

10 Anos do Projeto de  
Juros e Spread Bancário

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Estabilidade Financeira e  
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Banco Central do Brasil*

# **Default Correlation: An Empirical Investigation of Brazilian Retail Loans**

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# Motivation

- Relevance of Default Correlation (DC) for risk diversification
- Influence of the DC in the capital requirement (Basel II – Advanced Models)
- Papers about DC for retail loans are rare
  - Cowan e Cowan (2004) – Proposes the "first research" for Empirical DC in the Subprime Loans.

# Objectives

- Calculate DC for Brazilian retail loans
- Different stages of the Economic Cycle will be presented
- Losses on Recession x Losses on Expansion (Credit VaR)
- Identify another segmentation for credit transactions different from the traditional one (Risk Rating)

# Literature Review

- Merton (1974)
  - Asset Correlation concept
  - Seminal analytical model for credit risk
  - Equity Price - Proxy for asset correlation => not valid for retail credits
- Lucas (1995) – Impact of DC as a function of variables
  - Macro and Micro (Environment of the business)
- Servigny & Renault (2002) – Empirical Methodology
  - Analysis in the DC at business cycles
  - S&P Data Base – 21 years, Americans Companies, 916.000 Observations.
- Lando & Skodeberg (2002) - Critique of the discrete time

# Data

- Data from the Central Bank Credit Information System (CIS)
- Period: jan/2003 – jul/2008 (Semiannual Frequency)
- Retail operations: R\$ 5,000 – R\$ 50,000 (Contracting Date)
- Two relevant retail credit modalities
  - Consumer Credit
  - Vehicle Financing
- Two most relevant financial institutions in these modalities
  - Consumer Credit (31% together)
  - Vehicle Financing (38% together)

# Some Descriptive Statistics

- In both modalities:
  - Mostly male:
    - 61% Consumer
    - 68% Vehicle
  - Concentrated in intermediate age groups:
    - 34% between 45-60 years for Consumer
    - 27% between 45-60 years for Vehicle
  - Private Sector accounts for the largest number of borrowers:
    - 25% Consumer and 19% Vehicle
  - Highly concentrated in SE region:
    - 71% Consumer
    - 61% Vehicle

# Methodology

- Transition and Correlation Matrices (Servigny & Renault - Annual) - 11 semesters
- Loans were grouped by Credit Ratings – Res. 2682/99
- Default Class – (Between D e H e Write-off)
- Univariates or Marginals Transitions
- Bivariates or Joint Transitions
- Transition matrices were calculated for both phases of the business cycle (Contraction semester: 2003/1 , Expansion semesters: 2003/2 – 2008/1)



# Methodology

## Marginal Transition Frequencies

$$f_i^k = \frac{T_i^k}{N_i}$$

where:

$f_i^k$  = marginal transition frequency from rating  $i$  to rating  $k$  in one semester;

$T_i^k$  = total number of transactions transiting from rating  $i$  at the beginning of the semester to rating  $k$  at the end of the same semester;

$N_i$  = total number of transactions belonging to rating  $i$  at the beginning of the semester.

# Methodology

## Marginal Transition Frequencies

$$f_{i,j}^{k,l} = \frac{T_i^k * T_j^l}{N_i * N_j}$$

where:

$f_{i,j}^{k,l}$  = joint transition frequency from ratings  $i$  and  $j$  respectively to ratings  $k$  and  $l$ , in one semester;

$T_i^k$  = total number of transactions transiting from rating  $i$  at the beginning of the semester to rating  $k$  at the end of the same semester;

$T_j^l$  = total number of transactions transiting from rating  $j$  at the beginning of the semester to rating  $l$  at the end of the same semester;

$N_i$  = total number of transactions belonging to rating  $i$  at the beginning of the semester;

$N_j$  = total number of transactions belonging to rating  $j$  at the beginning of the semester.

# Methodology

## Transition Correlations

$$\rho_{i,j}^{k,l} = \frac{f_{i,j}^{k,l} - f_i^k * f_j^l}{\sqrt{f_i^k * (1 - f_i^k) * f_j^l * (1 - f_j^l)}}$$

where:

$\rho_{i,j}^{k,l}$  = correlation coefficient between a pair of loan transactions transiting from ratings  $i$  and  $j$  at the beginning of one semester respectively to ratings  $k$  and  $l$  at the end of the semester.

# Results

## Marginal Transitions – Contraction and Expansion

		Consumer Credit- Contraction					Vehicle Financing- Contraction					
		Final Rating					Final Rating					
		AA	A	B	C	Default	AA	A	B	C	Default	
Classe Inicial	AA	40.03%	35.15%	3.35%	17.41%	4.07%	AA	77.40%	12.62%	6.85%	1.66%	1.47%
	A	2.02%	61.06%	14.84%	8.58%	13.50%	A	0.02%	84.40%	6.50%	3.40%	5.68%
	B	0.13%	9.52%	49.76%	6.44%	34.16%	B	0.11%	22.46%	45.64%	7.59%	24.19%
	C	0.04%	0.74%	1.85%	56.45%	40.92%	C	0.03%	23.45%	8.41%	14.82%	53.30%
	Default	0.00%	0.26%	0.68%	0.32%	98.74%	Default	0.01%	4.08%	1.73%	1.66%	92.52%
		Consumer Credit- Expansion					Vehicle Financing- Expansion					
		Final Rating					Final Rating					
		AA	A	B	C	Default	AA	A	B	C	Default	
Classe Inicial	AA	48.97%	43.36%	2.45%	2.04%	3.17%	AA	88.92%	1.81%	3.05%	2.88%	3.33%
	A	1.25%	77.38%	11.57%	2.30%	7.50%	A	10.28%	75.61%	5.89%	4.06%	4.16%
	B	0.07%	8.77%	60.24%	4.44%	26.48%	B	9.29%	17.95%	44.94%	10.96%	16.87%
	C	0.14%	3.24%	9.03%	46.93%	40.67%	C	10.50%	11.01%	6.37%	35.17%	36.95%
	Default	0.01%	0.52%	2.91%	0.80%	95.76%	Default	2.92%	3.35%	1.81%	2.65%	89.28%

# Results

## Default Correlations

Consumer Credit						Vehicles Financing					
	AA	A	B	C	Default		AA	A	B	C	Default
AA	1.67%	1.03%	2.40%	-0.46%	2.27%	AA	0.75%	0.51%	1.13%	0.47%	-6.82%
A	1.03%	-2.77%	-3.68%	-3.63%	-17.69%	A	0.51%	0.01%	0.23%	0.96%	-0.83%
B	2.40%	-3.68%	-3.07%	-3.04%	-22.83%	B	1.13%	0.23%	0.96%	1.12%	-7.42%
C	-0.46%	-3.63%	-3.04%	-6.34%	15.86%	C	0.47%	0.96%	1.12%	-2.04%	-20.40%
Default	2.27%	-17.69%	-22.83%	15.86%	23.88%	Default	-6.82%	-0.83%	-7.42%	-20.40%	32.86%

- Great dispersion in both modalities
- Weak correlations in most cases
- Similar results in Lucas (1995), Nagpal & Bahar (2001), Rosch (2003) and Servigny & Renault (2002)

# Methodology

## Losses estimation using VaR

$$L = \sum_{i=1}^n EAD_i * LGD_i * Y_i$$

where:

$L$  = total portfolio loss at the end of the semester, equal to the sum of individual losses;

$EAD_i$  = exposure at default for the  $i$ -th credit transaction (equal to R\$1);

$LGD_i$  = loss given default for the  $i$ -th credit transaction;

$Y_i$  = default indicator variable (Bernoulli) for the  $i$ -th credit transaction.

# VaR Results

## Consumer Credit

	95,0%	99,0%	99,9%
Expansion	18,85%	18,89%	18,91%
Recession	21,55%	21,61%	21,62%

## Vehicle Financing

	95,0%	99,0%	99,9%
Expansion	12,27%	12,31%	12,32%
Recession	12,82%	12,88%	12,90%

- Contraction VaR higher than Expansion VaR for both modalities
- Consumer Credit VaR higher than Vehicles Financing VaR
- Consumer Credit
  - Contraction VaR 14% higher than Expansion VaR
- Vehicles Financing
  - Contraction VaR only 4% higher than Expansion VaR

# The Probit Model

Our main objective:

- ❖ Identify a most appropriate grouping to measure default correlation.

Underlying Theoretical Model

- ❖ When he takes a loan, the borrower wants to implement a project.
- ❖ The project's return should depend on three main elements:
  1. The borrower's personal characteristics (Gender, Age, Type of Occupation, Region of Residence)
  2. The macroeconomic environment
  3. The default rates in other risk ratings. Assuming there is an interdependence of the projects in the economy (Default Rate per Risk Rating)



# Probit Results

## Consumer Credit and Vehicle Financing

- None of the 2 macroeconomic variables were significant
- As expected, the coefficients of the Default Rate variable of a certain rating were positives and significant for explaining the default of that same rating
- Type of Occupation was the only significant variable in all of its sub-categories for all of the risk ratings. It may suggest a better discrimination

# Probit Results

## Default Correlations by Type of Occupation

### Consumer Credit

	Private	Public	Self-Employed	Company Owner	Pensioner	Others
Private	23.52%	29.71%	29.18%	28.21%	25.69%	36.95%
Public	29.71%	36.87%	36.60%	35.19%	31.82%	45.71%
Self-Employed	29.18%	36.60%	36.16%	34.90%	31.62%	45.62%
Company Owner	28.21%	35.19%	34.90%	33.60%	30.30%	43.92%
Pensioner	25.69%	31.82%	31.62%	30.30%	27.31%	39.52%
Others	36.95%	45.71%	45.62%	43.92%	39.52%	56.95%

### Vehicle Financing

	Private	Public	Self-Employed	Company Owner	Pensioner	Others
Private	2.04%	1.99%	3.16%	2.58%	1.89%	3.53%
Public	1.99%	1.93%	3.06%	2.50%	1.81%	3.52%
Self-Employed	3.16%	3.06%	4.98%	4.01%	2.85%	5.75%
Company Owner	2.58%	2.50%	4.01%	3.26%	2.32%	4.66%
Pensioner	1.89%	1.81%	2.85%	2.32%	1.70%	3.31%
Others	3.53%	3.52%	5.75%	4.66%	3.31%	6.40%

- Unlike before:
  - positive correlations in the main diagonal
  - stronger coefficients in general
- Correlation values for Consumer Credit higher than for Vehicle Financing

# Conclusions

- Empirical correlation matrices grouped by risk rating, as commonly seen in literature, presented weak and highly dispersed coefficients in both credit modalities
- The probit model suggested that a segmentation of transactions by type of occupation could be more appropriate for the calculation of correlations. Analogy between retail and corporate
- New results: positive correlations, less dispersed and higher in both modalities.

# Conclusions

- In a recession, there is an increase of transition probabilities for the class of default and losses estimated by VaR.
- Lower default correlations for Vehicles Financing credits than for Consumer credits => it might be the existence of collaterals

# Shortcomings

- Initial Work.
- It is not proposed to use Probit to predict DC.
- Results not comparable with Basel II. Default Criteria and transition horizon are different.
- Short period in recession.
- There is no significance test for DC.

# Suggestions and Future Studies

- But we can improve the model (personal income, loan interest rates) and the database (other credit modalities, a longer time period).
- Clusters in the retail area..

**THANK YOU**

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# The Probit Model

- ❖ Then, 
$$y_{i,j,t}^* = x_i' \beta + m_t' \gamma + \sum_{j=1}^J \theta_j z_{j,t} + u_{i,j,t}$$
- $i$  represents the borrower;  $j = 1, \dots, J$  is the risk rating
  - $y_{i,j,t}^*$  is the project return of borrower  $i$ , who belongs to the risk rating  $j$  at time  $t$
  - $x_{i,t}$  is a vector with personal characteristics of borrower  $i$
  - $m_t$  are macroeconomic variables at time  $t$
  - $z_{j,t}$  is the default rate in rating  $j$  at time  $t$
  - $u_{i,j,t}$  is a shock affecting the project's return, and  $u_{i,j,t} \sim N(0,1)$
- ❖ In order for the borrower to repay the loan, the project must have a minimum return equal to  $\alpha$ .

$$y_{i,j,t}^* < \alpha \Leftrightarrow \text{Default} \Leftrightarrow y_{i,j,t} = 1$$

$$y_{i,j,t}^* > \alpha \Leftrightarrow \text{No Default} \Leftrightarrow y_{i,j,t} = 0$$