.061	GAVIV	0.045	CAPINDE	0.066	MBICOMLU	0.094	CRBTES	0.034
ITCRIPPT	0.058	CAPINDE	0.044	ITCRIPCN	0.061	UEMATCO	0.093	BASEMON	0.031
MBKEQTRA	0.055	ACTPROD	0.042	NCREGUL	0.054	EXPAUMPR	0.088	MBICOMLU	0.027
CRBBAN	0.054	NCREGUL	0.042	AEIMAN	0.045	M1	0.084	GASAL	0.026
EXPAUMPR	0.053	M1	0.038	EXPPRO	0.042	NCTRAN	0.082	DEPCTAHO	0.025
CREDBR	0.052	PPI	0.038	GACUL	0.042	CREDBR	0.081	GAEDU	0.020

Numbers correspond to the posterior inclusion probabilities

Variables with higher posterior probability - Marginal likelihood

h=1		h=3		h=6		h=9		h=12	
ISNCOMIN	0.984	GAOTGA	0.692	EXPSITEC	0.817	ISNIMAOB	0.906	CDT90DBA	1.000
CPI	0.942	GAVIV	0.570	TASACTIV	0.739	EFECTIV	0.835	TASACTIV	1.000
TERMINTE	0.543	CRBTES	0.505	ITCRIPCN	0.539	PPI	0.772	GAOTGA	1.000
VOLACTPE	0.506	VOLACTPE	0.491	ITCRIPCT	0.539	NCTRAN	0.728	UECINTER	1.000
NCTRAN	0.496	MBISI	0.419	ITCRIPPN	0.497	ITCRIPCN	0.508	ISNIMAOB	0.998
GASAL	0.407	TASACTIV	0.408	ITCRIPPT	0.460	ITCRIPPT	0.313	GAVES	0.002
ITCRIPPT	0.161	CDT90DBA	0.407	EXPAUMPR	0.337	GAOTGA	0.098	TOTALDEP	0.000
UECFINAL	0.142	DTFNO90D	0.270	SECONOM	0.121	DEPCTAHO	0.097	MBKSI	0.000
PPI	0.133	AEMIN	0.205	GAVES	0.068	DTFNO90D	0.096	PPI	0.000
DTFNO90D	0.107	M1	0.197	CDT90DBA	0.067	PBPRODCO	0.091	CRDOBPRI	0.000
NCNOTRAN	0.085	ITCRIPPT	0.178	PBM	0.063	ITCRIPPN	0.072	GAVIV	0.000
ITCRIPCT	0.054	PBPRODCO	0.178	UEFORK	0.059	AEIMAN	0.070	NCTRAN	0.000
AEA	0.042	UEMATCO	0.082	TCNMPROM	0.038	ISRSINT	0.062	GATRAN	0.000
M1	0.030	TERMINTE	0.081	CRDOBPRI	0.029	RESNETAS	0.040	MBKMATCO	0.000
M3	0.028	UECINTER	0.066	EFECTIV	0.028	UECFINAL	0.035	CRBCORP	0.000
AEMIN	0.027	NCTRAN	0.065	GATRAN	0.028	PCNOVIS	0.025	TCNMPROM	0.000
CRDOBPRI	0.027	AEIMAN	0.034	UEMATCO	0.025	NCREGUL	0.022	DTFNO90D	0.000
ITCRIPCN	0.026	GAEDU	0.032	EXPPRO	0.025	EXISTEN	0.019	M1	0.000
PBM	0.018	DEPCTCOR	0.013	AEA	0.017	SECONOM	0.016	M2	0.000
GAOTGA	0.014	GACUL	0.013	TIBPROME	0.014	EXPPRO	0.016	UEFORK	0.000

Numbers correspond to the posterior inclusion probabilities

Models with higher posterior probability - Predictive likelihood - h=12

Variable	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14	M15	M16	M17	M18	M19	M20
GAOTGA	0,1,2	0	1	0,1	0	0	0,1	0	0,1	1	0,1	0	0	0,2	1	0,1	0	0	0	1
AEA	0	0,2	0,2	0,2	0,2	0	0	0	0	0,2	0	0,1	0	0,2	2	0	0,2	0,1	0	0
NCREGUL	1	0	0	1	0	1		1	1	0	1	0		0	0	0	0	0		1
UECFINAL		2	0,2	0	2	0	1	0		2		1	0	0	2		2	1		
TERMINTE	0	1				0					0	0	0			0		0		
BASEMON		1			1	1	1						1			1	1			
DEPCTAHO				1			2		0	1									0	0
EXPSITEC				1				1	2			2					1			2
GATRAN		2					0						1						1	2
EFECTIV			2		2	2		2							2					
GAVES							0						0		1				1	
M3											1			2		2				2
CDT90DBA								0			1						0			
EXISTEN										2				2					0,2	
PCNOVIS			0												1					
MBICOMLU						1														
CREDBR															2					
GASAL	0,1																			
Posterior Prob.	0.120	0.070	0.059	0.058	0.055	0.049	0.049	0.047	0.046	0.044	0.042	0.042	0.042	0.041	0.040	0.040	0.040	0.040	0.039	0.038

* the numbers in cells are the lags of each variable in the model

Forecast Evaluation - RSME

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
BMA-pl	***0.338***	0.807	1.777	***1.083***	1.114	***0.874***	***1.000***	1.560	1.804	1.022	0.962	1.021
BMA-ml	0.457	1.066	1.169	1.601	1.557	1.667	1.880	2.201	1.153	1.518	1.167	1.284
ITMA1	***0.358***	1.007	1.819	1.636	1.762	1.355	***1.527***	2.017	2.110	1.402	1.050	1.088
ITMA2	***0.360***	***0.641***	0.950	***0.916***	0.856	***0.746***	***0.789***	0.932	0.913	0.724	0.671	0.739
Simple average	***0.358***	1.007	1.819	1.635	1.762	1.355	***1.527***	2.017	2.110	1.402	1.050	1.088
Dynamic factors	***0.307***	***0.728***	***1.071***	1.400	1.732	1.974	2.003	2.097	2.203	2.213	2.189	2.326
Random Walk	0.473	0.851	1.175	1.457	1.742	2.024	2.278	2.473	2.608	2.712	2.789	2.801

Numbers in bold and italic correspond to the cases where MDM test for equal forecast ability compared to the random walk forecast is rejected

BMA-pl refers to BMA combination using predictive likelihood

BMA-ml refers to BMA combination using marginal likelihood

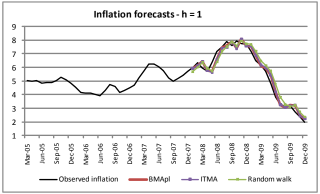
ITMA1 refers to the information theoretic model averaging combination of models selected by BMA

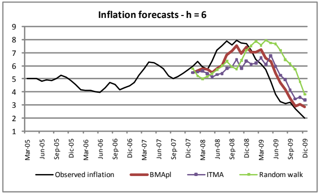
ITMA2 refers to the information theoretic model averaging combination of models selected by ITMA

Bootstrapping forecasting errors

Forecast Combination	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
BMA_PL	***1.000***	0.528	0.154	0.806	***0.924***	***0.994***	***0.990***	0.898	0.734	***1.000***	***0.998***	***0.998***
BMA_ml	0.458	0.304	0.360	0.374	0.578	0.692	0.578	0.456	0.290	0.542	***0.970***	***0.988***
ITMA1	***0.986***	0.352	0.140	0.284	0.450	0.770	0.756	0.520	0.542	0.488	***0.950***	***0.946***
ITMA2	***0.996***	0.884	0.710	0.768	***0.982***	***1.000***	***1.000***	***0.998***	***1.000***	***1.000***	***1.000***	***0.998***
simple average	***0.990***	0.348	0.150	0.304	0.396	0.798	0.760	0.542	0.574	0.480	***0.944***	***0.940***
Dinamic Factors	***1.000***	***0.984***	***0.960***	0.550	0.810	0.726	0.602	0.390	0.320	0.280	0.356	0.356

% of samples with reduction in RMSE of at least 5% relative to the random walk forecast.


Inflation forecasts - h = 1
9
8
7
6
5
4
3
2
1
Mar-05 Jun-05 Sep-05 Dec-05 Mar-06 Jun-06 Sep-06 Dec-06 Mar-07 Jun-07 Sep-07 Dec-07 Mar-08 Jun-08 Sep-08 Dec-08 Mar-09 Jun-09 Sep-09 Dec-09
Observed inflation
BMApl
ITMA
Random walk


Inflation forecasts - h = 6
9
8
7
6
5
4
3
2
1
Mar-05 Jun-05 Sep-05 Dic-05 Mar-06 Jun-06 Sep-06 Dic-06 Mar-07 Jun-07 Sep-07 Dic-07 Mar-08 Jun-08 Sep-08 Dic-08 Mar-09 Jun-09 Sep-09 Dic-09
Observed inflation
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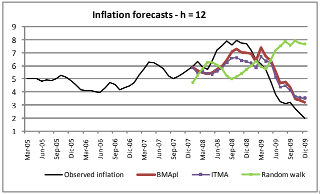
Inflation forecasts - h = 12
9
8
7
6
5
4
3
2
1
Mar-05 Jun-05 Sep-05 Dic-05 Mar-06 Jun-06 Sep-06 Dic-06 Mar-07 Jun-07 Sep-07 Dic-07 Mar-08 Jun-08 Sep-08 Dic-08 Mar-09 Jun-09 Sep-09 Dic-09
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BMApl
ITMA
Random walk

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- The variables chosen as best predictors for inflation have change to some extent over the last two years, especially for the predictive likelihood (40 % of the variables are selected with the full sample). For marginal likelihood, it seems that the forces driving inflation have not changed over time (70 % of the variables are selected with the full sample).

Concluding Remarks ...

for future research,

- How often the selection of variables and models should be done in order to continue applying this methodology on a regular basis, given that the results are influenced by the sample, specially when using predictive likelihood.
- Which priors and which algorithm should be used to select the variables and models, having into account the findings of Ohara and Sillampaa,(2009), "the performance of the method depends on the priors and how it is implemented"
- How many models to combine?
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Thanks!