Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk*

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Abstract

We study the functioning and possible breakdown of the interbank market in the presence of counterparty risk. We allow banks to have private information about the risk of their assets. We show how banks' asset risk affects funding liquidity in the interbank market. Several interbank market regimes can arise: i) normal state with low interest rates; ii) turmoil state with adverse selection and elevated rates; and iii) market breakdown with liquidity hoarding. We provide an explanation for observed developments in the interbank market before and during the 2007-09 financial crisis (dramatic increases of unsecured rates and excess reserves banks hold, as well as the inability of massive liquidity injections by central banks to restore interbank activity). We use the model to discuss various policy responses.

JEL classification: G01, G21, D82

Keywords: Financial crisis; Interbank market; Liquidity; Counterparty risk; Asymmetric information

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Neither the recent massive money injections, the coordinated lowering of interest rates nor the use of public funds to recapitalize banks have done much to restart interbank lending. This action did not solve the underlying problem preventing interbank lending: extreme information asymmetry.

Financial Times, November 9, 2008

1 Introduction

Interbank markets play a key role in banks' liquidity management and the transmission of monetary policy. They provide benchmark rates for the pricing of fixed-income securities throughout the economy (e.g. LIBOR). In normal times, interbank markets are among the most liquid in the financial sector. Since August 2007, however, the functioning of interbank markets has become severely impaired around the world. As the financial crisis deepened in September 2008, liquidity in the interbank market has further dried up as banks preferred hoarding cash instead of lending it out even at short maturities. Central banks' massive injections of liquidity did little to restart interbank lending. The failure of the interbank market to redistribute liquidity has become a key feature of the 2007-09 crisis (see, for example, Allen and Carletti, 2008, and Brunnermeier, 2009).

Figure 1 illustrates the unprecedented extent of the turbulence. It plots the spread between the three-month unsecured rate and the overnight index swap in three months' time,[1] a standard measure of interbank market tensions (red line), and the amounts of excess reserves banks hold with the European Central Bank (light and dark blue bars).[2] A notable feature is the build up of tensions in the interbank market. Until August 9, 2007, the unsecured euro interbank market is characterized by a very low spread, around five basis points, and infinitesimal amounts of excess reserves with the European Central Bank (ECB). In *normal* times, banks prefer to lend out excess cash since the interest rate on excess reserves is punitive relative to rates available in interbank markets. The *turmoil* phase between August 9, 2007 and the last weekend of September 2008

[1] The overnight index swap is a measure of what the market expects the overnight unsecured rate to be over a three-month period and thus controls for interest rate expectations.

[2] Banks can hold excess reserves with the European Central Bank in two ways. First, they can access the deposit facility, which is a standing facility available for banks on a continuous basis for overnight deposits. These are renumerated at a punitive rate, usually 100 basis points below the policy rate. Second, the ECB occasionally offers banks to deposit funds for a short period of time at the policy rate (liquidity-absorbing fine tuning operations).

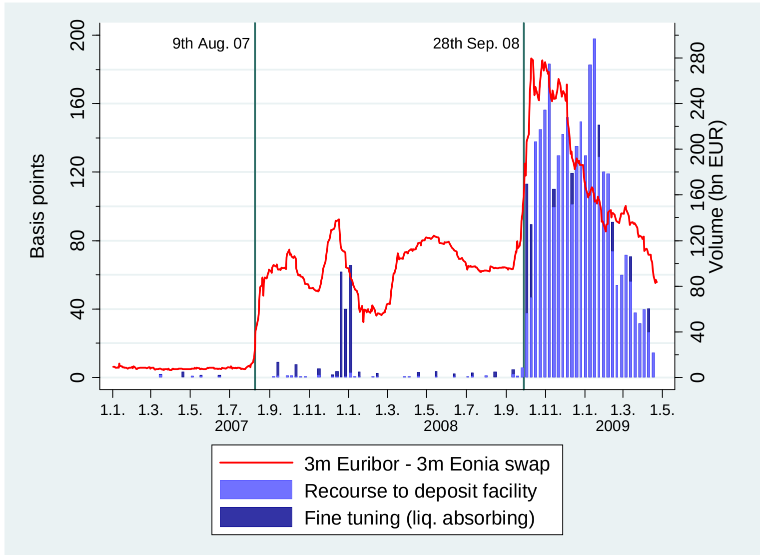

Figure 1: Interbank spread and excess reserves (recourses to the ECB deposit facility and liquidity-absorbing fine tuning operations), daily average per week, 01/2007 - 04/2009

is characterized by a significantly higher spread, yet excess reserves remain virtually nil.[3] As of September 28, 2008, the spread increases even further to a maximum of 186 basis points. But the distinguishing feature of this *crisis* phase is a dramatic increase in excess reserves. Banks are hoarding liquidity. At the same time, the average daily volume in the overnight unsecured interbank market halved.[4] A similar pattern of three distinct phases can be observed in the spread for the United States (Figure 2).[5]

What caused the interbank market to seize up? Why has the market been dysfunctional for so long despite massive interventions by public authorities? What frictions can explain these developments and how do they relate to the broader roots of the financial crisis? And how do the policy responses that were discussed or implemented around the world hold up against these frictions?

This paper provides a model of how the risk of banks' long-term assets can lead to the evaporation of liquidity in the unsecured interbank market. The key friction in the model is counterparty risk. Asymmetric information amplifies the friction. We use the model to understand the qualita-

[3]Except a year-end effect whereby the ECB helps banks to balance their books at the end of every year.

[4]We examine the events of September and October 2008 in more detail in Section 5.

[5]Unlike in Europe, we do not have daily information on the evolution of excess reserves in the US as the Federal Reserve did not renumerate them until late in the crisis.

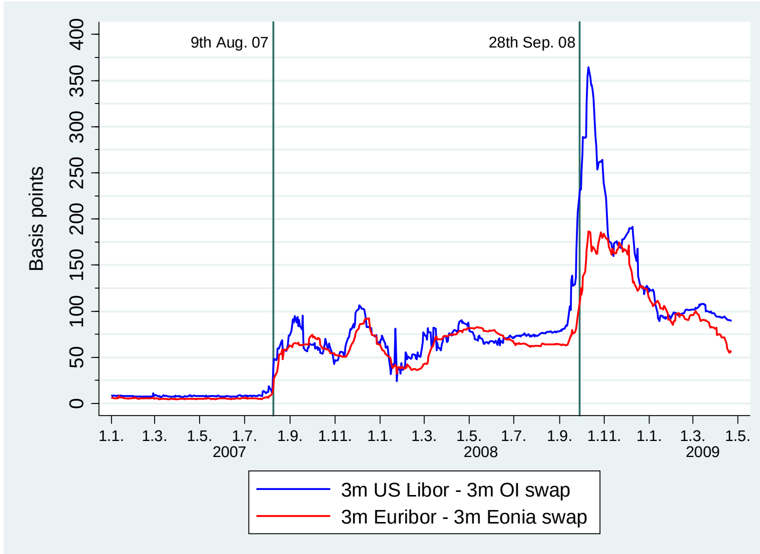

Figure 2: Interbank spreads US and euro area, 01/2007 - 04/2009

tive developments prior to and during the financial crisis, and to shed light on policy responses. We model banks as maturity transformers that face a trade-off between liquidity and return. Banks trade in the interbank market to smooth out idiosyncratic liquidity shocks. Lending banks face counterparty risk stemming from the risk of borrowing banks' assets. Each bank knows the distribution of risk in the banking sector and is privately informed about the risk of its own assets but banks cannot observe the risk of their counterparties.

Various interbank market regimes arise depending on the level and distribution of counterparty risk. First, when the level and dispersion of risk are low, the unsecured interbank market functions smoothly despite counterparty risk and asymmetric information. The interest rate for unsecured loans is low and all banks manage their liquidity using the interbank market. Riskier banks exert an externality on safer banks as the latter subsidize the liquidity of the former. But the cost is small compared to the cost of obtaining liquidity outside the unsecured market. Second, for higher levels of risk there can be adverse selection in the interbank market. The externality on safer banks is so costly that they leave the unsecured market. Liquidity is still traded but the interest rate rises to reflect the presence of riskier banks. Third, the interbank market may break down when the dispersion of risk is high. Liquidity rich banks prefer to hoard liquidity instead of lending it out to an adverse selection of borrowers. Finally, it is possible that even riskier borrowers find the unsecured interest rate too high and prefer to obtain liquidity elsewhere. Moreover, when the dispersion of risk is high, multiple equilibria are possible and which regime occurs depends on self-fulfilling expectations.

The outcomes of our model resemble the observed three phases in Figure 1: i) normal times, ii) turmoil with elevated spreads but no excess reserves, and iii) crisis with a further increase in spreads and substantial excess reserves. The points of transition across the phases are in line with changes in the level and distribution of counterparty risk: a market-wide reassessment of risk in the summer of 2007, after subprime-mortgage backed securities were discovered in portfolios of banks and bank-sponsored conduits, and a further dramatic revision of expected default probabilities following the collapse of Lehman Brothers. Asymmetric information as an underlying friction can also rationalize the prolonged nature of interbank market tensions despite an unprecedented level of liquidity provision by central banks.

Our modeling assumptions are designed to reflect the insights from broad analyses of the 2007-09 financial crisis. First, asymmetric information about the size and location of risk, and the accompanying fear of counterparty default, which was created by the complexity of securitization, are at the heart of the financial crisis (see Gorton, 2008, 2009). Second, maturity mismatch is a key factor contributing to the fragility of modern financial systems that can become clogged by illiquid securities (see, for example, Diamond and Rajan, 2009a, and Brunnermeier, 2009). Hence, we employ a version of the standard model of banking introduced by Diamond and Dybvig (1983) that allows for a trade-off between the liquidity of and the return on assets. Banks may need to realize cash quickly due to demands of customers who draw on committed lines of credit or on their demandable deposits. Banks in need of liquidity can borrow from banks with a surplus of liquidity as in Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994).[6] We allow for banks' profitable but illiquid assets to be risky. Asset risk implies counterparty risk in the interbank market since banks may not be able to repay their loan.[7] Asymmetric information about counterparty risk amplifies the frictions in the interbank market as suppliers of liquidity who cannot distinguish safer and riskier banks protect themselves against lending to "lemons".

[6] An important complement to liquidity within the financial sector is the demand and supply of liquidity within the real sector (see Holmström and Tirole, 1998).

[7] Our model therefore applies to money market segments in which credit risk concerns play a role, namely unsecured (term) markets and markets secured by risky collateral.

The effects of private information on the functioning of debt markets were first examined by Stiglitz and Weiss (1981). In their analysis banks with market power do not raise the interest rate to clear away an excess demand for loans since they fear being left with an adverse selection of borrowers. In our analysis, the interbank market is perfectly competitive and banks are price takers. Liquidity hoarding may occur precisely when liquidity rich banks face an adverse selection of borrowers and the interest rate is high.[8] Broecker (1990) and Flannery (1996) examine a situation in which banks do not know their competitors' underwriting abilities. Each bank fears a winner's curse, i.e., lending to borrowers who have been rejected elsewhere. To protect themselves, banks only lend at high rates. Freixas and Holthausen (2005) show how the integration of interbank markets may fail when cross-border information about banks is less precise than home-country information.

Asymmetric information between short-term and long-term investors is a key friction in Bolton, Santos, and Scheinkman (2009). Longer-term investors, as potential buyers of assets, do not know whether short-term investors sell because the asset failed to produce a return or because they need liquidity and the asset has not yet matured. Delaying the sale deepens the information problem and adverse selection may inefficiently accelerate asset liquidation. They distinguish between outside and inside liquidity (asset sales versus cash), which connects to our analysis where banks hold liquid and illiquid securities and the former can be traded in exchange for risky claims on the latter. Brunnermeier and Pedersen (2009) distinguish between market liquidity and funding liquidity. In our model, banks can obtain funding liquidity in the interbank market by issuing claims on assets with limited market liquidity.

In Diamond and Rajan (2009b), illiquidity can depress lending and low prices for illiquid assets go hand in hand with high returns on holding liquidity. They do not consider asymmetric information. Instead, potential buyers may want to wait for asset prices to decline further. At the same time, the managers of selling banks may want to gamble for resurrection. These two effects feed on each other and may lead to a market freeze.

Allen, Carletti, and Gale (2009) present a model of a repo market freeze without asymmetric information or counterparty risk. Banks can stop trading due to aggregate liquidity risk, i.e.

[8] In this sense our paper is also related to Myers and Majluf (1984). Our outcome in which lenders face an adverse selection of borrowers and the interest rate is high is comparable to their outcome in which investors face an adverse selection of equity issuing firms and the stock price is low.

banks hold similar rather than offsetting positions. Aggregate shortages are also examined in Diamond and Rajan (2005) where bank failures can be contagious due to a shrinking of the pool of available liquidity. Freixas, Parigi, and Rochet (2000) analyze systemic risk and contagion in a financial network and its ability to withstand the insolvency of one bank. In Allen and Gale (2000), the financial connections leading to contagion arise endogenously as a means of insurance against liquidity shocks.

Rationales for central bank intervention in the interbank market are studied in Acharya, Gromb, and Yorulmazer (2008) and Freixas, Martin, and Skeie (2009). In Acharya et al., market power makes it possible for liquidity rich banks to extract surplus from liquidity poor banks. A central bank provides an outside option for the banks suffering from such liquidity squeezes. In Freixas et al., multiple equilibria exist in interbank markets, some of which are more efficient than others. By steering interest rates, a central bank can act as a coordination device for market participants and ensure that a more efficient equilibrium is reached. Freixas and Jorge (2008) examine how financial imperfections in the interbank market affect the monetary policy transmission mechanism beyond the classic money channel. Bruche and Suarez (2009) explore implications of deposit insurance and spatial separation for the ability of money markets to smooth out regional differences in savings rates.

The remainder of the paper is organized as follows. In Section 2, we describe the model setup. In Section 3, we analyze the benchmark case in which there is no asymmetric information about counterparty risk. In Section 4, we analyze the case in which there is asymmetric information. In Section 5, we discuss the empirical implications of the model and relate them to the developments during the financial crisis. In Section 6, we employ the model to discuss policy responses. In Section 7, we offer concluding remarks. All proofs are in the Appendix.

2 The model

There are three dates, $t = 0, 1$, and 2, and a single homogeneous good that can be used for consumption and investment. There is no discounting and no aggregate uncertainty.

Banks. There is a $[0, 1]$ continuum of identical, risk neutral banks. Banks manage the funds on behalf of risk neutral customers with future liquidity needs. To meet the liquidity needs of customers, banks offer them claims worth d_1 and d_2 that can be withdrawn at $t = 1$ and $t = 2$, respectively, e.g. demand deposits or lines of credit. We assume that the liquidity needs are strictly positive at each date so that $d_1 > 0$ and $d_2 > 0$.

The aggregate demand for liquidity is certain: a fraction λ of customers withdraws their claims at $t = 1$. The remaining fraction $1 - \lambda$ withdraws at $t = 2$. At the individual bank level, however, the demand for liquidity is uncertain. A fraction π_h of banks face a high liquidity demand $\lambda_h > \lambda$ at $t = 1$ and the remaining fraction $\pi_l = 1 - \pi_h$ of banks faces a low liquidity demand $\lambda_l < \lambda$. Hence, we have $\lambda = \pi_h\lambda_h + \pi_l\lambda_l$. Let the subscript $k = l, h$ denote whether a bank faces a low or a high need for liquidity at $t = 1$. We assume that banks' idiosyncratic liquidity shocks are not contractible. A bank's liabilities cannot be contingent on whether it faces a high or a low liquidity shock at $t = 1$ and $t = 2$. This will give rise to an interbank market.

Assets and banks' portfolio decision. At $t = 0$, banks can invest in two types of real assets, a long-term illiquid asset and a short-term liquid asset. We assume that each bank has one unit of the good under management at $t = 0$. Each unit invested in the short-term asset offers a return equal to 1 after one period (costless storage). Each unit invested in the long-term asset yields an uncertain payoff at $t = 2$. The long-term asset can either succeed and return R or fail and generate a loss $Z \leq 0$. We assume that the loss is high enough to render a bank insolvent in which case its liabilities are assumed by the regulator. To prevent any risk-shifting behavior due to limited liability, the regulator dictates the structure of banks' liabilities, i.e. he imposes d_1 and d_2.[9] Banks must honor their liabilities as long as they are solvent. Given d_1 and d_2, banks choose their portfolio of short-term and long-term assets to maximize profits. Let α denote the fraction invested in the long-term asset at $t = 0$. The remaining fraction $1 - \alpha$ is invested in the short-term asset.

Banks are uncertain about the riskiness of their long-term investment when they make their portfolio choice at $t = 0$. With probability q, the long-term investment succeeds with probability p_s and with probability $1 - q$, it succeeds with probability $p_r < p_s$.[10] Let p denote the expected success probability: $p \equiv qp_s + (1 - q)p_r$. Each bank becomes privately informed about the risk of its long-term investment at $t = 1$. While the overall level of risk, p, is known, banks have

[9] The regulator sets d_1 and d_2 such that banks are solvent as long as their long-term asset succeeds.

[10] This is a simple case of the monotone likelihood ratio property, see, for example, Laffont and Martimort (2002), p.164.

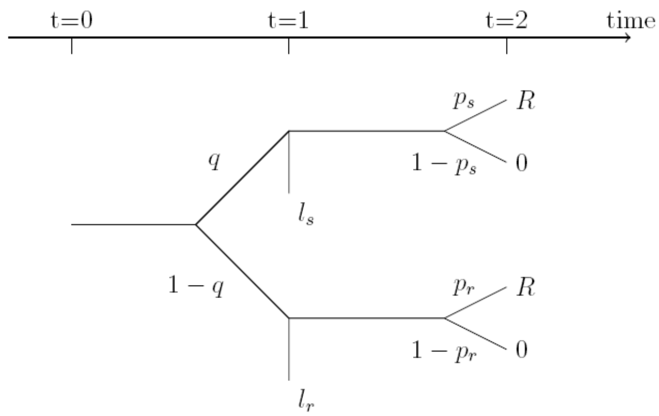

Figure 3: Pay-off structure of the long-term, illiquid asset

private information whether their long-term investment is safer, $p_s > p$, or riskier, $p_r < p$, than expected. The uncertainty about the risk of the long-term asset is assumed to be independent of the uncertainty about liquidity demand. Let the subscript $\theta = s, r$ denote whether a bank's long-term asset is safer or riskier than expected.

The investment in the long-term asset is ex ante efficient: $pR > 1$. This does not, however, preclude a long-term investment that turns out to be riskier than expected to be unprofitable ex post: $p_r R < 1$. Any fraction α^L of the long-term investment can be converted into liquidity at $t = 1$ using a private liquidation technology that yields a constant unit return of less than one (costly liquidation). We interpret this broadly as a cost of accessing sources of funding other than unsecured borrowing. We assume that safer investments are easier to convert into liquidity, $1 > l_s > l_r$. This structure makes riskier assets also more illiquid, a feature particularly pronounced in the current crisis.[11] Our results would be qualitatively unchanged if we instead assumed that the safer long-term asset returns less than the riskier asset, $R_s < R_r$, and that both types of long-term assets can be converted into liquidity at the same rate, $l_s = l_r = l$. We show this in Appendix B. What matters is that the opportunity cost of liquidation, $\frac{R}{l}$, is higher for a riskier bank.

In case the riskier investment is unprofitable ex post, $p_r R < 1$, we will assume that $p_r R > l_r$ so that banks prefer to keep long-term investment to maturity even if it turns out to be riskier than expected. In sum, banks face a trade-off between liquidity and return when making their portfolio

[11] This would be the case if the liquidation technology realizes a constant fraction γ of the long-term asset's expected value: $l_\theta = \gamma p_\theta R$.

decision. The long-term asset is ex ante more productive but it is costly to convert it into liquidity at $t = 1$.

Interbank market and liquidity management. Given that banks face differing liquidity demands at $t = 1$, an interbank market can develop. Banks with low withdrawals at $t = 1$ can lend any excess liquidity to banks with high withdrawals. Let L_l and L_h denote the amount lent and borrowed, respectively, and let r denote the interest rate on interbank loans.[12]

Due to the risk of the long-term asset, borrowers as well as lenders in the interbank market may be insolvent at $t = 2$ when the loan repayment is due. Solvent borrowers must always repay their interbank loans. If their lender is insolvent, the repayment goes to the regulator. In contrast, solvent lenders are only repaid if their borrowers are solvent, too. Hence, lenders in the interbank market are exposed to the possibility that their loans are not repaid, i.e. they are exposed to *counterparty risk*. We denote the probability that an interbank loan is repaid by $\hat{p}$.

We assume that the interbank market is competitive, i.e. banks act as price takers, and that banks are completely diversified across interbank loans. Hence, $\hat{p}$ is also the proportion of interbank loans made by a lender that will be repaid at $t = 2$.

In sum, a bank can manage its liquidity at $t = 1$ in three ways: 1) by borrowing/lending in the interbank market, 2) by converting the long-term asset into liquidity, and 3) by investing in the short-term asset for another period.

The sequence of events is summarized in Figure 4.

3 Benchmark: No asymmetric information

In this section we assume that the shock to the risk of the long-term asset at $t = 1$, $\theta = \{s, r\}$, is *publicly* observable. This case provides a useful benchmark in order to evaluate the impact of asymmetric information later on. We proceed backwards by first examining banks' liquidity management at $t = 1$ and then considering banks' portfolio choice at $t = 0$. Since the risk of banks' long-term assets is publicly known, there will be two interbank markets: one for safer borrowers, $\theta = s$, and one for riskier borrowers, $\theta = r$. We denote the gross interest rate in each market by $1 + r_\theta$.

[12] The screening of borrowers is not possible in this set-up as all banks demand the same loan size and there is no readily available collateral they can pledge.

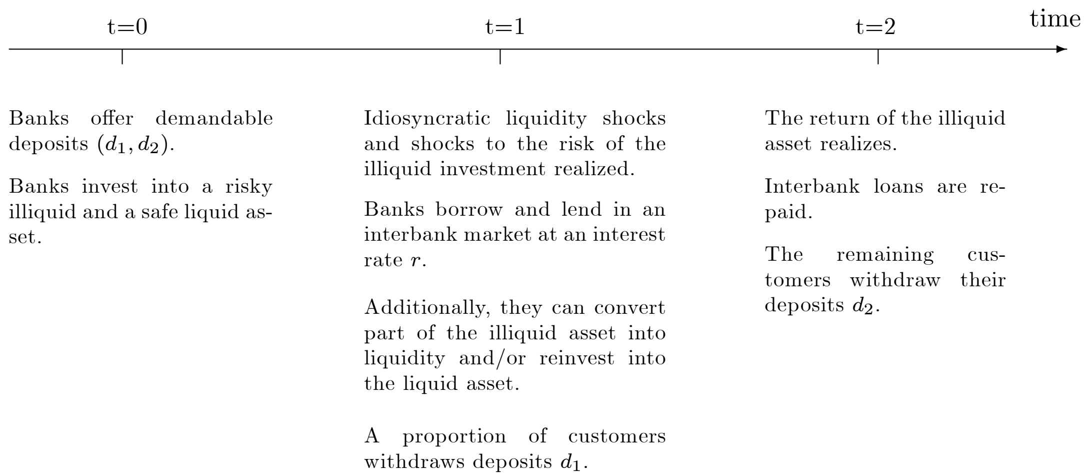

Figure 4: The timing of events

Having received the liquidity shock, $k = \{l, h\}$, and the shock to the risk of their long-term asset, $\theta = \{s, r\}$, banks manage liquidity at $t = 1$ in order to maximize expected profits at $t = 2$ while taking their asset allocation $(\alpha, 1-\alpha)$ as given. A bank that received a high liquidity shock, type-(h, θ), has a liquidity shortage at $t = 1$. It can obtain liquidity by borrowing an amount $L_{h,\theta}$ in the interbank market and by liquidating a fraction $\alpha^L_{h,\theta}$ of its long-term investment. It can also reinvest a fraction $\alpha^R_{h,\theta}$ of its liquidity in the short-term asset. The optimization problem at $t = 1$ of a bank with a liquidity shortage is:

$$\max_{L_{h,\theta},\alpha^L_{h,\theta},\alpha^R_{h,\theta}} p_\theta[R\alpha(1-\alpha^L_{h,\theta}) + \alpha^R_{h,\theta}(1-\alpha+\alpha^L_{h,\theta}\alpha l_\theta) - (1+r_\theta)L_{h,\theta} - (1-\lambda_h)d_2] \tag{1}$$

subject to the resource constraint

$$\lambda_h d_1 + \alpha^R_{h,\theta}(1-\alpha+\alpha^L_{h,\theta}\alpha l_\theta) \leq 1-\alpha+\alpha^L_{h,\theta}\alpha l_\theta + L_{h,\theta} \tag{2}$$

and the feasibility constraints $L_{h,\theta} \geq 0$, $0 \leq \alpha^L_{h,\theta} \leq 1$, $0 \leq \alpha^R_{h,\theta} \leq 1$.

With probability p_θ a bank is solvent at $t = 2$ and its profits are composed of: 1) the return on its long-term investments, i.e., R per unit invested and not liquidated; 2) the proceeds from reinvesting into the short-term asset; minus 3) the repayment of the interbank loans plus interest; and 4) the payout to $t = 2$ customers. With probability $1 - p_\theta$ a bank is insolvent and is taken over by the regulator. As it is protected by limited liability, its profits are zero in this case. The resource constraint requires that the outflow of liquidity at $t = 1$ (withdrawals and reinvestment into the short-term asset) be matched by the inflow (return on the short-term asset, proceeds from liquidation, and the amount borrowed).

A bank that received a low liquidity shock, type-(l, θ), has a liquidity surplus. It can lend to safer and riskier banks in the interbank market (an amount $L^s_{l,\theta}$ and $L^r_{l,\theta}$, respectively), reinvest in the short-term asset (fraction $\alpha^R_{l,\theta}$ of its liquidity) and liquidate part of its long-term asset (fraction $\alpha^L_{l,\theta}$). The optimization problem at $t = 1$ of a bank with a liquidity surplus is:

$$\max_{L^s_{l,\theta}, L^r_{l,\theta}, \alpha^L_{l,\theta}, \alpha^R_{l,\theta}} p_\theta[R\alpha(1-\alpha^L_{l,\theta}) + \alpha^R_{l,\theta}(1-\alpha+\alpha^L_{l,\theta}\alpha l_\theta) + \hat{p}_s(1+r_s)L^s_{l,\theta} + \hat{p}_r(1+r_r)L^r_{l,\theta} - (1-\lambda_l)d_2] \quad (3)$$

subject to the resource constraint

$$\lambda_l d_1 + \alpha^R_{l,\theta}(1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta) + L^s_{l,\theta} + L^r_{l,\theta} \leq 1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta \quad (4)$$

and the feasibility constraints $L^s_{l,\theta} \geq 0$, $L^r_{l,\theta} \geq 0$, $0 \leq \alpha^L_{l,\theta} \leq 1$, $0 \leq \alpha^R_{l,\theta} \leq 1$.

The key difference between (1) and (3) is that a bank with a liquidity surplus is exposed to counterparty risk when lending in the unsecured interbank market. A lender who is fully diversified across interbank loans of type θ collects the repayment on just a fraction $\hat{p}_\theta$ of his interbank loan portfolio since some borrowers are insolvent at $t = 2$. The difference between (2) and (4) is that the amounts lent in the interbank market appear as outflows in the resource constraint of a bank with a liquidity surplus.

Banks' liquidity management at $t = 1$ determines at which interbank interest rates i) banks with a liquidity surplus are willing to lend and ii) banks with a liquidity shortage are willing to borrow.

Proposition 1 (Liquidity management, no asymmetric information) *Banks with a liquidity surplus are willing to lend to type-(h, θ) banks (those with a liquidity shortage) if and only if the interbank interest rate r_θ satisfies:*

$$\frac{1}{\hat{p}_\theta} \leq 1 + r_\theta. \quad (5)$$

Suppose that banks with a liquidity surplus are willing to lend. Then type-(h, θ) banks borrow in the *interbank market if and only if the interbank interest rate r_θ satisfies:*

$$1 + r_\theta \leq \frac{R}{l_\theta}. \tag{6}$$

The lower bound on the interest rate $1 + r_\theta$ is given by the participation constraint of banks with a liquidity surplus. Their outside opportunity is to reinvest in the short-term asset. If the expected return on risky lending, $\hat{p}_\theta(1 + r_\theta)$, is lower than the riskless return on the short-term asset, then banks with a liquidity surplus do not lend in the unsecured interbank market.

The upper bound is given by the participation constraint of banks with a liquidity shortage. Their outside opportunity to borrowing in the interbank market is to convert part of their long-term asset into liquidity. The opportunity cost is the foregone return on the long-term asset, R, times the illiquidity premium, $\frac{1}{l_\theta}$. If the interest rate is too high, then banks with a liquidity shortage do not borrow in the unsecured interbank market.

The participation constraint of banks with a liquidity surplus does not depend on the risk of their own long-term asset. It only depends on the counterparty risk of unsecured lending. In contrast, the participation constraint of banks with a liquidity shortage depends on the type of their long-term investment: The opportunity cost of liquidating is lower for the safer long-term asset.[13]

We concentrate on the case in which all banks manage their liquidity in the unsecured interbank market. Proposition 1 determines a range of feasible interest rates:

$$\frac{1}{\hat{p}_\theta} \leq 1 + r_\theta \leq \frac{R}{l_\theta}. \tag{7}$$

The following corollary to Proposition 1 characterizes banks' liquidation and reinvestment decisions at $t = 1$ when all banks participate in the interbank market.

Corollary 1 *If all types (k, θ) of banks manage their liquidity at $t = 1$ using the interbank market, then they do not reinvest in the short-term asset and they do not liquidate their long-term*

[13] The opportunity cost is determined by $\frac{R}{l}$. Hence, it is not essential whether the return R or liquidation value l is type-specific. An alternative set-up with the *same* liquidation proceeds $l_s = l_r = l$ and type-specific returns $R_s < R_r$, with $p_s > p_r$ (safer asset succeeds more often but returns less) would also deliver the prediction that the opportunity cost of liquidating the safer long-term asset is lower: $\frac{R_s}{l} < \frac{R_r}{l}$. The participation decisions would be unchanged. See Appendix B for a detailed derivation of this alternative set-up.

investment, $\alpha^R_{k,\theta} = \alpha^L_{k,\theta} = 0$*.*

A bank with a liquidity shortage that participates in the interbank market finds the cost of liquidation too high (equation 6) and does not liquidate. Moreover, borrowing and reinvesting in the short-term asset is not profitable. The cost of borrowing is larger than the benefit of reinvesting. This is because $1 + r_\theta > \hat{p}_\theta(1 + r_\theta) \geq 1$ where the latter inequality holds since banks with a liquidity surplus participate in the interbank market. Similarly, a bank with a liquidity surplus that participates in the interbank market prefers not to reinvest in the short-term asset (equation 5). Moreover, the expected return on risky lending is not high enough to warrant the costly liquidation of the long-term asset by lenders. If banks with a liquidity shortage prefer not to liquidate, then banks with a liquidity surplus do not liquidate either: $\hat{p}_\theta(1 + r_\theta) < 1 + r_\theta \leq \frac{R}{l_\theta}$ where the latter inequality holds since banks with a liquidity shortage participate in the interbank market.

At $t = 0$ banks make their portfolio choice α. When making the choice, they take the price of unsecured interbank loans, $1 + r_\theta$, as given. The price in turn must be consistent with banks' portfolio choice. Banks will not invest everything in the short-term asset nor will they invest everything in the long-term asset. The profitability of the long-term asset, $pR > 1$, implies that $\alpha < 1$. The need to meet withdrawals at $t = 1$, $d_1 > 0$, implies that $\alpha > 0$.

Suppose that all banks manage their liquidity at $t = 1$ using the interbank market. In this case, an interior portfolio allocation α solves

$$\begin{aligned}\max_{0<\alpha<1} \quad & \pi_l p \left[R\alpha + \hat{p}_s(1 + r_s)L^s_l + \hat{p}_r(1 + r_r)L^r_l - (1 - \lambda_l)d_2\right] \\ + \quad & \pi_h \left[qp_s\left(R\alpha - (1 + r^s)L_h - (1 - \lambda_h)d_2\right) + (1 - q)p_r\left(R\alpha - (1 + r^r)L_h - (1 - \lambda_h)d_2\right)\right]\end{aligned} \tag{8}$$

subject to

$$L^s_l + L^r_l = 1 - \alpha - \lambda_l d_1 \tag{9}$$

$$L_h = \lambda_h d_1 - (1 - \alpha). \tag{10}$$

Banks are identical at $t = 0$ since the shocks to liquidity and to the riskiness of the long-term asset have not yet materialized. The objective function of banks at $t = 0$ is therefore the expectation over $k = \{l, h\}$ and $\theta = \{s, r\}$ of their objective function at $t = 1$. The optimization problem makes use of Corollary 1. When all banks manage their liquidity using the interbank market, then there is no liquidation and no reinvestment in the short-term asset at $t = 1$. Banks' resource constraints are binding. Type-(h, θ) banks borrow the entire liquidity shortfall and type-(l, θ) banks lend their entire liquidity surplus. Since the risk of a bank's long-term asset does not affect the amount it borrows or lends, we drop the subscript θ from $L_{h,\theta}$, $L^s_{l,\theta}$ and $L^r_{l,\theta}$.

When both safer and riskier banks with a liquidity shortage borrow in the interbank markets, then banks with a liquidity surplus must expect the same return from lending to each type of borrower (see the proof of Proposition 1 for the formal derivation):

$$\hat{p}_s(1 + r_s) = \hat{p}_r(1 + r_r). \tag{11}$$

Since lenders hold a fully diversified portfolio of interbank loans, the proportion of loans to type-(h, θ) banks that will be repaid is given by the proportion of type-(h, θ) banks that are solvent at $t = 2$:

$$\hat{p}_\theta = p_\theta. \tag{12}$$

The following Proposition states the interbank interest rates that are consistent with an interior portfolio allocation at $t = 0$:

Proposition 2 (Pricing of liquidity) *If all types (k, θ) of banks manage their liquidity using the interbank market, then the interest rate for unsecured loans to type-(h, θ) banks is*

$$1 + r_\theta = \frac{p}{p_\theta}\left(\frac{1}{p\pi_l + \pi_h}\right) R \tag{13}$$

where $p \equiv qp_s + (1 - q)p_r$ is the expected probability that the long-term asset succeeds.

The interest rate, i.e., the price of liquidity traded in the unsecured interbank market, is effectively given by a no-arbitrage condition. Equation (13) can be written as

$$(\pi_l p p_\theta + \pi_h p_\theta)(1 + r_\theta) = pR. \tag{14}$$

The right-hand side is the expected return from investing an additional unit into the long-term asset.

At $t = 0$, banks expect their long-term asset to succeed with probability p. The left-hand side is the expected return from investing an additional unit into the short-term asset. With probability π_h, a bank will have a liquidity shortage at $t = 1$. One more unit of the short-term asset saves on borrowing at an expected cost of $p_\theta(1 + r_\theta)$, where p_θ is the probability that a borrower of type θ has to repay his loans. With probability π_l, a bank will have a liquidity surplus. One more unit of the short-term asset can be lent out at an expected return $pp_\theta(1 + r_\theta)$. A lender receives the loan repayment only when both he and his counterparties are solvent, which occurs with probability pp_θ.

We can write (13) as:

$$1 + r_\theta = \frac{p}{p_\theta} \frac{1}{\delta} R \tag{15}$$

where

$$\frac{1}{\delta} \equiv \frac{1}{p\pi_l + \pi_h} > 1 \tag{16}$$

denotes a common premium on risky interbank debt. Lenders require a compensation for counterparty risk. The premium δ is "common" since it depends only on the average success probability of the long-term asset, p. The price of liquidity is adjusted downwards for safer borrowers since $\frac{p}{p_s} < 1$. Conversely, it is adjusted upwards for riskier borrowers since $\frac{p}{p_r} > 1$.

The next proposition shows when the pricing of liquidity (15) is consistent with the range of feasible interest rates (7).

Proposition 3 (No asymmetric information) *When the risk of the long-term asset is publicly observable and all banks manage their liquidity in the interbank market, the adjusted premium on risky interbank debt is smaller than the illiquidity premium of the long-term asset:* $\frac{p}{p_\theta}\frac{1}{\delta} \leq \frac{1}{l_\theta}$ *for* $\theta = \{s, r\}$.

Banks with a liquidity shortage will borrow in the interbank market only if the premium on risky interbank debt - adjusted for their observable risk - is smaller than the illiquidity premium. Banks with a liquidity surplus, however, are always willing to lend since

$$p_s (1 + r_s) = p_r (1 + r_r) = \frac{pR}{\delta} > pR > 1.$$

The amounts invested in the short-term and long-term asset are determined by market clearing in the interbank market. When all banks participate in the interbank market, the market clearing conditions are

$$\begin{aligned} \pi_l L_l^s &= \pi_h q L_h \\ \pi_l L_l^r &= \pi_h (1-q) L_h. \end{aligned}$$

Using (9) and (10), market clearing yields:

$$1 - \alpha = \lambda d_1. \tag{17}$$

The amount invested in the short-term asset exactly covers the amount of expected withdrawals. The interbank market fully smoothes out banks' idiosyncratic liquidity shocks, $k = \{l, h\}$.

Banks' expected profits when everybody participates in the interbank market are given by (using (13) and (17)):

$$\Pi(d_1, d_2) = p[R(1 - \hat{\lambda} d_1) - (1 - \lambda) d_2], \tag{18}$$

where $\hat{\lambda} \equiv \hat{\pi}_l \lambda_l + \hat{\pi}_h \lambda_h$, $\hat{\pi}_l \equiv \frac{p}{\delta} \pi_l$, and $\hat{\pi}_h \equiv \frac{1}{\delta} \pi_h$ (note that $\hat{\pi}_l + \hat{\pi}_h = 1$).

In order to give an interpretation for this expression, it is useful to first consider the case when there is no counterparty risk, $p_\theta = 1$, i.e. when the long-term investment is safe. Without counterparty risk, banks with a shortage of liquidity are always willing to participate in the interbank market as well. There is no premium on interbank debt, $\delta = 1$. The condition in Proposition 3 reduces to $l_\theta \leq 1$, which is always satisfied since liquidation is costly. Banks' expected profits without counterparty risk are

$$\Pi^*(d_1, d_2) = R(1 - \lambda d_1) - (1 - \lambda) d_2. \tag{19}$$

Banks invest as much as possible in the profitable long-term asset provided that they are able to satisfy average withdrawals at $t = 1$, $\alpha = 1 - \lambda d_1 < 1$. The interbank market smoothes out the idiosyncratic deviations in withdrawals. Banks keep the return from the long-term asset, $R(1 - \lambda d_1)$, minus average withdrawals at $t = 2$, $(1 - \lambda) d_2$.

Comparing the profits in (19) with those in (18) shows that the risk of the long-term asset reduces banks' profits. More risk not only makes it less likely that the bank survives, but it also makes it more expensive to smooth out the liquidity shocks in the unsecured interbank market since lenders must be paid a premium. It is as if banks needed to hold more liquidity in order to satisfy higher average withdrawals at $t = 1$, $\hat{\lambda} > \lambda$.

4 Asymmetric information

In this section we assume that the shock to the risk of the long-term asset at $t = 1$, $\theta = \{s, r\}$ is *privately* observed. While each bank gets to know the risk of its own long-term investment after it chose its portfolio, banks no longer know the risk type of their counterparty when they borrow and lend in the unsecured interbank market. As before, we solve the model backwards by first examining banks' liquidity management at $t = 1$ and then their portfolio choice at $t = 0$.

We derive different regimes in the unsecured interbank market under asymmetric information. First, there can be full participation of all banks in the market. Second, there can be adverse selection when safer banks with a liquidity shortage prefer not to borrow in the unsecured interbank market. Third, the market can break down. The break down can occur because banks with a liquidity surplus prefer to hoard liquidity instead of lending it out or because no bank wants to borrow anymore.

4.1 Liquidity management

The objective function at $t = 1$ of a bank with a liquidity shortage is

$$\max_{L_{h,\theta}, \alpha^L_{h,\theta}, \alpha^R_{h,\theta}} p_\theta [R\alpha(1 - \alpha^L_{h,\theta}) + \alpha^R_{h,\theta}(1 - \alpha + \alpha^L_{h,\theta}\alpha l_\theta) - (1 + r)L_{h,\theta} - (1 - \lambda_h)d_2]. \tag{20}$$

The difference compared to the objective function of a type-(h, θ) bank in the benchmark case (equation (1)) is that the interest rate r can no longer be indexed by the type θ since the risk of the long-term asset is not publicly observable.

The objective function at $t = 1$ of bank with a liquidity surplus is

$$\max_{L_{l,\theta},\alpha^L_{l,\theta},\alpha^R_{l,\theta}} p_\theta[R\alpha(1-\alpha^L_{l,\theta}) + \alpha^R_{l,\theta}(1-\alpha+\alpha^L_{l,\theta}\alpha l_\theta) + \hat{p}(1+r)L_{l,\theta} - (1-\lambda_l)d_2]. \tag{21}$$

The difference to the objective function of a type-(l,θ) bank in the benchmark case (equation (3)) is the expected loan repayment. A bank with a liquidity surplus can no longer distinguish between safer and riskier borrowers. Neither the amount lent, $L_{l,\theta}$, nor the interest rate r, nor the expected fraction of repaid loans, $\hat{p}$, can be indexed by the risk type of borrowers (the θ in the subscript of L denotes the risk type of lenders).

The changes in the objective function of banks at $t = 1$ with asymmetric information lead to the following analogue of Proposition 1:

Proposition 4 (Liquidity management under asymmetric information) *Banks with a liquidity surplus are willing to lend if and only if the interbank interest rate r satisfies:*

$$\frac{1}{\hat{p}} \leq 1+r. \tag{22}$$

Suppose that banks with a liquidity surplus are willing to lend. Then type-(h,θ) banks borrow in the interbank market if and only if the interbank interest rate r satisfies:

$$1+r \leq \frac{R}{l_\theta}. \tag{23}$$

As in the case without asymmetric information, the lower bound on the interest rate is given by the participation constraint of banks with a liquidity surplus while the upper bound is given by the participation constraint of banks with a liquidity shortage (see Proposition 1). The key difference is that the interest rate r and counterparty risk p are no longer indexed.

Proposition 4 implies that there are four possible outcomes: i) all banks participate in the unsecured interbank market; ii) all banks with a liquidity surplus and riskier banks with a liquidity shortage participate; iii) banks with a liquidity surplus do not participate; and iv) banks with liquidity shortage do not participate. In the following sections, we analyze each of the four cases.

4.2 Regime 1: Full participation of all banks

In order to derive the interest rate and characterize the regime with full participation of all banks in the interbank market (Regime 1), we start by assuming that there is indeed full participation and then derive the parameter restrictions this entails.

Let r_1 denote the interest rate in Regime 1. According to Proposition 4, the interest rate must lie in the following interval when all banks manage their liquidity in the interbank market:

$$\frac{1}{\hat{p}} \leq 1 + r_1 \leq \frac{R}{l_s}. \tag{24}$$

The upper bound is determined by safer banks with a liquidity shortage since they have a lower opportunity cost of liquidation than riskier banks. Analogous to the benchmark case, the next result follows directly from Proposition 4 and characterizes banks' liquidation and reinvestment decisions at $t = 1$ in Regime 1.

Corollary 2 *If all types (k, θ) of banks manage their liquidity at $t = 1$ using the interbank market, they do not reinvest in the short-term asset and they do not liquidate their long-term investment, $\alpha^R_{k,\theta} = \alpha^L_{k,\theta} = 0$.*

Let α_1 denote the fraction that banks invest in the long-term asset at $t = 0$. As in the benchmark case, banks take the price of unsecured interbank loans as given when choosing their portfolio. When all banks manage their liquidity using the the interbank market, α_1 solves:

$$\begin{aligned} \max_{0<\alpha_1<1} \quad & \pi_l \left(qp_s + (1-q)p_r\right)\left[R\alpha_1 + \hat{p}(1+r_1)L_l - (1-\lambda_l)d_2\right] \\ + \quad & \pi_h \left(qp_s + (1-q)p_r\right)\left[R\alpha_1 - (1+r_1)L_h - (1-\lambda_h)d_2\right] \end{aligned} \tag{25}$$

subject to

$$L_l = 1 - \alpha_1 - \lambda_l d_1 \tag{26}$$

$$L_h = \lambda_h d_1 - (1 - \alpha_1) \tag{27}$$

where we have used Corollary 2. The change relative to the benchmark case without asymmetric information is that there is now a single interbank market for all borrowers' risk types.

The first-order condition for a bank's optimal portfolio allocation across the short-term and long-term assets requires that:

$$(\pi_l p\hat{p} + \pi_h p)(1 + r_1) = pR \tag{28}$$

where $p \equiv qp_s + (1-q)p_r$. As in (14), the interbank interest rate r_1 is given by a no-arbitrage condition. The right-hand side is the expected return from investing an additional unit into the long-term asset, pR. The left-hand side is the expected return from investing an additional unit into the short-term asset that can either be lent out or saves on borrowing. Lenders expect to receive the loan repayment with probability $\hat{p}$. Banks' own ex-ante probability of being solvent at $t = 2$, p, affects the expected return of both the short-term and the long-term asset and therefore cancels out.

Given that all banks with a liquidity shortage borrow in the interbank market, lenders expect the proportion of repaid interbank loans or, equivalently, the probability of loan repayment to be equal to the expected proportion of solvent banks at $t = 2$:

$$\hat{p} = p \tag{29}$$

where $p \equiv qp_s + (1-q)p_r$.

Using (16), (28), (29) and Proposition 4, we can state the following proposition:

Proposition 5 (Regime 1) *Under asymmetric information about the risk of the long-term asset, if all banks manage their liquidity in the interbank market, the interest rate is given by*

$$1 + r_1 = \frac{R}{\delta} \tag{30}$$

and the common premium for risky interbank debt is smaller than the illiquidity premium for the safer long-term asset, $\frac{1}{\delta} \leq \frac{1}{l_s}$.

The interest rate in Regime 1 is determined solely by the common premium for risky interbank debt, $\frac{1}{\delta}$, in contrast to the interest rates in the benchmark case without asymmetric information (equation (15)). It is no longer possible to adjust for the risk type θ of borrowers. Safer borrowers pay a higher interest rate compared to the benchmark whereas riskier borrowers pay a lower rate:

$$r_s < r_1 < r_r.$$

Riskier borrowers exert an externality on safer borrowers in Regime 1 since the latter subsidize the former.

As in the benchmark case, banks with a liquidity surplus are always willing to lend when all banks participate in the interbank market since $p(1 + r_1) = \frac{pR}{\delta} > 1$. As long as the common premium for risky interbank debt is not too high, all banks with a liquidity shortage prefer to borrow. As the premium becomes higher, the first borrowers to drop out of the interbank market are the safer ones (Corollary 4). Hence, Regime 1 ceases to be an equilibrium when $\frac{1}{\delta} > \frac{1}{l_s}$.

Asymmetric information shrinks the set of parameters for which all banks manage liquidity in the interbank market. Suppose full participation in the interbank market is an equilibrium in the benchmark case. Then $\frac{p}{p_s}\frac{1}{\delta} \leq \frac{1}{l_s}$ holds (Proposition 3). If, however, $\frac{1}{\delta} > \frac{1}{l_s}$ holds, then full participation is not an equilibrium under asymmetric information (Proposition 5).

Since all banks manage their liquidity in the interbank market in Regime 1, the amounts invested in the short-term and long-term asset $(1 - \alpha_1, \alpha_1)$ are as in the benchmark case. Market clearing in the interbank market, $\pi_l L_l = \pi_h L_h$, yields

$$1 - \alpha_1 = \lambda d_1 \tag{31}$$

where we have used (26) and (27). The amount invested in the short-term asset exactly covers the amount of expected withdrawals. Using the interest rate (30) and the portfolio choice (31), we can also show that banks' expected profits in Regime 1 are the same as in the benchmark case (18), $\Pi_1(d_1, d_2) = \Pi(d_1, d_2)$.

In sum, there is no impairment to market functioning in Regime 1 despite asymmetric information. All banks manage their liquidity in the unsecured interbank market and there is no costly liquidation of the long-term asset. Banks' portfolio allocation and expected profits are the same as in the benchmark case. Asymmetric information, however, shrinks the set of parameters for which all banks participate in the market. It therefore amplifies the fundamental friction in the model, i.e., counterparty risk of unsecured interbank lending.

4.3 Regime 2: Adverse selection in the interbank market

The previous section showed that asymmetric information shrinks the set of parameters for which all banks manage their liquidity in the interbank market. Safer banks with a liquidity shortage may find the mispricing caused by the riskier borrowers too large, i.e. the interest rate too high, relative to the cost of obtaining liquidity outside the unsecured market. Lenders then face an adverse selection of riskier borrowers. We follow the same steps as in the previous sections. We start by assuming that there is adverse selection in the interbank market (Regime 2) and then derive the parameter restrictions this entails.

When banks with a liquidity surplus lend to an adverse selection of riskier banks, then Proposition 4 specifies the following range for the interbank interest rate r_2:

$$\frac{1}{\hat{p}} \leq 1 + r_2 \leq \frac{R}{l_r} \tag{32}$$

$$\frac{R}{l_s} < 1 + r_2. \tag{33}$$

The interest rate must be high enough for banks with a liquidity surplus to be willing to lend and for safer banks with a liquidity shortage not to be willing to borrow. At the same time, the interest rate must be low enough so that riskier banks with a liquidity shortage prefer to borrow.

The next result characterizes banks' liquidation and reinvestment decisions at $t = 1$.[14]

Corollary 3 *If all banks except safer banks with a liquidity shortage manage their liquidity using the interbank market, then no bank reinvests in the short-term asset: $\alpha^R_{k,\theta} = 0$. Banks with a liquidity surplus and riskier banks with a liquidity shortage do not liquidate: $\alpha^L_{l,\theta} = \alpha^L_{h,r} = 0$. Safer banks with a liquidity shortage liquidate part of their long-term asset in order to satisfy withdrawals at $t = 1$: $\alpha^L_{h,r} > 0$.*

Let α_2 denote the fraction that banks invest in the long-term asset at $t = 0$. When banks expect

[14] Under adverse selection, it may be the case that safer banks with a liquidity surplus liquidate their long-term asset in order to profit from lending at high rates, in particular at rates such that $p_r(1 + r_2) > \frac{R}{l_s}$. This case, however, rules out liquidity hoarding, which we document in Figure 1 (since $p_r(1 + r_2) > \frac{R}{l_s} > 1$, see Section 4.4 for details). Hence, it does not seem relevant for the 2007-09 crisis. We proceed under the assumption that lenders do not liquidate their long-term asset to be able to lend more at $t = 1$: $p_r(1 + r_2) \leq \frac{R}{l_s}$.

adverse selection in the interbank market at $t = 1$, their portfolio choice α_2 solves:

$$\begin{aligned}\max_{0<\alpha_2<1} \quad & \pi_l p[R\alpha_2 + \hat{p}(1+r_2)L_l - (1-\lambda_l)d_2] \\ & +\pi_h q p_s[R\alpha_2(1-\alpha^L_{h,s}) - (1-\lambda_h)d_2] \\ & +\pi_h(1-q)p_r[R\alpha_2 - (1+r_2)L_{h,r} - (1-\lambda_h)d_2]\end{aligned} \tag{34}$$

subject to

$$\begin{aligned} L_l &= 1 - \alpha_2 - \lambda_l d_1 \\ L_{h,r} &= \lambda_h d_1 - (1-\alpha_2) \\ \alpha^L_{h,s}\alpha_2 l_s &= \lambda_h d_1 - (1-\alpha_2) \end{aligned}$$

where we have used Corollary 3.

In contrast to Regime 1 (full participation), a safer bank with a liquidity shortage no longer borrows in the interbank market and instead liquidates part of its long-term asset. This occurs with probability $\pi_h q p_s$. Instead of interbank loans L_h the liquidation proceeds $\alpha^L_{h,s}\alpha_2 l_s$ have to make up for the shortfall in liquidity at $t = 1$.

The first-order condition for an optimal portfolio allocation under adverse selection is:

$$\pi_l p\hat{p}(1+r_2) + \pi_h\left[(1-q)p_r(1+r_2) + qp_s\frac{R}{l_s}\right] = pR. \tag{35}$$

One more unit of the short-term asset saves a bank with a liquidity shortage the cost of borrowing, $1 + r_2$, or liquidation, $\frac{R}{l_s}$. The return on the short-term asset for a bank with a liquidity surplus depends on the level of counterparty risk, i.e., the probability of not being repaid $1 - \hat{p}$. Since only riskier banks borrow, the level of counterparty risk is higher than under full participation:

$$\hat{p} = p_r < p.$$

We can rewrite condition (35) to obtain the interbank interest rate in Regime 2:

$$1 + r_2 = \frac{R}{l_s} \left(\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} \right) \tag{36}$$

where $\frac{1}{\delta_2}$ denotes the premium for risky interbank debt under adverse selection:

$$\frac{1}{\delta_2} \equiv \frac{1}{\pi_l p_r + \pi_h}.$$

The premium is higher than in the full participation case, $\frac{1}{\delta_2} > \frac{1}{\delta}$, since counterparty risk is higher under adverse selection.

Using (36), we can write the condition that safer banks with a liquidity shortage drop out of the interbank market (33) as

$$\frac{1}{\delta_2} > \frac{1}{l_s}. \tag{37}$$

A necessary condition for adverse selection in the interbank market to be an equilibrium is that the premium for risky interbank debt under adverse selection is higher than the illiquidity premium for the safer long-term asset. Condition (33) also implies that the equilibrium interest rate with adverse selection is indeed higher than with full participation (see condition (24)):

$$1 + r_2 > \frac{R}{l_s} \geq 1 + r_1.$$

Using the expression for the interest rate (36), the interval (32) becomes:

$$\frac{l_s}{p_r R} \leq \frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} \leq \frac{l_s}{l_r}. \tag{38}$$

A necessary condition for riskier banks with a liquidity shortage to borrow in the interbank market (i.e. for the upper bound to hold) is that the premium for risky interbank debt under adverse selection is smaller than the illiquidity premium for the riskier long-term assets, $\frac{1}{\delta_2} < \frac{1}{l_r}$.

Unlike in the case of full participation, banks with a liquidity surplus may not want to lend since their participation constraint (the lower bound in (38)) is no longer satisfied automatically. A sufficient condition for their participation constraint to hold is that the expected return on the riskier long-term asset is larger than the cost of liquidation for the safer long-term asset, $p_r R > l_s$.[15]

The following proposition summarizes the results:

Proposition 6 (Regime 2) *If there is adverse selection in the unsecured interbank market, i.e. safer banks with a liquidity shortage do not borrow, the interbank interest rate is given by (36) and conditions (37) and (38) hold.*

The set of parameters for which conditions (37) and (38) hold is non-empty: $l_r < l_s$ and the lower bound of (38) involves R, which is not present in the upper bound of (38) or in (37).

The amounts invested in the short-term and long-term asset are determined by market clearing in the interbank market, $\pi_l L_l = \pi_h (1-q) L_{h,r}$:

$$\alpha_2 = 1 - d_1(\tilde{\pi}_l \lambda_l + \tilde{\pi}_h \lambda_h) \tag{39}$$

where $\tilde{\pi}_l \equiv \frac{\pi_l}{\pi_l + \pi_h(1-q)} > \pi_l$ and $\tilde{\pi}_h \equiv \frac{\pi_h(1-q)}{\pi_l + \pi_h(1-q)} < \pi_h$ (note that $\tilde{\pi}_l + \tilde{\pi}_h = 1$). Since $\lambda_l < \lambda_h$, we have $\alpha_2 > \alpha_1$. Banks choose a less liquid portfolio when they expect adverse selection in the interbank market since fewer banks will demand liquidity in the interbank market.

When compared to full participation, adverse selection in the interbank market: i) leads to a higher interest rate for unsecured interbank loans, ii) may lead to a binding participation constraint for banks with a liquidity surplus, and iii) has banks choose a less liquid portfolio ex ante. Moreover, it may lead to lower expected bank profits:

Proposition 7 (Bank profits under adverse selection) *Adverse selection leads to lower expected bank profits than under full participation,* $\Pi_2(d_1, d_2) < \Pi_1(d_1, d_2) = \Pi(d_1, d_2)$*, if and only if* $l_s < \frac{p_s}{p}\delta$.

Note that if all banks participate in the interbank market in the benchmark case of full information, the condition above is always satisfied (see Proposition 3). Moreover, a simple sufficient condition for profits under adverse selection to be lower is $l_s \leq p_s$. The condition is therefore satisfied if safer banks succeed with high probability.

[15] Sicne condition (37) must hold under adverse selection, we have $l_s - \pi_h \frac{qp_s}{p} > \delta_2 - \pi_h \frac{qp_s}{p}$ and hence $\frac{l_s}{p_r R} < 1$ is sufficient for the lower bound of (38) to hold.

4.4 Breakdown of the interbank market

Figure 1 showed that as of the end of September 2008 banks seemed to be hoarding liquidity. In the context of our model, banks always prefer to lend out any excess liquidity in the full participation regime (Regime 1). The interest rate, $1+r_1 = \frac{R}{\delta}$, is high enough for banks with a liquidity surplus to prefer lending in the interbank market to reinvesting in the short-term asset. But liquidity hoarding becomes possible once safer banks with a liquidity shortage drop out of the interbank market. Although the interest rate increases to reflect the higher counterparty risk, the increase need not be large enough to compensate lenders for lending to an adverse selection of borrowers.

When the interest rate under adverse selection is such that

$$p_r(1+r_2) < 1, \tag{40}$$

then banks with a liquidity surplus prefer to reinvest in the short-term asset, i.e., to hoard liquidity. Substituting for the interest rate from (36) and using the fact that adverse selection requires $\frac{1}{\delta_2} > \frac{1}{l_s}$, a necessary condition for liquidity hoarding is

$$p_r R < l_s. \tag{41}$$

Since $l_s < 1$, lenders only hoard liquidity if the riskier long-term investment turns out to be unprofitable. Note that (41) is compatible with the assumption about the ex ante efficiency of the long-term investment, $pR > 1$.

Liquidity hoarding leads to a breakdown of the interbank market since banks with a liquidity surplus no longer lend. But the market can also break down if banks with a liquidity shortage no longer borrow. We established that safer banks stop borrowing at lower rates than riskier banks causing adverse selection in the interbank market. But even riskier banks may choose to leave the unsecured market segment if adverse selection drives up the interest rate too much. This occurs when

$$1 + r_2 > \frac{R}{l_r}.$$

Substituting for the interest rate using (36), a sufficient condition for no borrowing is

$$\frac{1}{\delta_2} > \frac{1}{l_r},$$

i.e., the premium for risky interbank debt under adverse selection is higher than the illiquidity premium for the riskier long-term asset.

4.5 Multiple equilibria

We use the conditions derived in the previous sections to discuss the possibility of multiple equilibria in the interbank market. We provide proofs of the statements below in the Appendix.

Full participation in the interbank market (Regime 1) is the unique equilibrium if and only if

$$\frac{1}{\delta_2} \leq \frac{1}{l_s}.$$

The condition is equivalent to $1 + r_2 \leq \frac{R}{l_s}$ where r_2 is the interest rate under adverse selection (see equation (36)). If the interest rate that would arise under adverse selection is relatively low, safer borrowers prefer to stay in the market and hence Regime 2 cannot be an equilibrium.

Adverse selection in the interbank market is the unique equilibrium if and only if

$$\frac{1}{l_s} < \frac{1}{\delta}$$

and (38) holds. The condition is equivalent to $\frac{R}{l_s} < 1 + r_1$ where r_1 is the interest rate under full participation. In this case, Regime 1 is not an equilibrium since the interest rate is too high for safer borrowers to participate in the interbank market.

Liquidity hoarding is the unique equilibrium if and only if $\frac{1}{l_s} < \frac{1}{\delta}$ and the lower bound in (38) is violated. Similarly, no borrowing is the unique equilibrium if and only if $\frac{1}{l_s} < \frac{1}{\delta}$ and the upper bound in (38) is violated.

Since the premia on risky interbank debt are such that $\delta_2 < \delta$, our model admits multiple equilibria when

$$\frac{1}{\delta} \leq \frac{1}{l_s} < \frac{1}{\delta_2}. \tag{42}$$

Both full participation and adverse selection in the interbank market coexist as equilibria if conditions (42) and (38) hold. If banks expect full participation in the interbank market, the resulting gross interest rate $1 + r_1$ is smaller than $\frac{R}{l_s}$, which justifies banks' expectations. However, if banks expect adverse selection in the interbank market, the gross interest rate is $1 + r_2$, which is larger than $\frac{R}{l_s}$. Safer banks with a liquidity shortage drop out of the interbank market and banks' expectations are justified.

Both full participation and liquidity hoarding are equilibria when condition (42) holds and the lower bound in (38) is violated. Similarly, full participation and no borrowing coexist when (42) holds and the upper bound in (38) is violated.

The possibility of multiple equilibria creates scope for policy interventions that coordinate banks' expectations. We discuss this further in Section 6.2.

5 Discussion and empirical implications

Depending on parameters, three different outcomes are possible in our model: i) full participation and no impairment to the functioning of the interbank market, ii) adverse selection and higher interest rates, and iii) market breakdown. Figure 5 shows which outcome occurs under different values for the average success probability, p, and the dispersion of risk, $\Delta p \equiv p_s - p_r$. Since banks have private information about the risk of the illiquid asset, Δp is a measure of the severity of the asymmetric information problem.

When the average level of counterparty risk is low (high p), full participation is always an equilibrium in the interbank market. It is the unique equilibrium as long as the dispersion of risk is low as well. Asymmetric information about the risk of long-term assets does not impair the functioning of the interbank market in this case. However, an increase in the dispersion of risk alone, without an increase in the level of risk, leads to the full participation equilibrium coexisting with the adverse selection equilibrium (for intermediate dispersion levels) or with liquidity hoarding (for high dispersion levels). Hence, expectations start to matter and can be self-fulfilling.

If average counterparty risk is higher but dispersion remains low, adverse selection arises as the unique equilibrium. Safer banks with a liquidity shortage find the interest rate in the interbank market too high and prefer to obtain liquidity outside the unsecured interbank market. Only an

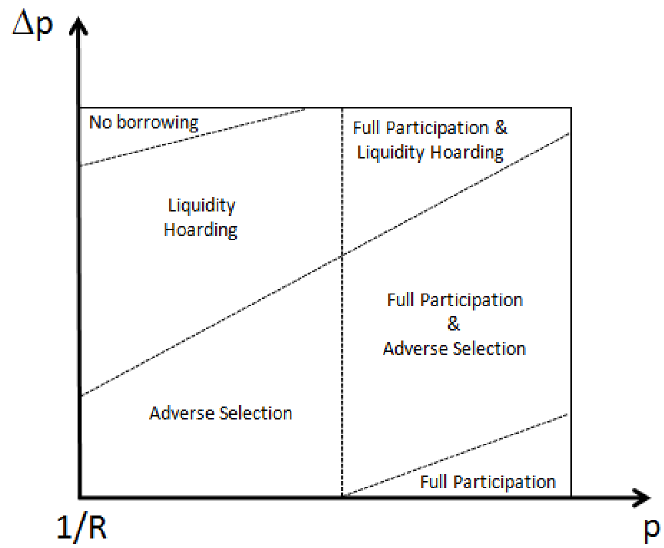

Figure 5: Equilibrium outcomes as a function of the level and dispersion of counterparty risk (drawn for the following parameter values: $R = 1.5$, $q = 0.66$, $\pi_l = 0.3$, $l_s = 0.95$, $l_r = 0.3$)

adverse selection of riskier banks keeps borrowing unsecured, causing the interest rate to increase. When higher average counterparty risk is combined with high dispersion, liquidity hoarding ensues. Lenders prefer to keep liquidity instead of lending it out despite the high rates borrowers would be willing to pay. Finally, when both the level and the dispersion of risk are high, the market breaks down because no bank wants to borrow.

The different outcomes in our model, i) no impairment, ii) adverse selection, and iii) liquidity hoarding, resemble three phases described in Figure 1: i) normal times, ii) turmoil with elevated spreads but no excess reserves, and iii) crisis with a further increase in spreads and substantial excess reserves. Interbank interest rates suddenly increased in August 2007. At that time, subprime-mortgage backed securities were discovered in portfolios of banks and bank-sponsored conduits (SIVs) leading to a reassessment of risk. The extent of exposures was unknown and counterparties could not distinguish safe from risky banks. In the context of our model, the rise in the interest rate can be attributed either to an increase in the perceived dispersion of risk, or to a deterioration in the underlying level of risk, or a combination of the two. Interbank rates rose to record-high levels and trading activity declined significantly following the dramatic events surrounding the last weekend of September 2008, when the financial crisis spread outside the realm of investment banking and into the global financial system.[16] These events can be interpreted as a further increase in the level and, importantly, in the dispersion of counterparty risk making the adverse selection problem more severe.[17] Moreover, one can view the effect of the rescue of Bear Stearns as initially placing a lower bound on the perceived probability of default of counterparties. But letting Lehman fail led to a drastic revision of expected default probabilities.

Since the possibility of a market breakdown due to liquidity hoarding by lenders is an important feature of our model, we examine the empirical evidence more closely. The major developments at the end of September are depicted in relation to flows and stocks of liquidity using daily data in Figures 6 and 7. Excess reserves start rising after the collapse of Washington Mutual, ten days after the Lehman failure (September 15, 2008). Importantly, the rise precedes the ECB announcement of a change in its liquidity provision policy on October 8, 2008.[18] In the week of September 29, 2008, the daily amounts of excess reserves averaged more than €169 billion (Figure 6).

At exactly the same time as excess reserves rose, the average daily volume in the overnight unsecured interbank market (Eonia) halved and the net amount of central bank liquidity outstanding dropped significantly (Figure 7).[19] The net amount of central bank liquidity outstanding is the total stock of liquidity provided minus the amount absorbed in all open market operations and recourses to the standing facilities. The Figures show that although the ECB provided large amounts of liquidity (see the spikes in the net stock of liquidity) throughout September 2008, banks were not holding excess reserves until the end of the month. Moreover, there is evidence that the set of banks holding excess reserves is not the same as the set of banks borrowing from the ECB. It

[16] Before the weekend of September 27-28, 2008 Washington Mutual, the largest S&L institution in the US was seized by the FDIC and sold to JPMorgan Chase. At the same time, negotiations on the TARP rescue package stalled in US Congress. Over the weekend, it was reported that British mortgage lender Bradford & Bingley had to be rescued and Benelux announced the injection of €11.2 billion into Fortis Bank. On the following Monday, Germany announced the rescue of Hypo Real Estate, and Iceland nationalized Glitnir.

[17] The fact that banks no longer trust each other amid perceptions that other banks are at risk of default was also pointed out by market commentators at the time, see, for example, "Central Banks Add Funds to Money Markets," The Wall Street Journal, September 29, 2008 and "Why the ECB Can't Fix Europe," Business Week, October 8, 2008.

[18] As of October 9, the deposit facility rate was increased from 100 to 50 basis points below the policy rate, thus making deposits relatively more attractive. Moreover, as from the operation settled on October 15, 2008, the weekly main refinancing operation was carried out through a fixed rate tender procedure with full allotment at the policy rate. Previously, banks had to bid demand schedules given an announced aggregate allotment.

[19] At the onset of the crisis in August 2007, the Eonia saw an *increase* in volume. The average daily volume was €40.91 billion in the year prior to August 9, 2007. It increased by 27%, to an average of €52.12 billion, between August 9, 2007 and September 26, 2008. This increase could reflect a substitution towards more short-term financing in the interbank market in Regime 2 as liquidity in longer-term segments of the market dried up. The drop in overnight volumes of more than €29 billion observed at the end of September 2008 is thus even more dramatic.

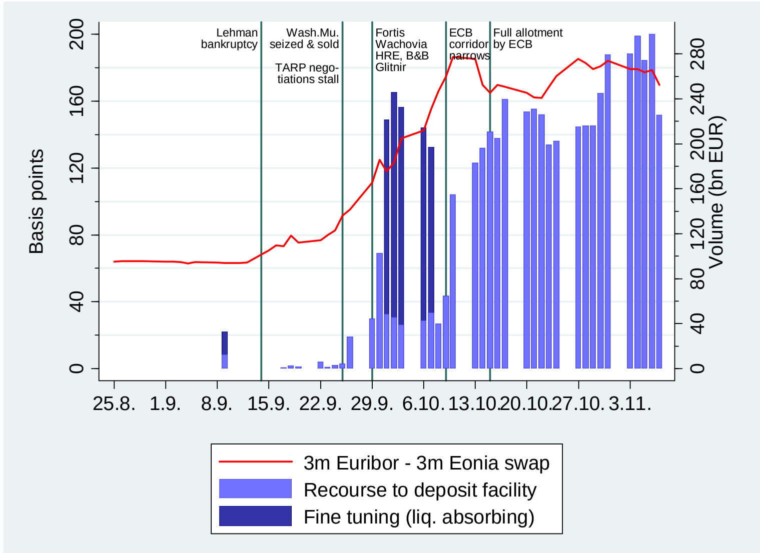

Figure 6: Interbank spread and excess reserves (recourses to the ECB deposit facility and liquidity-absorbing fine tuning operations), 08/2008 - 11/2008

follows that, as of the last weekend of September 2008, banks were hoarding liquidity and parking it as excess reserves rather than lending it out.

If the interbank market suffers from liquidity hoarding, two further implications follow from our model. First, a necessary condition for liquidity hoarding is that some banks are insolvent, i.e. $p_r R < 1$ (as implied by (41)). Tackling the roots of the problem therefore requires finding out who these banks are and recapitalizing (or closing) them. Indeed, the US government and banking regulators were assessing banks' risk and viability through a comprehensive "stress testing" exercise as of February 25, 2009. Second, increasing the rate at which excess reserves are renumerated reinforces liquidity hoarding. To see this, consider an increase in the right-hand side of (40) from 1 to $1 + \tau$, where τ is the interest earned on excess reserves. After making excess reserves more attractive by increasing the deposit facility rate from 100 to 50 basis points below the policy rate on October 9, 2008, the ECB lowered it back to 100 basis points on January 21, 2009.

Our model suggests that interbank interest rates can rise due to adverse selection. Safer banks leave the unsecured market since they have better alternatives for obtaining liquidity than riskier banks. This is consistent with anecdotal evidence about the reluctance of banks to borrow at high rates since the onset of the crisis in order to avoid "signaling" that they are bad banks. Moreover, there is evidence of "tiering" in interbank markets consistent with our model where differences

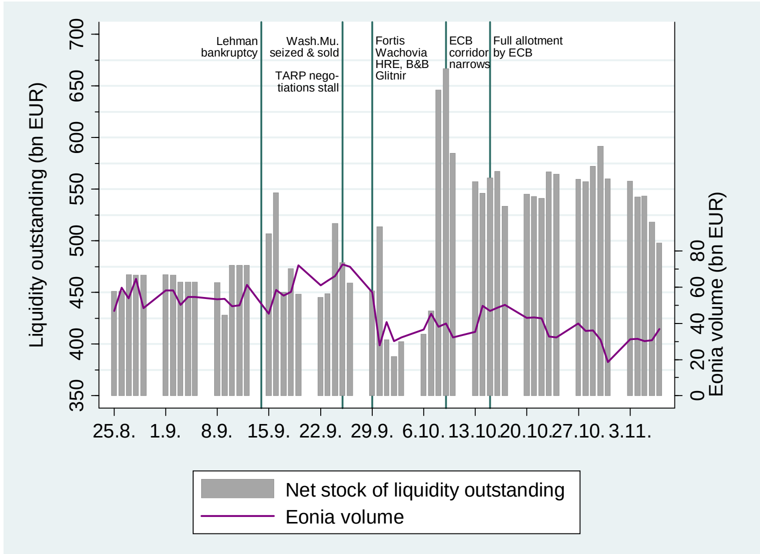

Figure 7: Net stock of central bank liquidity outstanding (left scale) and overnight unsecured market volumes (right scale), 08/2008 - 11/2008

in the risk of banks' long-term assets translates into differences in their alternatives to unsecured borrowing. With the onset of the financial crisis in August 2007, the spread between the rate in the interbank market secured by government bonds (Eurepo) and the rate of secured borrowing in ECB auctions rose significantly. Banks with high quality collateral could borrow more cheaply than banks bidding in the ECB auctions where a larger set of collateral is accepted (Tapking and Weller, 2008).

Prior to September 2008 and in light of committed credit lines to SIVs, aggregate liquidity risk was also suggested as a reason for the high level of interbank rates.[20] However, aggregate liquidity risk by itself cannot explain why banks with sufficient liquidity refused to lend funds in the market even at short maturities. Moreover, since the ECB moved to fully satisfy banks' demand for liquidity against a wide set of collateral and committed itself to uphold the full allotment for a considerable amount of time, concerns about aggregate liquidity shortages are greatly reduced (see also Taylor, 2009).

[20] A number of studies assess the relative importance of credit and liquidity risk in interbank interest rate spreads (see, for example, Taylor and Williams, 2009, and Schwarz, 2009). Acharya and Merrouche (2008) establish a causal link between aggregate liquidity held by large settlement banks in the UK and interest rates in secured and unsecured interbank markets.

6 Policy responses

The aim of this section is to shed light on some of the policy responses that were discussed or implemented in order to relieve the tensions observed in interbank markets since August 2007. We examine five responses through the lens of our model: market transparency, liquidity requirements, central bank liquidity provision, loan guarantees, and asset purchases.

6.1 Transparency

Asymmetric information about the riskiness of the illiquid asset is at the heart of the adverse selection problem in our model. Safer borrowers subsidize riskier ones since lenders cannot tell them apart.

A natural regulatory response is therefore to improve transparency in the banking sector.[21] If, for example, bank supervisors could assess banks' risk and communicate it to the market, then lenders would be able to distinguish safer and riskier borrowers, as in our benchmark model without asymmetric information. Two markets would emerge, one for riskier banks with an interest rate r_r, and one for safer banks with an interest rate r_s, with $r_s < r_r$ (see Proposition 2).[22]

Transparency could ensure that safer borrowers remain in the interbank market while they would drop out in its absence. In particular, if $\frac{1}{l_s} < \frac{1}{\delta}$ holds, then full participation is not an equilibrium under asymmetric information (Proposition 5). However, it can be an equilibrium under full information since only the weaker condition $\frac{p}{p_s}\frac{1}{\delta} \leq \frac{1}{l_s}$ is required.

6.2 Liquidity requirements

In the wake of the financial crisis, bank regulators are investigating a strengthening of liquidity requirements. These requirements are supposed to ensure that financial institutions are able to withstand liquidity stresses of varying magnitude and duration.

[21] Increased transparency is a key recommendation of the de Larosière report, which examines the organization of supervision of financial institutions and markets in the EU. Similar recommendations are made by the UK's Turner Review and the Group of 30 Report by Paul Volcker.

[22] Assessing banks' risk is indeed the aim of the "stress testing" exercise undertaken by the US government and banking regulators (Board of Governors of the Federal Reserve System, 2009). Such an exercise can also help to restore normal trading conditions in the interbank market by reducing the degree of asymmetric information, Δp. Moreover, it can help to bring back the supply of funds that withdrew in fear of lending to unprofitable "lemons" (see equation (41)). Either the regulator is able to find out which bank is unprofitable and close it down, or it can convince market participants that there are no such banks around.

In our model, requiring banks to hold a certain amount of liquidity can also act as a coordination device when multiple equilibria in the interbank market are possible. Full participation is not a unique equilibrium when $\delta \geq l_s > \delta_2$. A regulator can ensure full participation and a low interest rate in the interbank market if he requires banks to hold the following amount of liquidity at $t = 0$:

$$1 - \alpha_1 = \lambda d_1.$$

This amount is a higher than the one that banks would choose if they anticipated adverse selection, $\alpha_1 < \alpha_2$ (see Section 4.3).

6.3 Liquidity provision by the central bank

A central bank can offer to provide liquidity directly to banks in need. Indeed, increased liquidity provision was a common reaction by central banks around the world to the 2007-2009 financial crisis.[23]

Suppose that an unanticipated adverse shock to counterparty risk, p, moves the economy from full participation to adverse selection.[24] Assuming that a central bank has no informational advantage over the market, it has to offer liquidity to all banks at the same rate, r_{CB}. The highest rate at which safer banks are willing to borrow from the central bank is:

$$1 + r_{CB} = \frac{R}{l_s}.$$

[23] At the onset of the crisis on 9 August 2007, when overnight rates temporarily spiked up by 60 basis points, the Eurosystem provided €94 billion of liquidity via collateralized, overnight lending. From August 2007 until September 2008, the Eurosystem was able to stabilize the overnight interbank rates without increasing the aggregate supply of liquidity by adjusting the time path of its liquidity provision ("frontloading" liquidity within each maintenance period). From October 2008, it introduced a full allotment procedure in its market operations which led to a significant increase in the liquidity provision. As a result, the size of the ECB's balance sheet temporarily increased by roughly €600 billion. The Federal Reserve introduced the Term Auction Facility (TAF), which allowed the auctioning of term funds to all depository institutions. In early 2009, the outstanding volume in the TAF was almost $500 billion, and the total short-term liquidity provided by the Federal Reserve to financial institutions totalled around $850 billion (Bernanke 2009).

[24] Since we assume that the shock to counterparty risk is unanticipated, the regulatory response to the crisis is also unexpected. Thus, we abstract from moral hazard issues that can be an important consideration when examining policy responses to crises (for a recent analysis see, e.g., Diamond and Rajan, 2009c).

The central bank's net return from lending (an amount $\pi_h L_h$) to *all* banks is:

$$\pi_h L_h \left(p\frac{R}{l_s} - 1 \right),$$

which is positive since $pR > 1 > l_s$. Even though the central bank lends at a subsidized rate, it makes a profit. The reason is that a central bank can raise liquidity at unit cost. That is, it can "print money". In contrast, the private supply of liquidity is costly since banks have to forgo investing in the long-term asset if they want to be able to provide liquidity at $t = 1$. Moreover, banks have to bear liquidity and counterparty risk. Condition (30) shows that the cost of private liquidity is $\frac{R}{\delta} > R > 1$.

If a central bank provides liquidity to banks with a liquidity shortage, it crowds out the private supply of liquidity. Banks with excess liquidity are no longer able to find a counterparty. In order to have a more balanced intervention, the central bank can offer to take on the excess liquidity and, possibly, offer a return on it. The central bank would effectively become an intermediary. It would be the counterparty for all liquidity transactions and replace the interbank market.[25]

A central bank can always provide liquidity at a lower cost than the interbank market. This is true even without a crisis. While such an intervention may seem desirable ex post (thus disregarding any moral hazard issues), it can have substantial costs ex ante. One important consideration is the role of interbank markets in information aggregation, price discovery, and peer monitoring (see, for example, Rochet and Tirole, 1996).

6.4 Interbank loan guarantees

Several countries have introduced loan guarantees in order to revive the interbank market.[26] Depending on their scope, loan guarantees reduce or even eliminate counterparty risk, thus lowering the interbank interest rate and inducing safer banks to borrow again.

Consider first the case of full interbank loan guarantees. Counterparty risk is eliminated and all

[25] See also Buiter (2008). As of October 15, 2008 the ECB was de facto intermediating: it fully satisfied demand for liquidity in its weekly Main Refinancing Operations and, at the same time, banks deposited significant amounts with the ECB (see also the discussion in section 5).

[26] One example is Italy, where the Banca d'Italia and the owners of the e-Mid trading platform have established the Mercato Interbancario Collateralizzato (MIC). Even though its trading activity is in principle collateralized, the Banca d'Italia guarantees timely repayment of all loans in MIC. The reason is that the crisis also affected secured interbank lending as there were credit risk concerns due to uncertain collateral values.

banks participate in the interbank market. The interest rate in the interbank market is $1+r_{FG}=R$, where r_{FG} denotes the interest rate under full guarantees. The cost of this intervention to the guarantor is

$$p\left(1+r_{FG}\right)\pi_h L_h-\left(1+r_{FG}\right)\pi_h L_h$$

or, equivalently,

$$-R\pi_h L_h\left(1-p\right). \tag{43}$$

The guarantor has to pay for all losses due to the risk of the illiquid investment.

Consider next partial guarantees that increase the probability of repayment from p to $\tilde{p}$, where $\tilde{p}$ is high enough to guarantee full participation in the interbank market:

$$1+r_{PG}=\frac{R}{l_s},$$

and where r_{PG} is the interest rate under partial loan guarantees.[27] The cost to the guarantor is:

$$p\left(1+r_G\right)\pi_h L_h-\tilde{p}\left(1+r_G\right)\pi_h L_h$$

or, equivalently,

$$-\frac{R}{l_s}\pi_h L_h\left(\tilde{p}-p\right). \tag{44}$$

The following proposition shows that interbank loan guarantees should be sufficiently comprehensive to be cost-efficient for the public sector.

Proposition 8 (Partial guarantees) *The cost of partial guarantees that yield an interest rate just ensuring full participation,* $1+r_{PG}=\frac{R}{l_s}$*, always exceeds the cost of full guarantees.*

A guarantee covers both principal and interest. While a partial guarantee reduces the cost on the principal it increases the cost on the interest as the interest rate rises to compensate lenders for the remaining counterparty risk.

[27] To ensure that lenders are willing to lend, the guarantee must be sufficiently high: $\tilde{p}(1+r_{PG})>1$.

6.5 Asset purchases

An alternative to borrowing in the interbank market is to convert the risky long-term asset into liquidity. One way to do this is to acquire high quality collateral that can be used in the repo market or with central banks. Selling long-term assets is costly in the context of our model, $l_\theta < 1$. In a financial crisis, this cost is particularly acute due to "fire-sale" prices. If banks bring more long-term assets to the market than there are funds available to buy them, the market will be characterized by "cash-in-the-market pricing". In other words, long-term assets are particularly subject to market liquidity risk (as in, for example, Shleifer and Vishny, 1992, Allen and Gale, 2004, or Gorton and Huang, 2004).

A central bank or a government authority does not face liquidity risk. Since liquidity risk does not need to be priced in, they can offer to buy long-term assets from banks at a higher price, $P > l_\theta$.[28,29] The price only needs to reflect the credit risk of assets. Moreover, by setting the price appropriately, the central bank or government can attract both safer and riskier borrowers and take advantage of pooling assets.

In particular, the price P could be set equal to the expected return on the long-term asset, pR. This ensures that the central bank or government does not suffer losses on average. Such pricing effectively lowers the opportunity cost of liquidity to 1. This is beneficial for borrowers who would otherwise have to pay a premium for obtaining liquidity in the interbank market since they have to compensate lenders for counterparty risk.

7 Conclusion

Interbank markets underwent dramatic changes during the financial crisis of 2007-09, with interest rates rising to previously unseen levels and trading activity declining significantly in some market segments. Unsecured, longer-term interbank lending and lending secured with risky collateral were particularly affected. Motivated by these facts, we present a model of the interbank market in

[28] The Eurosystem has for example widened the set of eligible collateral for its refinancing operations. While the Federal Reserve cannot purchase assets other than Treasuries, agencies and agency MBS, the US government is purchasing assets via the Troubled Asset Relief Program (TARP). For an analysis of how the availability of collateral affects money markets see, for example, Heider and Hoerova (2009).

[29] For an analysis of public interventions to alleviate debt overhang among private firms when the government has limited information and limited resources see Philippon and Schnabl (2009).

which lenders are exposed to counterparty risk. We show that depending on parameters, reflecting in particular the level and distribution of risk among banks, an equilibrium in which all banks participate in the interbank market and in which liquidity is reallocated smoothly may not be reached. The functioning of the interbank market can be impaired by adverse selection, possibly leading to a market breakdown. The interbank market regimes obtained in the model echo the developments prior to and during the financial crisis.

Although a number of factors affect banks' decisions to trade in interbank markets, our model highlights the role of counterparty risk as an important ingredient to explain the observed events. At the same time, asymmetric information about risk can rationalize the prolonged nature of interbank market tensions despite an unprecedented liquidity provision by central banks. We use the model to shed light on various policy responses that were put in place to relieve the tensions.

The model can be extended along a number of dimensions. In particular, potential spill-overs between the secured and unsecured money market segments can be examined. What led to the significant degree of interbank market segmentation during the crisis? What determines the willingness of banks to pay at central bank refinancing operations? How broad should the list of eligible collateral be? These questions are left for future research.

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Appendix A

Proof of Proposition 1

At $t = 1$, type-(l, θ) banks solve (3) subject to (4) and the following feasibility constraints:

$$\begin{array}{ll} L^s_{l,\theta} \geq 0 & [\mu_2^{l,\theta,s}] \\ L^r_{l,\theta} \geq 0 & [\mu_2^{l,\theta,r}] \\ 0 \leq \alpha^L_{l,\theta} \leq 1 & [\mu_3^{l,\theta}, \mu_4^{l,\theta}] \\ 0 \leq \alpha^R_{l,\theta} \leq 1 & [\mu_5^{l,\theta}, \mu_6^{l,\theta}] \end{array} \tag{A.1}$$

where the corresponding Lagrange multipliers are in square brackets. Let $\mu_1^{l,\theta}$ be the Lagrange multiplier on the resource constraint (4).

Type-(h, θ) banks solve (1) subject to (2) (with Lagrange multiplier $\mu_1^{h,\theta}$) subject to the following feasibility constraints:

$$\begin{array}{ll} L_{h,\theta} \geq 0 & [\mu_2^{h,\theta}] \\ 0 \leq \alpha^L_{h,\theta} \leq 1 & [\mu_3^{h,\theta}, \mu_4^{h,\theta}] \\ 0 \leq \alpha^R_{h,\theta} \leq 1 & [\mu_5^{h,\theta}, \mu_6^{h,\theta}] \end{array} \tag{A.2}$$

The first-order conditions for a type-(l, θ) with respect to $L^s_{l,\theta}, L^r_{l,\theta}, \alpha^L_{l,\theta}$ and $\alpha^R_{l,\theta}$ are:

$$p_\theta \hat{p}_s(1 + r_s) - \mu_1^{l,\theta} + \mu_2^{l,\theta,s} = 0 \tag{A.3}$$

$$p_\theta \hat{p}_r(1 + r_r) - \mu_1^{l,\theta} + \mu_2^{l,\theta,r} = 0 \tag{A.4}$$

$$-p_\theta R\alpha + p_\theta \alpha^R_{l,\theta} \alpha l_\theta + \mu_1^{l,\theta} \alpha l_\theta \left(1 - \alpha^R_{l,\theta}\right) + \mu_3^{l,\theta} - \mu_4^{l,\theta} = 0 \tag{A.5}$$

$$\left(1 - \alpha + \alpha^L_{l,\theta} \alpha l_\theta\right) \left(p_\theta - \mu_1^{l,\theta}\right) + \mu_5^{l,\theta} - \mu_6^{l,\theta} = 0 \tag{A.6}$$

Similarly, the first-order conditions for a type-(h, θ) bank are given by:

$$-p_\theta(1 + r_\theta) + \mu_1^{h,\theta} + \mu_2^{h,\theta} = 0 \tag{A.7}$$

$$-p_\theta R\alpha + p_\theta \alpha^R_{h,\theta} \alpha l_\theta + \mu_1^{h,\theta} \alpha l_\theta \left(1 - \alpha^R_{h,\theta}\right) + \mu_3^{h,\theta} - \mu_4^{h,\theta} = 0 \tag{A.8}$$

$$(1 - \alpha + \alpha^L_{h,\theta} \alpha l_\theta) \left(p_\theta - \mu_1^{h,\theta}\right) + \mu_5^{h,\theta} - \mu_6^{h,\theta} = 0 \tag{A.9}$$

We proceed in a number of steps. We first derive the marginal value of liquidity in the interbank market.

Lemma A.1 *The marginal value of liquidity for a type-(l, θ) bank that offers funds to safer borrowers, $L^s_{l,\theta} > 0$, is $\mu_1^{l,\theta} = p_\theta \hat{p}_s(1 + r_s)$ and for a type-(l, θ) bank that offers funds to riskier borrowers, $L^r_{l,\theta} > 0$, it is $\mu_1^{l,\theta} = p_\theta \hat{p}_r(1+r_r)$. The marginal value of liquidity for a type-(h, θ) bank that demands funds is $\mu_1^{h,\theta} = p_\theta(1 + r_\theta)$.*

The Lemma follows from equations (A.3), (A.4) and (A.7) where $\mu_2^{l,\theta,s} = 0$ since $L^s_{l,\theta} > 0$, $\mu_2^{l,\theta,r} = 0$ since $L^r_{l,\theta} > 0$ and $\mu_2^{h,\theta} = 0$ since $L_{h,\theta} > 0$. The marginal value of liquidity is lower for a bank that offers funds than for a bank that demands funds due to counterparty risk.

The following Corollary follows immediately from Lemma A.1.

Corollary A.1 *If a type-(l,θ) bank offers funds to both safer and riskier borrowers, $L^s_{l,\theta} > 0, L^r_{l,\theta} > 0$, then it must make the same expected return on both groups of borrowers, $\hat{p}_s(1+r_s) = \hat{p}_r(1+r_r)$.*

Next, we characterize the decision to offer funds in the interbank market.

Lemma A.2 *Type-(l,θ) banks offer funds to safer banks, $L^s_{l,\theta} > 0$, and not to riskier banks, $L^r_{l,\theta} = 0$, if and only if $\hat{p}_s(1+r_s) \geq 1 > \hat{p}_r(1+r_r)$. They offer funds to riskier banks, $L^r_{l,\theta} > 0$, and not to safer banks, $L^s_{l,\theta} = 0$, if and only if $\hat{p}_r(1+r_r) \geq 1 > \hat{p}_s(1+r_s)$. They offer funds to both if and only if $\hat{p}_s(1+r_s) = \hat{p}_r(1+r_r) \geq 1$.*

We first show that if $L^s_{l,\theta} > 0$ and $L^r_{l,\theta} = 0$, then $\hat{p}_s(1+r_s) \geq 1 > \hat{p}_r(1+r_r)$. Suppose not. Since $L^s_{l,\theta} > 0$, we have $\mu_1^{l,\theta} = p_\theta \hat{p}_s(1+r_s) > 0$ and the resource constraint binds. Substituting into (A.6) yields:

$$p_\theta \left(1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta\right)\left(1 - \hat{p}_s(1+r_s)\right) + \mu_5^{l,\theta} - \mu_6^{l,\theta} = 0.$$

We have that $1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta > 0$ since otherwise the resource constraint cannot hold ($L^s_{l,\theta} > 0$ and $d_1 > 0$). Since we assume $\hat{p}_s(1+r_s) < 1$, it must be the case that $\mu_6^{l,\theta} > 0$ implying $\alpha^R_{l,\theta} = 1$. But if a lender reinvests all of his liquidity in the short-term asset, he has nothing left to offer in the interbank market. Again, this contradicts $L^s_{l,\theta} > 0$.

We now show that if $\hat{p}_s(1+r_s) \geq 1 > \hat{p}_r(1+r_r)$, then $L^s_{l,\theta} > 0$ and $L^r_{l,\theta} = 0$. Suppose not. Equation (A.6) is then written as:

$$p_\theta \left(1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta\right)\left(1 - \hat{p}_s(1+r_s)\right) - \mu_2^{l,\theta,s}\left(1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta\right) + \mu_5^{l,\theta} - \mu_6^{l,\theta} = 0.$$

We have that $1 - \alpha + \alpha^L_{l,\theta}\alpha l_\theta > 0$ since otherwise the resource constraint is violated (as $d_1 > 0$). Then, it must be that $\mu_5^{l,\theta} > 0$ since all the other terms in the condition are negative. Hence, $\alpha^R_{l,\theta} = 0$. But if a lender does not reinvest his liquidity surplus into the short-term asset, he must be offering it in the interbank market since his resource constraint binds ($\mu_1^{l,\theta} > 0$). It follows that $L^r_{l,\theta} > 0$ and hence $\mu_2^{l,\theta,r} = 0$ implying

$$p_\theta \hat{p}_r(1+r_r) = \mu_1^{l,\theta}$$

by equation (A.4). Combining with equation (A.3), we get that

$$p_\theta \hat{p}_s(1+r_s) + \mu_2^{l,\theta,s} = p_\theta \hat{p}_r(1+r_r)$$

and hence $\hat{p}_s(1+r_s) < \hat{p}_r(1+r_r)$, a contradiction.

The proof that $L^r_{l,\theta} > 0$ and $L^s_{l,\theta} = 0$ if and only if $\hat{p}_r(1+r_r) \geq 1 > \hat{p}_s(1+r_s)$ proceeds analogously (by simply exchanging the s and r labels above).

We now show that $L^s_{l,\theta} > 0$ and $L^r_{l,\theta} > 0$ if and only if $\hat{p}_s(1+r_s) = \hat{p}_r(1+r_r) \geq 1$. We know that if $L^s_{l,\theta} > 0$ and $L^r_{l,\theta} > 0$, then $\hat{p}_s(1+r_s) = \hat{p}_r(1+r_r)$ (Corollary A.1). It remains to show that $\hat{p}_\theta(1+r_\theta) \geq 1$. Suppose not. Using equation (A.6) again, we get that $\mu_6^{l,\theta} > 0$ implying $\alpha^R_{l,\theta} = 1$. This is in contradiction with offering funds in the interbank market.

Finally, we show that $\hat{p}_s(1+r_s) = \hat{p}_r(1+r_r) \geq 1$ implies $L^s_{l,\theta} > 0$ and $L^r_{l,\theta} > 0$. Suppose not. Using equation (A.6), we again have that $\mu_5^{l,\theta} > 0$ and $\alpha^R_{l,\theta} = 0$. But if a lender does not reinvest his liquidity surplus into the short-term asset, he must be offering it in the interbank market since his resource constraint binds ($\mu_1^{l,\theta} > 0$). This is incompatible with $L^s_{l,\theta} = L^r_{l,\theta} = 0$.

Note that the decision to offer funds in the interbank market is independent of the risk type θ of a lender. We therefore write $L_{l,\theta}^{s} = L_l^s$ and $L_{l,\theta}^{r} = L_l^r$. Moreover, banks with a liquidity surplus only offer funds when they are compensated for counterparty risk: $r_\theta > 0$ must hold when $\hat{p}_\theta < 1$.

Next, we show that if a bank offers or demands funds in the interbank market, then it does not reinvest in the short-term asset.

Lemma A.3 *If a bank offers or demands funds in the interbank market, $L_l^s > 0$, $L_l^r > 0$ or $L_{h,\theta} > 0$, then it does not reinvest in the short-term asset, $\alpha_{k,\theta}^R = 0$.*

Consider first the case of a bank that demands funds. Since $L_{h,\theta} > 0$, we have $\mu_1^{h,\theta} = p_\theta(1+r_\theta)$. Substituting into (A.9) yields:

$$p_\theta \left(1 - \alpha + \alpha_{h,\theta}^L \alpha l_\theta\right)(-p_\theta r_\theta) + \mu_5^{h,\theta} - \mu_6^{h,\theta} = 0. \tag{A.10}$$

Note that $1-\alpha+\alpha_{h,\theta}^L \alpha l_\theta \geq 0$ holds with equality if and only if $\alpha = 1$ and $\alpha_{h,\theta}^L = 0$. The case $\alpha = 1$ and $\alpha_{h,\theta}^L = 0$ cannot be optimal since a type-(h,θ) bank would have to finance its entire need for liquidity by borrowing in the interbank market at a rate $1 + r_\theta > 1$ whereas it could get liquidity at unit cost using the short-term asset. Hence, $1-\alpha+\alpha_{h,\theta}^L \alpha l_\theta > 0$ implying $\mu_5^{h,\theta} > 0$ and $\alpha_{h,\theta}^R = 0$.

Next, consider the case of a bank that offers funds to safer borrowers. Since $L_l^s > 0$, we have $\mu_1^{l,\theta} = p_\theta \hat{p}_s (1+r_s)$. Substituting into (A.6) yields:

$$p_\theta \left(1 - \alpha + \alpha_{l,\theta}^L \alpha l_\theta\right) p_\theta (1 - \hat{p}_s (1+r_s)) + \mu_5^{l,\theta} - \mu_6^{l,\theta} = 0. \tag{A.11}$$

Note that $1 - \alpha + \alpha_{l,\theta}^L \alpha l_\theta > 0$ holds since otherwise the resource constraint cannot hold ($L_l^s > 0$ and $d_1 > 0$). Since $\hat{p}_s(1+r_s) \geq 1$ (Lemma A.2 with $L_l^s > 0$), we have $\mu_5^{l,\theta} > 0$ and hence $\alpha_{l,\theta}^R = 0$.

The proof is analogous for a bank that offers funds to riskier borrowers.

Next, we characterize banks' decision to liquidate part of the long-term asset when they offer funds in the interbank market.

Lemma A.4 *Suppose a type-(l,θ) bank offers funds in the interbank market to safer banks, $L_l^s > 0$. Then it does not liquidate, $\alpha_{l,\theta}^L = 0$, if and only if $\hat{p}_s(1+r_s) \leq \frac{R}{l_\theta}$. Similarly, suppose a type-(l,θ) bank offers funds to riskier banks, $L_l^r > 0$. Then it does not liquidate, $\alpha_{l,\theta}^L = 0$, if and only if $\hat{p}_r(1+r_r) \leq \frac{R}{l_\theta}$.*

Consider first the case of a bank that offers funds to safer borrowers, $L_l^s > 0$ (and hence $\mu_2^{l,\theta,s} = 0$). Substituting $\mu_1^{l,\theta} = p_\theta \hat{p}_s (1+r_s)$ and $\alpha_{l,\theta}^R = 0$ (Lemma A.3) into (A.5) yields:

$$-p_\theta \alpha \left(R - \hat{p}_s (1+r_s) l_\theta\right) + \mu_3^{l,\theta} - \mu_4^{l,\theta} = 0$$

It must be that $\alpha > 0$. If not, borrowers will never be able to repay interbank loans, which is inconsistent with $L_l^s > 0$. It follows that $\alpha_{l,\theta}^L = 0$ if and only if $\hat{p}_s(1+r_s) l_\theta \geq R$. The same argument establishes that $\alpha_{l,\theta}^L = 0$ if and only if $\hat{p}_r(1+r_r) l_\theta \geq R$ when banks offer funds to riskier borrowers, $L_l^r > 0$.

Next, we characterize a bank's demand for funds in the interbank market when it is being offered funds.

Lemma A.5 *Suppose that funds are offered to a type-(h,θ) bank, $L_l^\theta > 0$. Then it demands funds, $L_{h,\theta} > 0$, if and only if $1 + r_\theta \leq \frac{R}{l_\theta}$.*

We first show that $L_{h,\theta} > 0$ implies $1 + r_\theta \leq \frac{R}{l_\theta}$. Suppose not, i.e. $1 + r_\theta > \frac{R}{l_\theta}$. Substituting $\mu_1^{h,\theta} = p_\theta(1 + r_\theta)$ and $\alpha^R_{h,\theta} = 0$ (Lemma A.3) into (A.8) yields:

$$-p_\theta \alpha \left(R - (1 + r_\theta)l_\theta\right) + \mu_3^{h,\theta} - \mu_4^{h,\theta} = 0$$

It must be that $\alpha > 0$, since otherwise borrowers could never repay interbank loans. Since lenders would never be repaid, they would not lend, contradicting $L^\theta_l > 0$. Since we assume that $1+r_\theta > \frac{R}{l_\theta}$, the first term on the left-hand side is positive. This implies that $\mu_4^{h,\theta} > 0$ and hence $\alpha^L_{h,\theta} = 1$. But if a borrower fully liquidates the long-term asset and does not reinvest into the short-term asset, he will never be able to repay the interbank loan. Consequently, a lender would never offer the loan in the first place.

We now show that $1 + r_\theta < \frac{R}{l_\theta}$ implies that $L_{h,\theta} > 0$ (if $1 + r_\theta = \frac{R}{l_\theta}$, we assume that $L_{h,\theta} > 0$). Suppose not, i.e. $L_{h,\theta} = 0$. It must be that $1 - \alpha + \alpha^L_{h,\theta}\alpha l_\theta > 0$ and $\alpha^R_{h,\theta} < 1$ since $d_1 > 0$ in the resource constraint. Hence, $\mu_6^{h,\theta} = 0$ in equation (A.9) and it must be that $\mu_1^{h,\theta} \geq p_\theta > 0$. Hence, the resource constraint binds and $\alpha^L_{h,\theta} = \frac{\lambda_h d_1 - (1-\alpha)}{\alpha l_\theta} > 0$. This implies that $\mu_3^{h,\theta} = 0$ in equation (A.8). It follows from the same equation that

$$-p_\theta R\alpha + \mu_1^{h,\theta}\alpha l_\theta + \alpha^R_{h,\theta}\alpha l_\theta \left(p_\theta - \mu_1^{h,\theta}\right) - \mu_4^{h,\theta} = 0.$$

Using equation (A.7) to substitute for $\mu_1^{h,\theta}$ and collecting terms, we get

$$p_\theta \alpha \left((1 + r_\theta)l_\theta - R\right) - \mu_2^{h,\theta}\alpha l_\theta \left(1 - \alpha^R_{h,\theta}\right) - \alpha^R_{h,\theta}\alpha l_\theta p_\theta r_\theta - \mu_4^{h,\theta} = 0.$$

It must be that $\alpha > 0$. Otherwise a type-(l, θ) bank would not be offering funds to type-(h, θ) banks, contradicting $L^\theta_l > 0$. Also, we know that $\alpha^R_{h,\theta} < 1$. Hence, it must be that $1 + r_\theta \geq \frac{R}{l_\theta}$, contradicting $1 + r_\theta < \frac{R}{l_\theta}$.

Lemma A.6 *Suppose that funds are offered to a type-(h, θ) bank, $L^\theta_l > 0$. If a type-(h, θ) bank demands funds, $L_{h,\theta} > 0$, then it does not liquidate, $\alpha^L_{h,\theta} = 0$.*

Using Lemma A.3 and A.5 we write equation (A.8) as

$$-p_\theta \alpha(R - (1 + r_\theta)l_\theta) + \mu_3^{h,\theta} - \mu_4^{h,\theta} = 0.$$

It must be that $\alpha > 0$, since otherwise borrowers could never repay interbank loans. Since lenders would never be repaid, they would not lend, contradicting $L^\theta_l > 0$. Since $1 + r_\theta \leq \frac{R}{l_\theta}$ (Lemma A.5), the first term on the left-hand side is negative. This implies that $\mu_3^{h,\theta} > 0$ and hence $\alpha^L_{h,\theta} = 0$.

Proof of Corollary 1

It follows Lemma A.3, A.4 in conjunction with equation (7) and Lemma A.6.

Proof of Proposition 2

We use the no-arbitrage condition (11) (see Corollary A.1) and equation (12) to write the optimization problem (8) as

$$\begin{aligned}\max_{0<\alpha<1} \quad & \pi_l(qp_s +_l (1-q)p_r)[R\alpha + p_s(1+r_s)(1-\alpha-\lambda_l d_1) - (1-\lambda_l)d_2] \\ + \quad & \pi_h qp_s[R\alpha - (1+r_s)(\lambda_h d_1 - (1-\alpha) - (1-\lambda_h)d_2] \\ + \quad & \pi_h(1-q)p_r[R\alpha - \frac{p_s}{p_r}(1+r_s)(\lambda_h d_1 - (1-\alpha) - (1-\lambda_h)d_2]\end{aligned}$$

where we have substituted for $L_l^s + L_h^r$ and L_h using the binding resource constraint. The first order condition is

$$(qp_s + (1-q)p_r)R - (1+r_s)[\pi_l(qp_s + (1-q)p_r)p_s + \pi_h(qp_s + (1-q)p_s)] = 0$$

Rearranging yields equation (13) for safe borrowers ($\theta = s$). The interest rate for riskier borrowers, $1 + r_r$, is derived analogously.

Proof of Proposition 3

When all banks manage their liquidity in the interbank market then the interest rate is given by (15). The lower bound on the interest rate in (7) is always satisfied since

$$p_s\,(1+r_s) = p_r\,(1+r_r) = \frac{pR}{\delta} > pR > 1.$$

The upper bound is satisfied when $1 + r_\theta = \frac{p_\theta}{p}\frac{1}{\delta}R < \frac{R}{l_\theta}$, which simplifies to the condition in the proposition.

Proof of Proposition 4

The proof is the same as the proof of Proposition 1. The only change is that there is now a single interbank market for both risk-types of borrowers. Hence, one must replace r_θ with r everywhere. The first-order conditions remain otherwise the same, except for the first-order conditions (A.3) and (A.4), whereby there is now a single first-order condition with respect to $L_{l,\theta} \geq 0$:

$$p_\theta \hat{p}(1+r) - \mu_1^{l,\theta} + \mu_2^{l,\theta} = 0.$$

Proof of Corollary 3

No reinvestment into the short-term asset for banks that participate in the interbank market, $\alpha_{l,\theta}^R = \alpha_{h,r}^R = 0$, follows from Lemma A.3 (with the modification that r_θ is now r_2). No reinvestment into the short-term asset for banks that do not participate in the interbank market, $\alpha_{h,s}^R = 0$, follows from the following lemma.

Lemma A.7 *If a type-(h,θ) bank does not demand funds in the interbank market, $L_{h,\theta} = 0$, then it does not reinvest in the short-term asset, $\alpha_{h,\theta}^R = 0$.*

We first show that the resource constraint binds. Suppose not. Then, $\mu_1^{h,\theta} = 0$ and we can write the first-order condition with respect to $\alpha^R_{h,\theta}$ (equation A.9) as

$$(1 - \alpha + \alpha^L_{h,\theta}\alpha l_\theta)p_\theta + \mu_5^{h,\theta} - \mu_6^{h,\theta} = 0.$$

Since $1 - \alpha + \alpha^L_{h,\theta}\alpha l_\theta > 0$ (it cannot be that $\alpha = 1$ and $\alpha^L_{h,\theta} = 0$: a type-(h,θ) bank cannot satisfy withdrawals at $t = 1$ if it does not borrow, invests everything into the long-term asset and does not liquidate), it must be that $\mu_6^{h,\theta} > 0$ and hence $\alpha^R_{h,\theta} = 1$. But then the resource constraint is $\lambda_h d_1 < 0$, a contradiction. Hence, the resource constraint binds.

Since the bank has a liquidity shortage but does not borrow in the interbank market, it must liquidate some of its long-term asset in order satisfy the withdrawals at $t = 1$. Positive liquidation, $\alpha^L_{h,\theta} > 0$, implies $\mu_3^{h,\theta} = 0$ and the first-order condition with respect to $\alpha^L_{h,\theta}$ (equation A.8) becomes

$$-p_\theta R\alpha + \alpha l_\theta \left[p_\theta \alpha^R_{h,\theta} + \mu_1^{h,\theta}\left(1 - \alpha^R_{h,\theta}\right)\right] = \mu_4^{h,\theta} \geq 0. \tag{A.12}$$

Note that $p_\theta < \mu_1^{h,\theta}$ must hold. Suppose not and $p_\theta \geq \mu_1^{h,\theta}$. Then,

$$-p_\theta R\alpha + \alpha l_\theta \left[\alpha^R_{h,\theta}\left(p_\theta - \mu_1^{h,\theta}\right) + \mu_1^{h,\theta}\right] \leq -p_\theta R\alpha + \alpha l_\theta p_\theta = p_\theta\alpha\left(l_\theta - R\right) < 0,$$

contradicting equation (A.12).

We now show that there is no reinvestment, $\alpha^R_{h,\theta} = 0$. Suppose not, $\alpha^R_{h,\theta} > 0$ and hence $\mu_5^{h,\theta} = 0$. The first-order condition with respect to $\alpha^R_{h,\theta}$ (equation A.9) becomes

$$(1 - \alpha + \alpha^L_{h,\theta}\alpha l_\theta)(p_\theta - \mu_1^{h,\theta}) - \mu_6^{h,\theta} = 0. \tag{A.13}$$

As argued above, $1 - \alpha + \alpha^L_{h,\theta}\alpha l_\theta > 0$ and $p_\theta < \mu_1^{h,\theta}$. But then, equation (A.13) cannot hold, a contradiction. Hence, type-(h,θ) bank does not reinvest in the short-term asset, $\alpha^R_{h,\theta} = 0$.

No liquidation by riskier banks with a liquidity surplus, , follows from the participation in the interbank market of riskier banks with a liquidity shortage (equation (32) and Lemma A.4). No liquidation by safer banks with a liquidity surplus follows from the condition in the Corollary: $\hat{p}(1+r_2) \leq \frac{R}{l_s}$. No liquidation by riskier with a liquidity shortage banks follows from their participation in the interbank market (Lemma A.6). Safer banks with a liquidity shortage do not borrow in the interbank market and must therefore liquidate their long-term asset in order to satisfy the resource constraint at $t = 1$.

Proof of Proposition 7

Using (39) we can determine the amount that a type-(h,s) bank has to liquidate (from the last resource constraint in (34))

$$\alpha^L_{h,s} = \frac{1}{l_s}\frac{d_1(\lambda_h - \tilde{\lambda})}{1 - d_1\tilde{\lambda}}$$

where $\tilde{\lambda} = \tilde{\pi}_l \lambda_l + \tilde{\pi}_h \lambda_h$. Using also (36) and L_l and $L_{h,r}$ from the other resource constraints in (34), banks' expected profits are

$$\begin{aligned}\Pi_2(d_1, d_2) &= \pi_l p \left[R(1 - d_1\tilde{\lambda}) + p_r \frac{R}{l_s} \left(\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} \right) (d_1\tilde{\lambda} - \lambda_l d_1) - (1 - \lambda_l) d_2 \right] \\ &+ \pi_h q p_s \left[R(1 - d_1\tilde{\lambda}) \left(1 - \frac{1}{l_s} \frac{d_1(\lambda_h - \tilde{\lambda})}{1 - d_1\tilde{\lambda}} \right) - (1 - \lambda_h) d_2 \right] \\ &+ \pi_h (1 - q) p_r \left[R(1 - d_1\tilde{\lambda}) - \frac{R}{l_s} \left(\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} \right) (\lambda_h d_1 - d_1\tilde{\lambda}) - (1 - \lambda_h) d_2 \right].\end{aligned}$$

We further have

$$\begin{aligned}\tilde{\lambda} - \lambda_l &= \tilde{\pi}_h (\lambda_h - \lambda_l) \\ \lambda_h - \tilde{\lambda} &= \tilde{\pi}_l (\lambda_h - \lambda_l)\end{aligned}$$

so that

$$\begin{aligned}\Pi_2(d_1, d_2) &= p[R(1 - d_1\tilde{\lambda}) - (1 - \lambda_l) d_2] \\ &+ p d_1 \frac{R}{l_s} (\lambda_h - \lambda_l) \left[\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} \left(\pi_l \tilde{\pi}_h p_r - \pi_h \tilde{\pi}_l \frac{(1 - q) p_r}{p} \right) - \pi_h \tilde{\pi}_l \frac{qp_s}{p} \right].\end{aligned}$$

The condition that banks' expected profits under adverse selection are lower than under full participation, $\Pi_2(d_1, d_2)_2 < \Pi_1(d_1, d_2) = \Pi(d_1, d_2) = p[R(1 - \hat{\lambda} d_1) - (1 - \lambda) d_2]$ then becomes

$$(\hat{\lambda} - \tilde{\lambda}) + \frac{1}{l_s} (\lambda_h - \lambda_l) \left[\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} \left(\pi_l \tilde{\pi}_h p_r - \pi_h \tilde{\pi}_l \frac{(1 - q) p_r}{p} \right) - \pi_h \tilde{\pi}_l \frac{qp_s}{p} \right] < 0.$$

Using

$$\hat{\lambda} - \tilde{\lambda} = \pi_l \pi_h (\lambda_h - \lambda_l) \frac{1 - p + pq}{\delta \left(\pi_l + (1 - q) \pi_h \right)},$$

as well as $\tilde{\pi}_l \equiv \frac{\pi_l}{\pi_l + \pi_h(1-q)}$ and $\tilde{\pi}_h \equiv \frac{\pi_h(1-q)}{\pi_l + \pi_h(1-q)}$, the condition simplifies to

$$\frac{1}{\delta}[1 - p + pq] < \frac{1}{l_s} \left[\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta_2 - \pi_h \frac{qp_s}{p}} (1 - p) \frac{(1 - q) p_r}{p} + \frac{qp_s}{p} \right].$$

Using $\delta_2 = p_r \pi_l + \pi_h$, $\delta = p\pi_l + \pi_h$, and $qp_s = p - (1 - q) p_r$, we rewrite the condition as

$$\frac{1}{\delta}[1 - p + pq] - \frac{1}{l_s} \frac{qp_s}{p} < \frac{1}{l_s} \left[\frac{l_s - \pi_h \frac{qp_s}{p}}{\delta - \pi_h q} (1 - p)(1 - q) \right]$$

which simplifies to

$$\frac{\pi_h (1 - p(1 - q)) - \delta}{\pi_h q - \delta} \frac{l_s q - \frac{qp_s}{p} \delta}{l_s \delta} < 0$$

and further to $l_s < \frac{p_s}{p} \delta$ since $0 > -p \left(1 - \pi_h q \right) = \pi_h (1 - p(1 - q)) - \delta > \pi_h q - \delta$.

Proof of Equilibrium Uniqueness

We first show that if Regime 1 is the unique equilibrium, then $\frac{1}{\delta_2} \leq \frac{1}{l_s}$. Suppose, contrary to the claim, that Regime 1 (full participation of borrowers and lenders) is the unique equilibrium and $\frac{1}{\delta_2} > \frac{1}{l_s}$. Since Regime 1 is unique, lenders are always willing to participate in the interbank market and, by Proposition 4, $1 + r \geq \frac{1}{\tilde{p}}$. This rules out liquidity hoarding as an outcome.

Since Regime 1 is unique, borrowers are always willing to participate in the interbank market and, by Proposition 4, $1 + r \leq \frac{R}{l_s} < \frac{R}{l_r}$. Hence, market breakdown due to the drop-out of borrowers is not possible and $1 + r_2 \leq \frac{R}{l_s}$ must hold so that Regime 2 is not an equilibrium. This is equivalent to $\frac{1}{\delta_2} \leq \frac{1}{l_s}$. Contradiction.

To show sufficiency, suppose, contrary to the claim, that $\frac{1}{\delta_2} \leq \frac{1}{l_s}$ and Regime 1 is not unique. Suppose that Regime 2 is an equilibrium. Then, $1+r_2 > \frac{R}{l_s}$ or, equivalently, $\frac{1}{\delta_2} > \frac{1}{l_s}$. Contradiction.

The proof that Regime 2 is the unique equilibrium if and only if $\frac{1}{l_s} < \frac{1}{\delta}$ follows the same steps.

Proof of Proposition 8

Comparing (44) and (43), we see that the cost of partial guarantees exceeds the cost of full guarantees if and only if:

$$l_s > \frac{\tilde{p} - p}{1 - p}.$$

Since the participation constraint of safe borrowers is binding at the interest rate r_{PG}, we know that $l_s = \tilde{p}\pi_l + \pi_h$ (see Proposition 5). Thus, the condition above can be written as:

$$\tilde{p}\pi_l + \pi_h > \frac{\tilde{p} - p}{1 - p},$$

which simplifies to $\tilde{p} < 1$ and hence the claim in the Proposition follows.

Appendix B (not for publication)

In this Appendix, we show that the three regimes in the interbank market under asymmetric information also arise in a model in which the two types of long-term investments can be converted into liquidity at the same rate but differ in their long-run returns. That is, we assume that $R_s < R_r$ and $l_s = l_r = l < 1$. We retain the assumption that safer investments are more likely succeed: $p_s > p > p_r$ where $p \equiv qp_s + (1-q)\, p_r$. Hence, safer investments are characterized by a lower long-run return but a higher success probability than riskier investments. We let $E\,[R]$ denote the expected $t = 2$ return on the illiquid investments: $E\,[R] \equiv qp_sR_s + (1-q)\, p_rR_r$.

One possible interpretation of the liquidation technology is the ability to recover some of the original investment made at $t = 0$. This is costly in that the liquidation return is lower per unit than the original investment: l <1. By contrast, in the set-up developed above we interpret the liquidation technology as the ability to gather some of the future return on the long-term investments (R) already at $t = 1$. Since $p_sR > p_rR$, we also have $l_s > l_r$. Since the long-run returns are not yet realized, liquidation is costly: $l_s < 1$ and $l_r < 1$.

As before, we solve the model backwards by first examining banks' liquidity management at $t = 1$ and then their portfolio choice at $t = 0$.

Liquidity management

Having received liquidity shocks, $k = \{l, h\}$, and being privately informed about the risk of their illiquid investment, $\theta = \{s, r\}$, banks need to manage their liquidity at $t = 1$. A type-(l, θ) bank solves the following problem:

$$\begin{array}{ll} \max\limits_{L_{l,\theta},\alpha^L_{l,\theta},\alpha^R_{l,\theta}} p_\theta[R_\theta\alpha(1-\alpha^L_{l,\theta}) + \alpha^R_{l,\theta}\left(1-\alpha+\alpha^L_{l,\theta}\alpha l\right) + \hat{p}(1+r)L_{l,\theta} - (1-\lambda_l)d_2] & \\ \lambda_l d_1 + \alpha^R_{l,\theta}\left(1-\alpha+\alpha^L_{l,\theta}\alpha l\right) + L_{l,\theta} \leq 1-\alpha+\alpha^L_{l,\theta}\alpha l & [\mu_1^{l,\theta}] \\ L_{l,\theta} \geq 0 & [\mu_2^{l,\theta}] \\ 0 \leq \alpha^L_{l,\theta} \leq 1 & [\mu_3^{l,\theta}, \mu_4^{l,\theta}] \\ 0 \leq \alpha^R_{l,\theta} \leq 1 & [\mu_5^{l,\theta}, \mu_6^{l,\theta}] \end{array} \tag{B.1}$$

Similarly, a type-(h, θ) bank solves:

$$\begin{array}{ll} \max\limits_{L_{h,\theta}\alpha^L_{h,\theta},\alpha^R_{h,\theta}} p_\theta[R_\theta\alpha(1-\alpha^L_{h,\theta}) + \alpha^R_{h,\theta}\left(1-\alpha+\alpha^L_{h,\theta}\alpha l\right) - (1+r)L_{h,\theta} - (1-\lambda_h)d_2] & \\ \lambda_h d_1 + \alpha^R_{h,\theta}\left(1-\alpha+\alpha^L_{h,\theta}\alpha l\right) \leq 1-\alpha+\alpha^L_{h,\theta}\alpha l + L_{h,\theta} & [\mu_1^{h,\theta}] \\ L_{h,\theta} \geq 0 & [\mu_2^{h,\theta}] \\ 0 \leq \alpha^L_{h,\theta} \leq 1 & [\mu_3^{h,\theta}, \mu_4^{h,\theta}] \\ 0 \leq \alpha^R_{h,\theta} \leq 1 & [\mu_5^{h,\theta}, \mu_6^{h,\theta}] \end{array} \tag{B.2}$$

The first-order conditions for a type-(l, θ) bank with respect to $L_{l,\theta}$, $\alpha^L_{l,\theta}$, and $\alpha^R_{l,\theta}$ are:

$$\begin{aligned} p_\theta\hat{p}(1+r) - \mu_1^{l,\theta} + \mu_2^{l,\theta} &= 0 \\ -p_\theta R_\theta\alpha + p_\theta\alpha^R_{l,\theta}\alpha l + \mu_1^{l,\theta}\alpha l\left(1-\alpha^R_{l,\theta}\right) + \mu_3^{l,\theta} - \mu_4^{l,\theta} &= 0 \\ \left(1-\alpha+\alpha^L_{l,\theta}\alpha l\right)\left(p_\theta - \mu_1^{l,\theta}\right) + \mu_5^{l,\theta} - \mu_6^{l,\theta} &= 0 \end{aligned}$$

Similarly, the first-order conditions for a type-(h,θ) bank are given by:

$$\begin{aligned}
-p_\theta(1+r)+\mu_1^{h,\theta}+\mu_2^{h,\theta} &= 0 \\
-p_\theta R_\theta \alpha + p_\theta \alpha_{h,\theta}^R \alpha l + \mu_1^{h,\theta}\alpha l\left(1-\alpha_{h,\theta}^R\right)+\mu_3^{h,\theta}-\mu_4^{h,\theta} &= 0 \\
\left(1-\alpha+\alpha_{h,\theta}^L \alpha l\right)\left(p_\theta-\mu_1^{h,\theta}\right)+\mu_5^{h,\theta}-\mu_6^{h,\theta} &= 0
\end{aligned}$$

Optimal liquidity management decisions are derived following the same steps as in the proof of Proposition 1 in Appendix A. It is easy to see that the marginal value of liquidity for a type-(l,θ) bank that offers funds in the interbank market, $L_{l,\theta}>0$, is $\mu_1^{l,\theta}=p_\theta \hat{p}(1+r)$ and for a type-(h,θ) bank that demands funds, $L_{h,\theta}>0$, is $\mu_1^{h,\theta}=p_\theta(1+r)$. Similarly, type-$(l,\theta)$ banks offer funds in the interbank market if and only if $\hat{p}(1+r)\geq 1$. Suppose that funds are offered to a type-(h,θ) bank. Then it demands funds, $L_{h,\theta}>0$, if and only if $1+r\leq \frac{R_\theta}{l}$.

As for reinvestment decisions, if a bank offers or demands funds in the interbank market, then it does not reinvest in the short-term asset, $\alpha_{k,\theta}^R=0$. Similarly, if a type-(l,θ) bank offers funds in the interbank market, $L_{l,\theta}>0$, then it does not liquidate, $\alpha_{l,\theta}^L=0$, if and only if $\hat{p}\,(1+r)\leq \frac{R_\theta}{l}$. Suppose that funds are offered to a type-(h,θ) bank. If a type-(h,θ) bank demands funds, $L_{h,\theta}>0$, then it does not liquidate, $\alpha_{h,\theta}^L=0$.

In sum, it is easy to state the analogue of Proposition 4 in the case with different R's:

Result 1 *Banks with a liquidity surplus are willing to lend if and only if the interbank interest rate r satisfies:*

$$\frac{1}{\hat{p}}\leq 1+r. \tag{B.3}$$

Suppose that banks with a liquidity surplus are willing to lend. Then type-(h,θ) banks borrow in the interbank market if and only if the interbank interest rate r satisfies:

$$1+r\leq \frac{R_\theta}{l}. \tag{B.4}$$

As in the main text, the lower bound on the interest rate is given by the participation constraint of banks with a liquidity surplus while the upper bound is given by the participation constraint of banks with a liquidity shortage (see Proposition 4).

Interbank market regimes

Regime 1: Full participation of borrowers and lenders

Suppose there is full participation of borrowers and lenders in the interbank market so that $\hat{p}=p$. As before, let r_1 and α_1 denote the interest rate and portfolio choice in Regime 1, respectively.

The interval of feasible interbank interest rates is (by Result 1):

$$\frac{1}{p}\leq 1+r_1\leq \frac{R_s}{l},$$

where the the upper bound is given by the participation constraint of the safer banks with a liquidity shortage since they have a lower opportunity cost of liquidation than riskier banks, $\frac{R_s}{l}<\frac{R_r}{l}$.

Turning to the banks' optimization problem at $t = 0$, we have that:

$$\max_{0<\alpha_1<1} \quad \pi_l R\alpha_1 + \pi_l p\left[p(1+r_1)L_l - (1-\lambda_l)d_2\right] \\ +\pi_h R\alpha_1 - \pi_h p[(1+r_1)L_h + (1-\lambda_h)d_2]$$

where $E\left[R\right] \equiv qp_sR_s + (1-q)\,p_rR_r$, $L_l = 1-\alpha_1-\lambda_l d_1$, and $L_h = \lambda_h d_1 - (1-\alpha_1)$. Taking the first-order condition with respect to α_1, we get the analogue of equation (28):

$$\left(\pi_l pp + \pi_h p\right)(1+r_1) = E\left[R\right]. \tag{B.5}$$

The interest rate in Regime 1 is, as before, determined by the no-arbitrage condition.

Using (B.5), (16), and Result 1, we can characterize Regime 1 in terms of the interbank interest rate and the parameters for which it is an equilibrium.

Result 2 *If all banks manage their liquidity in the interbank market, the interest rate is*

$$1 + r_1 = \frac{E[R]}{\delta}\frac{1}{p} \tag{B.6}$$

and the common premium for risky interbank debt, $\frac{1}{\delta} \equiv \frac{1}{p\pi_l+\pi_h}$*, is smaller or equal to than* $\frac{1}{l}\frac{pR_s}{E[R]}$.

This condition ensures that the opportunity cost of liquidation for safer borrowers is higher than the cost of borrowing in the unsecured interbank market. The condition $\frac{1}{\delta} < \frac{1}{l}$, i.e. the common premium for risky interbank debt is smaller than the illiquidity premium, is either a necessary or a sufficient condition for this to be satisfied, depending on whether $pR_s \gtrless E\left[R\right]$. Moreover, with full participation in the interbank market, the participation constraint of lenders never binds since: $\frac{E[R]}{p\delta} > \frac{E[R]}{p} > \frac{1}{p}$ (compare to Proposition 5).

Regime 2: Adverse selection of borrowers in the interbank market

This is the case in which only riskier banks with a liquidity shortage borrow in the interbank market while safer banks with a liquidity shortage obtain liquidity outside the unsecured market. Hence, lenders in the interbank market are exposed to the adverse selection of borrowers and face counterparty risk $\hat{p} = p_r$. Let r_2 and α_2 denote the interest rate and portfolio choice in Regime 2, respectively.

The interval of feasible interbank interest rates is (by Result 1):

$$\frac{1}{p_r} \leq 1 + r_2 \leq \frac{R_r}{l}, \tag{B.7}$$

where the the upper bound is given by the participation constraint of the riskier banks with a liquidity shortage.

As in the analysis in the main text, there can be two cases in the adverse selection regime: 1) a case in which none of the lenders convert illiquid investments into liquidity, $1 + r_2 \leq \frac{R_s}{p_r l} < \frac{R_r}{p_r l}$; and 2) a case in which safer lenders choose to convert their illiquid investments and to lend excess liquidity in the interbank market, $1 + r_2 > \frac{R_s}{p_r l}$. As before, we will focus on the former case as the other case does not add any new features to the results. Moreover, it did not seem to play a central role in the interbank market developments in the 2007-09 crisis. This is because liquidity hoarding, which we document above, cannot occur in this case: $p_r\left(1+r_2\right) > \frac{R_s}{l} > 1$. We therefore proceed under the assumption that $p_r\left(1+r_2\right) \leq \frac{R_s}{l}$.

At $t = 0$, banks' optimization problem is:

$$\begin{aligned}\max_{0<\alpha_2<1} \quad & \pi_l R\alpha_2 + \pi_l p\left[p_r(1+r_2)L_l - (1-\lambda_l)d_2\right] \\ & +\pi_h q p_s\left[R_s\alpha_2\left(1-\alpha_{h,s}^L\right) - (1-\lambda_h)d_2\right] \\ & +\pi_h(1-q)\,p_r\left[R_r\alpha_2 - (1+r_2)L_{h,r} - (1-\lambda_h)d_2\right]\end{aligned}$$

where $E[R] \equiv qp_sR_s + (1-q)\,p_rR_r$, $L_l = 1-\alpha_2-\lambda_l d_1$, $L_{h,r} = \lambda_h d_1 - (1-\alpha_2)$, and $\alpha_{h,s}^L = \frac{\lambda_h d_1 - (1-\alpha_2)}{\alpha_2 l}$. Taking the derivative with respect to α_2, we get the analogue of equation (35):

$$(\pi_l pp_r + \pi_h(1-q)\,p_r)(1+r_2) + \pi_h qp_s\frac{R_s}{l} = E[R]. \tag{B.8}$$

We can rewrite condition (B.8) to obtain the interbank interest rate in Regime 2:

$$1+r_2 = \frac{\frac{E[R]}{p} - \frac{R_s}{l}\frac{\pi_h qp_s}{p}}{\delta_2 - \frac{\pi_h qp_s}{p}} \tag{B.9}$$

where $\frac{1}{\delta_2}$ denotes the premium for risky interbank debt under adverse selection, $\frac{1}{\delta_2} \equiv \frac{1}{\pi_l p_r + \pi_h}$.

Using (B.9), we can write the condition that safer banks with a liquidity shortage drop out of the interbank market as:

$$\frac{E[R]}{p} > \frac{R_s}{l}\delta_2 \tag{B.10}$$

so that $1+r_2 > \frac{R_s}{l}$ (compare to condition 37). Again, we have that:

$$1+r_2 > \frac{R_s}{l} \geq 1+r_1.$$

We now check under which conditions $\frac{1}{p_r} \leq 1+r_2 \leq \frac{R_r}{l}$ holds. We have that $p_r(1+r_2) \geq 1$ if and only if:

$$\frac{\pi_h qp_s}{p}\left(1-p_r\frac{R_s}{l}\right) \geq \delta_2 - p_r\frac{E[R]}{p}. \tag{B.11}$$

By B.10, we have that $p_r\frac{E[R]}{p} > p_r\frac{R_s}{l}\delta_2$ and hence a sufficient condition is

$$\frac{\pi_h qp_s}{p}\left(1-p_r\frac{R_s}{l}\right) \geq \delta_2 - p_r\frac{R_s}{l}\delta_2$$

or, equivalently,

$$\left(\delta_2 - \frac{\pi_h qp_s}{p}\right)\left(1-p_r\frac{R_s}{l}\right) \leq 0.$$

Since $\delta_2 - \frac{\pi_h qp_s}{p} > 0$, the sufficient condition for $p_r(1+r_2) \geq 1$ becomes

$$p_r\frac{R_s}{l} > 1$$

(compare to the similar sufficient condition in the main text: $p_rR > l_s$).

We have that $1 + r_2 \leq \frac{R_r}{l}$ if and only if:

$$\frac{\pi_h q p_s}{p} \frac{1}{l} (R_r - R_s) \leq \delta_2 \frac{R_r}{l} - \frac{E[R]}{p}. \tag{B.12}$$

Since $R_r > R_s$, a necessary condition for this to hold is:

$$\delta_2 \frac{R_r}{l} > \frac{E[R]}{p}$$

(compare to the necessary condition in the main text: $\frac{1}{\delta_2} < \frac{1}{l_r}$).[30]

The following results summarizes this discussion:

Result 3 *If there is adverse selection in the unsecured interbank market, i.e. safer banks with a liquidity shortage do not borrow, the interbank interest rate is given by (B.9) and conditions (B.11) and (B.12) hold.*

The set of parameters for which conditions (B.10), (B.11) and (B.12) hold is non-empty.

Regime 3: Market breakdown

Lenders drop out: Lenders prefer to hoard liquidity by reinvesting it in the liquid asset when the lower bound in (B.7) is violated, i.e.

$$p_r (1 + r_2) < 1,$$

or, equivalently,

$$\frac{\pi_h q p_s}{p} \left(1 - p_r \frac{R_s}{l}\right) < \delta_2 - p_r \frac{E[R]}{p}.$$

We know that the necessary condition for this to hold is:

$$p_r \frac{R_s}{l} < 1$$

since under adverse selection $p_r (1 + r_2) > p_r \frac{R_s}{l}$. It follows that it also has to be that

$$p\delta_2 - p_r E[R] > 0$$

which yields a condition analogous to (41):

$$p_r E[R] < p\delta_2 < 1.$$

As in the main text, the necessary condition for liquidity hoarding is that the net present value of a (hypothetical) asset returning $E[R]$ with probability p_r is negative.

Borrowers drop out: Risky borrowers choose to leave the unsecured interbank market if adverse selection drives the interest rate up too much. The upper bound on the interest rate in (B.7) is violated when:

$$1 + r_2 > \frac{R_r}{l},$$

[30] We can re-write condition (B.12) as $q (l - \pi_h) (p_s R_s - p_r R_r) \leq p_r R_r (\delta - l)$. If we have a mean-preserving spread, i.e. $p_s R_s = p_r R_r$, then the necessary and sufficient condition is simply $\delta \geq l$.

or, equivalently,

$$\frac{\pi_h q p_s}{p}\frac{1}{l}\left(R_r - R_s\right) > \delta_2 \frac{R_r}{l} - \frac{E\left[R\right]}{p}.$$

Since $R_r > R_s$, a sufficient condition for this to hold is:

$$\delta_2 \frac{R_r}{l} \leq \frac{E\left[R\right]}{p}.$$

Multiple equilibria

In this section, we summarize conditions under which a particular regime constitutes the unique equilibrium in the interbank market.

Regime 1 is the unique equilibrium if and only if

$$\frac{E\left[R\right]}{p} \leq \frac{R_s}{l}\delta_2.$$

This is equivalent to $1 + r_2 \leq \frac{R_s}{l}$. The interest rate that would arise under adverse selection is relatively low and safer borrowers prefer to stay in the market. Hence, adverse selection regime cannot be an equilibrium.

Similarly, Regime 2 is the unique equilibrium when

$$\frac{E\left[R\right]}{p} > \frac{R_s}{l}\delta$$

and conditions (B.11) and (B.12) hold. Then, $\frac{R_s}{l} < 1 + r_1$. The interest rate that would arise under full participation is so high that safer borrowers drop out of the market and hence full participation cannot be an equilibrium.

Liquidity hoarding is the unique equilibrium if and only if $\frac{E[R]}{p} > \frac{R_s}{l}\delta$ and condition (B.11) fails. Market breakdown due to the drop out of borrowers is the unique equilibrium if and only if $\frac{E[R]}{p} > \frac{R_s}{l}\delta$ and condition (B.12) fails.

There is an open set of parameters such that we have multiple equilibria in the model. This occurs when

$$\frac{R_s}{l}\delta_2 < \frac{E\left[R\right]}{p} \leq \frac{R_s}{l}\delta \tag{B.13}$$

(compare to the condition (42)).

Both Regimes 1 and 2 are equilibria if conditions (B.11), (B.12), and (B.13) hold. If banks expect Regime 1 to be an equilibrium, all banks participate in the interbank market and the resulting interest rate $1 + r_1$ is smaller than $\frac{R_s}{l}$, thus justifying banks' expectations. However, if banks expect Regime 2 to be an equilibrium, safer banks with a liquidity shortage drop out of the interbank market and the interest rate is given by $1 + r_2 > \frac{R_s}{l}$.

Similarly, both Regimes 1 and 3 (liquidity hoarding) are equilibria when conditions (B.13) and (B.12) hold while condition (B.11) fails. Both Regimes 1 and 3 (borrowers drop out) when conditions (B.13) and (B.11) hold while condition (B.12) fails.