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Estimating Strategic Complementarity in a State-Dependent Pricing Model

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Overview

Motivation

- Price rigidity is an important assumption in Macroeconomics

- Two classes of sticky price models:
  1. Time-Dependent Pricing Model (TDP)
     - Calvo (1983), New Keynesian Literature

  2. State-Dependent Pricing Model (SDP)
     - Caplin and Spulber (1987), Dotsey, King and Wolman (1999)
     - Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2009), Woodford (2009).

- A recent (old) discussion: **Can sticky price models generate large real effects from monetary shocks?**
  - Result: State-dependent model presents smaller real effects from monetary shocks than time-dependent pricing model

- **Can state-dependent pricing models behave like time-dependent pricing models?**
  - Golosov and Lucas (2007)
  - Gertler and Leahy (2008), Midrigan (2009), Woodford (2009)

- Menu cost model + strategic complementarity: Yes, they can!

- **Strategic complementarity:** Decisions of two or more players are called strategic complements if they mutually reinforce one another.

- Bils, Klenow and Malin (2009): Reset Price Inflation

  They found that a SDP model with no strategic complementarities aligns more closely with the data.

This paper

- We propose a methodology to estimate directly from microdata the structural parameter related to strategic complementarity in a SDP model.

- We estimate some parameters defining the (S,s) pricing rule, as well as the variance of shocks affecting the firms in each sector. We relate these parameters to the price rigidity behavior in each sector.

- We use microdata underlying the Brazilian CPI to estimate the model. Additionally, we document some stylized facts about price rigidity in Brazil and relate them to our results.

The Model

The model has three main elements:

- Households obtain utility from consumption goods. Firms supply differentiated goods in a monopolistically competitive environment. In the (segmented) labor markets households and firms behave competitively.

- Firms follow a state-dependent pricing rule.

- There are aggregate shocks and idiosyncratic productivity shocks.

Households: The representative household seeks to maximize

$$E_0\left\{\sum_{t=0}^{\infty} e^{-\rho t}\left[u(C_t;\xi_t)-\int_0^1 v(L_{i,t};\xi_t)di\right]\right\} \qquad (1)$$

where $C_t = \left[\int_0^1 C_{i,t}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$ and $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{1/(1-\theta)}$

The expenditure minimization problem implies that the demand for an individual product has the familiar form:

$$C_{i,t} = C_t\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \qquad (2)$$

The optimal quantity of labor is implicitly given by

$$\frac{v_L(L_{i,t};\xi_t)}{u_C(C_t;\xi_t)} = \frac{W_{i,t}}{P_t} \qquad (3)$$

Firms: There is a continuum of monopolistically competitive firms supplying differentiated goods. Each firm has the production function:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} M_t^{(1-\alpha)} \tag{4}$$

This leads to the following real marginal cost function:

$$\psi\left(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}\right) = \frac{\lambda}{A_{i,t}} \left\{ \frac{v_L\left(Y_{i,t}; \xi_t\right)}{u_C\left(Y_t; \xi_t\right)} \right\}^{\alpha} E_t^{1-\alpha} \tag{5}$$

Perfectly Flexible Prices: The firm maximizes

$$P_{i,t} Y_{i,t} - W_{i,t} L_{i,t} - E_t M_t \tag{6}$$

subject to (4), and perfect information about the cost structure (5) and the demand (2). This results in the following equation for the frictionless optimal price:

$$\frac{P_{i,t}^{*}}{P_t} = \mu \psi\left(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}\right) \tag{6}$$

A first-order log-linearization of equation (6) around the steady-state equilibrium with flexible prices leads to

$$\log P^{*}_{i,t} = \kappa + \zeta \log y_t + (1+\zeta)\log P_t + \frac{(1+\alpha)}{1+\alpha\omega\theta}\log E_t + a_{i,t} \tag{7}$$

where $a_{i,t} = \frac{1}{1+\alpha\omega\theta}\frac{\partial \log \psi_{i,t}}{\partial \xi_t}\xi_t - \frac{1}{1+\alpha\omega\theta}\log A_{i,t}$

- Strategic complementarity: The lower the value of $\zeta = \frac{\alpha\left(\omega+\sigma^{-1}\right)}{1+\alpha\omega\theta}$

We will write equation (7) as (variables in logs):

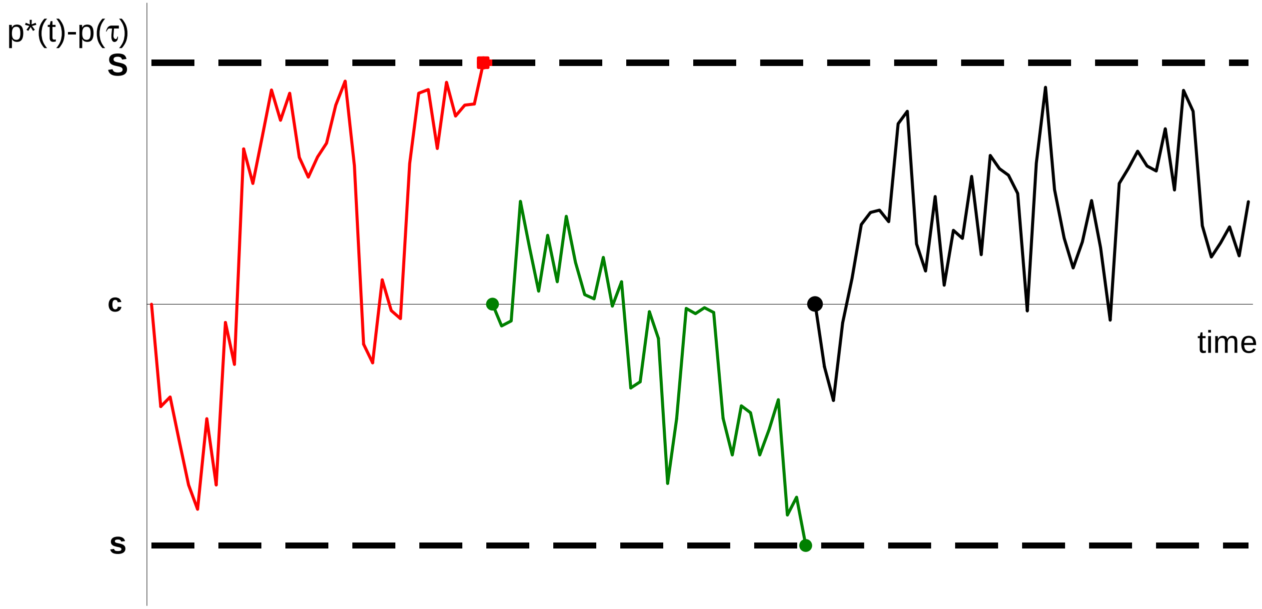
$$\begin{aligned} p^{*}_{i,t} &= \kappa + \zeta y_t + (1+\zeta)p_t + \frac{(1+\alpha)}{1+\alpha\omega\theta}e_t + a_{i,t} \\ &= \kappa + \mathbf{x}'_t\boldsymbol{\beta} + a_{i,t} \end{aligned} \tag{8}$$

We also assume that $a_{i,t} = \eta + a_{i,t-1} + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim N(0,\sigma^2)$

Price Rigidity

The firm follows a (s,S)-type rule:

Graph: Adjustment rule in a S,s model



The Econometric Model

Define the latent variable $y^*_{i,t} = p^*_{i,t} - p_{i,\tau}$ and $y_{i,t}$ as:

$$y_{i,t} = \begin{cases} 1, & if \quad p_{i,t} > p_{i,t-1} \\ 0, & if \quad p_{i,t} = p_{i,t-1} \\ -1, & if \quad p_{i,t} < p_{i,t-1} \end{cases}$$

By the pricing rule, at the moment of price change: $p^*_{i,\tau} - p_{i,\tau} = c$.

Then,

$$\begin{aligned} y^*_{i,t} \equiv p^*_{i,t} - p_{i,\tau} &= (\boldsymbol{x}'_t \boldsymbol{\beta} + a_{i,t}) - (\boldsymbol{x}'_\tau \boldsymbol{\beta} + a_{i,\tau} - c) \\ &= (\boldsymbol{x}_t - \boldsymbol{x}_\tau)' \boldsymbol{\beta} + c + (a_{i,t} - a_{i,\tau}) \\ &= \boldsymbol{z}'_{i,t} \boldsymbol{\beta} + c + (a_{i,t} - a_{i,\tau}) \end{aligned}$$

But, $a_{i,t} - a_{i,\tau} = \eta \delta_{i,t} + u_{i,t}$, where $u_{i,t}$ follows a $\mathrm{MA}(\delta_{i,t} - 1)$ process. $\delta_{i,t}$ is the difference between t and τ .Then,

$$\begin{aligned} y^*_{i,t} \equiv p^*_{i,t} - p_{i,\tau} &= \boldsymbol{z}'_{i,t} \boldsymbol{\beta} + c + (a_{i,t} - a_{i,\tau}) \\ &= \eta \delta_{i,t} + \boldsymbol{z}'_{i,t} \boldsymbol{\beta} + c + u_{i,t} \end{aligned}$$

and $u_{i,t} \sim N(0, \delta_{i,t} \sigma^2)$

Then, defining $\boldsymbol{w}_{i,t} = \left(\delta_{i,t}, \boldsymbol{z}_{i,t}\right)'$ we can derive the probability of observing a price increase:

$$\Pr\left[y_{i,t} = 1 \mid \boldsymbol{w}_{i,t}\right] = \Pr\left[y^{*}_{i,t} \geq S \mid \boldsymbol{w}_{i,t}\right] = \Pr\left[\eta\delta_{i,t} + \boldsymbol{z}'_{i,t}\boldsymbol{\beta} + c + u_{i,t} \geq S \mid \boldsymbol{w}_{i,t}\right]$$

$$= \Pr\left[\frac{u_{i,t}}{\sqrt{\delta_{i,t}}\,\sigma} \geq \frac{S - c - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta}}{\sqrt{\delta_{i,t}}\,\sigma}\right]$$

$$= 1 - \Phi\left(\frac{S-c}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma}\right)$$

$$= 1 - \Phi\left(\pi_1 \tilde{1}_{i,t} - \gamma\tilde{\delta}_{i,t} - \tilde{\boldsymbol{z}}'_{i,t}\theta\right)$$

We have the following Ordered Probit Model:

- Probability of Price Increase:

$$\Pr\left[y_{i,t}=1\,|\,\boldsymbol{w}_{i,t}\right]=1-\Phi\left(\frac{S-c}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}}-\frac{\eta}{\sigma}\sqrt{\delta_{i,t}}-\frac{\boldsymbol{z}_{i,t}^{'}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma}\right)$$

$$=1-\Phi\left(\pi_1\tilde{1}_{i,t}-\gamma\tilde{\delta}_{i,t}-\tilde{\boldsymbol{z}}_{i,t}^{'}\theta\right)$$

- Probability of Maintaining Price:

$$\Pr\left[y_{i,t}=0\,|\,\boldsymbol{w}_{i,t}\right]=\Phi\left(\pi_1\tilde{1}_{i,t}-\gamma\tilde{\delta}_{i,t}-\tilde{\boldsymbol{z}}_{i,t}^{'}\theta\right)-\Phi\left(\pi_0\tilde{1}_{i,t}-\gamma\tilde{\delta}_{i,t}-\tilde{\boldsymbol{z}}_{i,t}^{'}\theta\right)$$

- Probability of Price Decrease:

$$\Pr\left[y_{i,t}=-1\,|\,\boldsymbol{w}_{i,t}\right]=\Phi\left(\frac{s-c}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}}-\frac{\eta}{\sigma}\sqrt{\delta_{i,t}}-\frac{\boldsymbol{z}_{i,t}^{'}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma}\right)$$

$$=\Phi\left(\pi_0\tilde{1}_{i,t}-\gamma\tilde{\delta}_{i,t}-\tilde{\boldsymbol{z}}_{i,t}^{'}\theta\right)$$

Estimation and Identification

- **Estimation:** By Quasi-Maximum Likelihood and robust variance-covariance matrix for heteroskedasticity and autocorrelation.

- **Identification:**

Observe that: $\theta_1 = \dfrac{\zeta}{\sigma}$ and $\theta_2 = \dfrac{1-\zeta}{\sigma}$

Then,

$$\frac{\theta_1}{\theta_2} = \frac{\zeta}{1-\zeta} \quad \Rightarrow \quad \zeta = \frac{\theta_1}{\theta_1 + \theta_2}$$

Additionally, we have: $\theta_1 = \dfrac{\zeta}{\sigma} \quad \Rightarrow \quad \sigma = \dfrac{1}{\theta_1 + \theta_2}$

Dataset: Microdata

- First papers: Gouvea (2007), Barros at al (2009)
- Primary information of price quotes collected and used by IBRE-FGV to compute the CPI-FGV. Collected in 12 metropolitan regions.
- The CPI-FGV comprises 456 products and services grouped in 7 sectors. Approximately 2500 outlets.
- Typical item: black beans of type 1, of the brand Combrasil, in a package of 1kg, which is sold in the outlet number 16,352 in Belém.
- Our sample: A very representative sample of the overall CPI-FGV (around 85%), from 1996 to 2006.

Dataset: Some Information

	Original Dataset		Treated Dataset	
Sector	**# of observations**	**# of trajectories**	**# of observations**	**# of trajectories**
Food	3,973,527	36,400	1,324,589	22,809
Other Goods and Services	411,560	8,006	150,136	3,188
Education and Recreation	346,095	13,613	246,871	5,309
Housing	961,755	21,438	406,982	9,383
Medical and Personal Care	1,087,647	17,922	594,627	11,395
Transportation	149,185	5,026	111,203	2,335
Apparel	501,202	19,919	363,368	9,395
Total	7,430,971	122,324	3,197,776	63,814

Descriptive Statistics of the Price Changes

Information about the distribution of the price changes, conditional on adjustment

Sector	Duration (days)	Mean of $\|\Delta p\|$ %	% of Reduction	Average reduction %	Average increase %
All sectors	59.27	15.99	0.44	-16.67	15.45
Food	50.53	16.27	0.45	-16.70	15.92
Apparel	53.89	25.32	0.48	-25.46	25.19
Housing	61.59	13.27	0.42	-14.04	12.72
Other Goods and Services	63.42	10.75	0.43	-11.05	10.53
Transportation	64.82	7.58	0.39	-7.13	7.86
Medical and Personal Care	75.90	12.35	0.41	-13.37	11.67
Education and Recreation	134.61	15.82	0.36	-17.66	14.78

Sector	mean of Δp %	Std deviation	Kurtosis	% of small Δp
All sectors	1.31	19.38	4.15	37.94
Food	1.15	19.41	3.81	37.27
Apparel	1.15	30.08	3.45	33.57
Housing	1.60	15.80	3.89	38.63
Other Goods and Services	1.31	12.48	3.16	33.69
Transportation	2.06	8.93	4.34	40.59
Medical and Personal Care	1.53	14.84	3.80	39.30
Education and Recreation	2.95	18.70	4.60	32.62

Note: 1) p is defined as the natural logarithm of the item price

2) All statistics are calculated based on unweighted price changes. Kurtosis is calculated excluding the top and bottom 1% of observations

3) Small Δp is defined as any price change whose absolute value is lower than 0.5 of the mean of $|\Delta p|$

Some Stylized Facts

- **Fact 1:** Prices change frequently, but the degree of price rigidity is quite different among the sectors.
- **Fact 2:** On average price changes (in absolute values) are large.
- **Fact 3:** Price decreases are frequent events.
- **Fact 4:** On average price decreases are larger than price increases.
- **Fact 5:** A large percentage of price changes is of small changes.

Models Fit

Probit Models:

Dependent variable: y_t

Explanatory variables: $\boldsymbol{w}_{i,t} = \left(\frac{1}{\sqrt{\delta_{i,t}}}, \sqrt{\delta_{i,t}}, \frac{y_t - y_\tau}{\sqrt{\delta_{i,t}}}, \frac{p_t - p_\tau}{\sqrt{\delta_{i,t}}}, \frac{e_t - e_\tau}{\sqrt{\delta_{i,t}}} \right)$

Probability of a price change and price duration

Sector	**Pr[y_t= -1 or y_t=1|mean(w_t)]**	**Implied Duration (in days)**	**Duration (in days)**
All sectors	0.56	53.24	59.27
Food	0.63	47.35	50.53
Apparel	0.60	50.03	53.89
Housing	0.54	55.41	61.59
Other Goods and Services	0.53	56.82	63.42
Transportation	0.52	57.34	64.82
Medical and Personal Care	0.48	62.35	75.90
Education and Recreation	0.25	118.81	134.61

Estimation Results: Strategic Complementarity

Estimated Parameters of Probit Models and Strategic Complementarity

Sector	Y_t	P_t	e_t	ζ	Conf. interval for ζ
All sectors	0.92	9.15	-0.09	0.09	0.07 - 0.11
	(0.006)	(0.011)	(0.001)		
Food	0.20	8.79	0.07	0.02	0.01 - 0.03
	(0.027)	(0.020)	(0.004)		
Apparel	1.09	4.91	-0.17	0.18	0.15 - 0.22
	(0.057)	(0.071)	(0.012)		
Housing	1.36	11.02	0.11	0.11	0.10 - 0.12
	(0.039)	(0.106)	(0.008)		
Other Goods and Services	1.43	10.45	-0.36	0.12	0.09 - 0.15
	(0.078)	(0.358)	(0.027)		
Transportation	1.36	14.29	0.42	0.09	0.05 - 0.12
	(0.141)	(0.311)	(0.025)		
Medical and Personal Care	1.24	10.06	-0.70	0.11	0.10 - 0.12
	(0.026)	(0.118)	(0.007)		
Education and Recreation	6.09	11.81	0.32	0.34	0.32 - 0.36
	(0.122)	(0.382)	(0.024)		

Notes: The standard deviation is in parenthesis

The confidence interval is 95% of confidence

The standard deviation of ζ was obtained by the Delta method

Estimation Results: Pricing Rule

Estimated parameters related to the optimal pricing rule

Sector	π_0=(s-c)/σ	π_1=(S-c)/σ	σ	s-c	S-c	S-s	(S-s)/σ
All sectors	-0.93*	0.63*	0.10	-0.09	0.06	0.15	1.55
Food	-0.70*	0.48*	0.11	-0.08	0.05	0.13	1.18
Apparel	-0.72*	0.62*	0.17	-0.12	0.10	0.22	1.34
Housing	-1.05*	0.61*	0.08	-0.08	0.05	0.13	1.65
Other Goods and Services	-1.06*	0.70*	0.08	-0.09	0.06	0.15	1.76
Transportation	-1.21*	0.60*	0.06	-0.08	0.04	0.12	1.81
Medical and Personal Care	-1.30*	0.81*	0.09	-0.11	0.07	0.19	2.10
Education and Recreation	-2.12*	1.45*	0.06	-0.12	0.08	0.20	3.58

Note: Asterisk means significant at 1% of significance.

Preliminary Conclusions

- We propose a method to directly estimate strategic complementarity in pricing decisions. The methodology uses a microfounded model to derive a structural, non-standard ordered probit model.

- The results indicate that the parameter ζ is about 0.1, implying a substantial degree of strategic complementarity, in line with assumptions assumed in Gertler and Leahy (2008), for example.

- Differently from Bils, Klenow and Malin (2009), we did not find that a state-dependent pricing model with strategic complementarity is fundamentally at odds with the data.

- In addition, the methodology allows us to estimate some parameters related to the pricing rules. In general, the results seem to explain the stylized facts.

Further Research

- Do these results of substantial strategic complementarity come from the fact that we are using a state-dependent pricing model?
  - Development of similar methodology for estimating a time-dependent model and the degree of strategic complementarity.

- We would like to separate each parameter of the pricing rule: S, s and c.